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Research Article

DYNAMICS OF ANISOTROPIC BIANCHI UNIVERSE WITH BULK VISCOUS

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ABSTRACT

We have considered a innovative class of Bianchi type-III anisotropic cosmological models in which source of matter is bulk viscous fluid. To get the conclusive model of the universe, we have supposed that the coefficient of bulk viscosity is a power function of mass density. The exact solutions of the Einstein's field equations are achieved which represent an expanding, shearing, and anisotropic model of the universe. The models offered are tested for physically acceptable cosmologies by using constancy. In view of the accelerated expansion of the current universe, the dynamical and physical properties of the models have been analyzed.

Key Words:

Bianchi type-III space-time, Bulk viscous fluid, Effective pressure.

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INTRODUCTION

In recent years, there has been extensive interest in string cosmology. Cosmic strings are topologically steady objects which might be establish during a phase change in the early universe [1]. Cosmic strings play an significant role in the lessons of the early universe. These arise during the phase change after the big bang detonation as the temperature goes down under certain critical temperature as expected by grand unified theories [2-4]. It is supposed that cosmic strings give rise to density perturbations which direct to the creation of galaxies [5]. These cosmic strings have stress-energy and mingling to the gravitational field. Therefore it is motivating to study the gravitational effect that arises from strings.

It is well recognized that the solutions of Einstein's field equations representing cosmological models of relativity for space times goes to either Bianchi types or Kantowski-Sachs models [6]. The Friedman-Robertson-Walker (FRW) models are particular cases of anisotropic Bianchi type models. Currently cosmological observations give strong confirmation for homogeneous, isotropic and accelerating expansion of the universe. In its early phases of evolution, universe might have not so proper behavior. Experimental information and number of scientific arguments support the existence of anisotropic period of universe which is believed to be phased out during evolution of the universe. In order to Studies the early stages

behavior of evolution of universe, several authors have studied many Bianchi types and Kantowski-Sachs cosmological models. When we learn the Bianchi type models, we observed that the models have isotropic special cases and they allow arbitrarily small anisotropic levels at some moment of cosmic times. Bianchi type cosmological models are significant in the logic that these models are homogeneous and anisotropic, from which the method of isotropization of the universe is considered through the passage of time. Besides, from the theoretical point of view, anisotropic universe has a better generality than isotropic models. The cleanness of the field equations completed Bianchi space time helpful in making models of spatially homogeneous and anisotropic cosmologies. A number of authors have considered different anisotropic cosmological models in various contexts [7-25].

On the other hand, the matter circulation is satisfactorily explained by perfect fluids due to the large scale circulation of galaxies in our universe. Moreover, a realistic behavior of the problem requires the deliberation of material circulation other than the perfect fluid. It is well recognized that when neutrino decoupling happens, the matter acts as a viscous fluid in an early stage of the universe. Viscous fluid cosmological models of early universe have been broadly argued in the literature.

It has been recommended in the literature that bulk viscosity is related with grand unified theory, phase change and string

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creation. Moreover bulk viscosity behaves as a negative energy field in an escalating universe. Maartens [26] has considered exhaustive review of cosmological models with underlying thermodynamics. Singh and Kale [27] have considered bulk viscous Bianchi Type-V cosmological models with changeable gravitational and cosmological constant. Currently Verma [28] and Shriram [29] have studied Bianchi Types bulk viscous cosmological models with G and Λ . Singh [30] has considered some cosmological models gravitational cosmological constants. Pradhan *et al.* [31] have studied Bianchi type-I cosmological models with time reliant gravitational and cosmological constants.

Provoked by the above observations, we consider anisotropic space-time of Bianchi type-III model in a general form and obtained a general and exact solution of Einstein's field equations which is new and different from other author's solutions. The sketch of the paper is as follows: in Section 2, the metric and the field equations are described. Section 3 deals with the solutions of the field equations and their geometric and physical properties. Finally, conclusions are summarized in the last Section 4.

Metric and Field Equations

The field equations of Einstein are as:

$$R_i^j - \frac{1}{2} \delta_i^j R = -8\pi T_i^j$$

Here R_i^j is the Ricci tensor and R is Ricci Scalar.

We study the gravitational field known by five dimensional Bianchi Type-III cosmological model as select it in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{2ax} dy^2 + C^2 dz^2 \tag{1}$$

Where A, B, C are the functions of t and a is a constant.

