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Research Article

MEAN LABELING PATTERN OF C_n,[P_m;C_n] and $C_n \otimes P_n$

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ABSTRACT

The concept of mean labeling was introduced by Somasundaram and Ponraj in 2003. Many research papers have published in this topic. In this paper we have established a general format for labelling the cycle, [P_m; C_n], $C_n \otimes P_r$ and $K_{2, n}$ graphs.

INTRODUCTION

Let G = (V, E) be a graph which is finite, simple and undirected. The graph G has vertex set V = V(G) and edge set E = E(G). The graph labeling is an assignment of numbers to the vertices or edges or both subject to certain condition. If the domain of the mapping is the set of vertices /edges then the labeling is called a vertex / edge labeling .Graph labeling have enormous applications in mathematics as well as to several areas of computer science and communication network.

Definition

A graph G is an ordered pair (V(G),E(G)) consisting a non-empty set V(G) of vertices and a set E(G) disjoint from V(G) of edges, together with an incident function Ψ_G that associate with each edge of G, an unordered pair of vertices of G. If e is an edge u and v are vertices of G such that $\Psi_G(e) = \{u,v\}$ then e is said to joint u and v and the vertices u and v are called the ends of e.

Definition

A Path is a simple graph whose vertices can be arranged in a linear sequence in such a way that two vertices are adjacent if they are consecutive in the sequence and are non-adjacent.

A closed path is called a Cycle. A Cycle with n vertices is denoted by C_n.

Definition

A graph is bipartite if its vertex set can be partitioned into two subsets, X and Y so that every edge set has one end in X and one end in Y.

A bipartite graph G is said to be complete if every elements of X is adjacent with all elements of Y. A complete bipartite graph with m,n vertices is denoted by $K_{m, n.}$

Definition

The cartesian product of simple graphs G and H is the graph G⊗H whose vertex set is V(G)⊗V(H) and whose edge set of all pairs $(u_1,v_1),(u_2,v_2)$ such that either $u_1u_2 \in E(G)$ and $v_1 = v_2$ or $v_1v_2 \in E(H)$ and $u_1 = u_2$.

Definition

A labeling or valuation of a graph G is an assignment f of labels to the vertices of G that induced for each edge *xy* a label depending on the vertex labelled by f(x) and f(y).

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Definition

A function f is called a mean labeling of a graph G if f : V(G) → {0,1,2,…,q} is injective and the induced function f* : E(G) → {1,2,3,…,q} defined as,

$$f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2}, & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2}, & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

is bijective. A graph that admits mean labeling is called a mean graph.

Definition

Let G be a graph with fixed vertex v and let [P_m; G] be the graph obtained from m copies of G connected the common vertices of $v_i \in G_i$ by path P_m.

Theorem

Any cycle C_n of length n ≥ 3 is mean graph.

Proof

Let G = C_n be the cycle with n vertices and the vertex set of G is denoted by V(G) = $\{v_i / i = 1,2,3,\ldots,n\}$ and represented in (Figure : 1) as below we can label the vertices of G as

$L(V_{n-i})$ =

$$\begin{cases} n-(i-j) \ / \ i = n-1 \ to \ \left\lceil \frac{n+1}{2} \right\rceil \ and \ j = 2 \ to \ \left\lceil \frac{n+1}{2} \right\rceil \\ n-(j-i) \ / \ i = \left\lceil \frac{n-1}{2} \right\rceil \ to \ 0 \ and \ j = \left\lceil \frac{n+1}{2} \right\rceil \ to \ n \end{cases}$$

…..(A)

Hence, C_n for all n ≥ 3 is a mean graph.

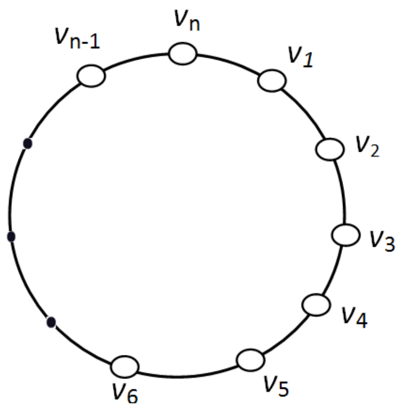

Figure 1

Note

The labeling pattern defined in (A) can be used to label any cycle in both directions.

Example

C_5 is labelled in (Figure: 2) by (A).
V = $\{v_1,v_2,v_3,v_4,v_5\}$
$L(v_{n-i})$ = n– (i –j), where n = 5, i = {4,3} and j = {2,3}.
$L(v_{5-4})$ = $v_1 = 5–(4–2) = 5–2 = 3$
$L(v_{5-3})$= $v_2 = 5–(3–3) = 5–0 = 5$
$L(v_{n-i})$ = n– (j–i), where i = {2,1,0} and j = {3,4,5}
$L(v_{5-2})$ = $v_3 = 5–(3–2) = 5 – 1 = 4$
$L(v_{5-1})$ = $v_4 = 5– (4–1) = 5–3 = 2$

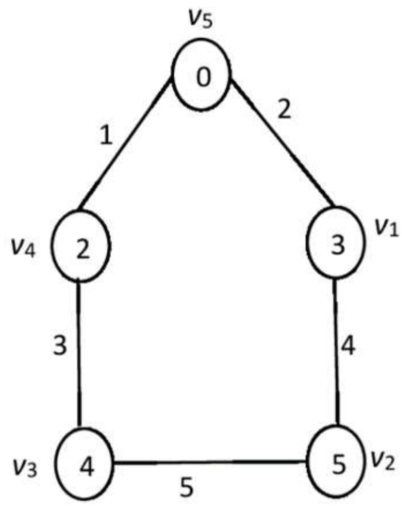

Figure 2

$L(v_{5-0})$ = $v_5 = 5– (5–0) = 5–5 = 0$

Example

C_8 is labelled in (Figure: 3) by (A)
V = $\{v_1,v_2,v_3,v_4,v_5,v_6,v_7\}$
$L(v_{n-i}) = n–(i–j)$; where n = 8, i ={7,6,5} and j = {2,3,4}.
$L(v_{8-7})$ = $v_1 = 8–(7–2) = 8–5 = 3$
$L(v_{8-6})$ = $v_2 = 8–(6–3) = 8–3 = 5$
$L(v_{8-5})$ = $v_3 = 8 – (5–4) = 8–1 = 7$
$L(v_{n-i}) = n–(j– i)$; where i ={4,3,2,1,0} and j = {4,5,6,7,8}
$L(v_{8-4})$ = $v_4 = 8–