The energy momentum tensor in the existence of bulk stress has the form

$$T_i^j = (\rho + \bar{p})u_i u^j + \delta_i^j \bar{p} \tag{2}$$

Where

$$\bar{p} = p - \xi u_{;i}^i \tag{3}$$

Here ρ , p, \bar{p} and ξ are the energy density, isotropic pressure, effective pressure,

Bulk viscous coefficient, respectively and u^i is the flow vector satisfying the relation

$$g_{ij} u^i u^j = -1$$

Here the comoving coordinates are taken to be $u^1 = u^2 = u^3 = 0$ and $u^4 = 1$

For the line element (1), the field equations (2) have been set up as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{a^2}{A^2} + \frac{a}{2} \left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A} \right) = -8\pi \bar{p} \tag{4}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{a^2}{A^2} + \frac{a}{2} \left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A} \right) = -8\pi \bar{p} \tag{5}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{B}}{AB} + \frac{a}{2} \left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A} \right) = -8\pi \bar{p} \tag{6}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} + \frac{a}{2} \left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A} \right) = 8\pi \rho \tag{7}$$

$$\frac{\dot{B}}{B} - \frac{\dot{A}}{A} = 0 \tag{8}$$

Here afterwards the dot over field variable represents ordinary differentiation with respect to t.

Solutions of the field Equations

Equation (8) gives

$$B = \mu A \tag{9}$$

Without loss of generality we have to take $\mu = 1$, so that

$$B = A \tag{10}$$

The field equations (4) to (7) with the equation (10) reduce as

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + \frac{a^2}{A^2} = -8\pi \bar{p} \tag{11}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{a^2}{A^2} = -8\pi \bar{p} \tag{12}$$

$$2 \frac{\ddot{A}}{A} + \left(\frac{\dot{A}}{A} \right)^2 = -8\pi \bar{p} \tag{13}$$

$$\left(\frac{\dot{A}}{A} \right)^2 + 2 \frac{\dot{A}\dot{C}}{AC} = 8\pi \rho \tag{14}$$

Equation (11) to (14) are four equations with four unknowns A, C, p and ρ . To get a determinate solution we need additional conditions. First we suppose that

$$p + \rho = 0 \tag{15}$$

i.e. the fluid is anti-stiff fluid.

So equations (11) to (14) together with equation (15) give

$$\frac{\ddot{A}}{A} - \frac{\dot{A}\dot{C}}{AC} = 0 \tag{16}$$

Secondly, we imagine that

$$A = C^n, \tag{17}$$

Where n is any real number.

The above equation becomes

$$\frac{\ddot{C}}{C} + (n - 2) \frac{\dot{C}}{C} = 0 \tag{18}$$

On integration, above equation gives

$$C = (n - 1)^{\frac{1}{n-1}} (pt + q)^{\frac{1}{n-1}} \tag{19}$$

Here p and q are constants of integration.

Hence we obtain

$$A^2 = (n - 1)^{\frac{2n}{n-1}} (pt + q)^{\frac{2n}{n-1}} \tag{20}$$

$$B^2 = (n - 1)^{\frac{2n}{n-1}} (pt + q)^{\frac{2n}{n-1}} \tag{21}$$

$$C^2 = (n - 1)^{\frac{2}{n-1}} (pt + q)^{\frac{2}{n-1}} \tag{22}$$

Therefore, the space-time (1) reduces to the form

$$ds^2 = -dt^2 + \left[(n-1)^{\frac{2n}{n-1}} (pt+q)^{\frac{2n}{n-1}} \right] dx^2 + \left[(n-1)^{\frac{2n}{n-1}} (pt+q)^{\frac{2n}{n-1}} e^{-2ax} \right] dy^2 + \left[(n-1)^{\frac{2}{n-1}} (pt+q)^{\frac{2}{n-1}} \right] dz^2 \quad (23)$$

After a suitable transformation of coordinates, the metric (23) takes the form

$$ds^2 = -\frac{dT^2}{a^2} + (n-1)^{\frac{2n}{n-1}} T^{\frac{2n}{n-1}} dX^2 + (n-1)^{\frac{2n}{n-1}} T^{\frac{2n}{n-1}} e^{-2ax} dY^2 + (n-1)^{\frac{2}{n-1}} T^{\frac{2}{n-1}} dZ^2 \quad (24)$$

Where $T = pt + q$

Physical and Geometrical Features of the Model

The properties of the physical and kinematical variables involved in this model are given as follows:

The effective pressure p and density ρ for the model (24) are given by

$$\bar{p} = (p - \xi\theta) = \frac{-a^2}{(n-1)^{\frac{2n}{n-1}} T^{\frac{2n}{n-1}}} - \frac{a^2(n+2)}{(n-1)^2 T^2} \quad (25)$$

$$\rho = \frac{8a^2(n+2)}{\pi(n-1)^2 T^2} \quad (26)$$

For the requirement of ξ , now we imagine that the fluid trails an equation of the state of the form

$$p = \gamma\rho \quad (27)$$

where γ ($0 \leq \gamma \leq 1$) is a constant.

In most of the reviews the bulk viscosity is supposed to be a simple power function of the energy density.

$$\xi(t) = \xi_0 \rho^r \quad (28)$$

Where ξ_0 and r are constants.

For $r = 1$, equation (28) may corresponding to a radiative fluid.

But, more practical models are based in the region, $0 \leq \gamma \leq 1/2$.

On using equations (25) and (28) we obtain

$$p = \xi_0 \rho^r \theta + \frac{-a^2}{8\pi(n-1)^{\frac{2n}{n-1}} T^{\frac{2n}{n-1}}} - \frac{a^2(n+2)}{8\pi(n-1)^2 T^2} \quad (29)$$

From equation (25), (26) and (28), it is viewed that ρ , p and \bar{p} vary as $1/T$. The models are singular at $T = 0$ and as they progress, the pressure and density decreases.

The scalar expansion θ is given by

$$\theta = \frac{(2n+1)a^2}{(n-1)T} \quad (30)$$

The factors of the shear tensor σ_i^i are given by

$$\sigma^2 = \frac{1}{9} \frac{(2n+1)^2 a^4}{(n-1)^2 T^2} \quad (31)$$

When $T \rightarrow 0$, the physical parameters θ and σ^2 diverge. With the raise in cosmic time T , θ and σ^2 decrease and finally they disappear when $T \rightarrow \infty$.

Now

$$\frac{\sigma}{\theta} = \frac{1}{3} \frac{\frac{(2n+1)a^2}{(n-1)T}}{\frac{(2n+1)a^2}{(n-1)T}} = \frac{1}{3} \quad (32)$$

Since $\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} \neq 0$, the model does not attain isotropy for large

values of T . Further we achieved $\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} \approx 0.3333$, which is larger than the present upper limit $(10)^{-5}$ attained by Collins *et al.* [32].

The volume is given by

$$V = \sqrt{-g} = \frac{e^{-ax}}{a} \left[(n-1)T \right]^{\frac{2n+1}{n-1}} \quad (33)$$

Hence the volume of the universe is zero at the initial epoch. With the increase in T volume increases and $V \rightarrow \infty$ as $T \rightarrow \infty$. The model is anisotropic. The model, in general, represents shearing and rotating universe. The model starts expanding with a Big bang at $T = 0$ and the expansion in the model increase as time decreases.

CONCLUSION

In this paper we have presented a new exact solution of Einstein's field equations for anisotropic Bianchi type-III space-time in presence of bulk viscous fluid which is different from the other author's solutions. The model starts with a big-bang at $T = 0$ and it goes on expanding until it comes out to rest at $T = \infty$. The model (24) in general represents shearing and expanding. In our models we have observed that they do not loom isotropy for large value of time t . The role of bulk viscosity is to obstruct expansion in the model. We can see from the above conversation that the bulk viscosity plays a important role in the evolution of the universe. Here the volume of the universe is zero at the initial epoch. With the increase in T , volume increases and $V \rightarrow \infty$ as $T \rightarrow \infty$. The ρ , p and \bar{p} start off with extremely large values, which continue to decrease with the expansion of the universe and ultimately tend

to zero for large time. Since $\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} \neq 0$, the model does not

attain isotropy for large values of T . Further we achieved $\lim_{t \rightarrow \infty} \frac{\sigma}{\theta} \approx 0.3333$, which is larger than the present upper limit $(10)^{-5}$ attained by Collins *et al.* [32].

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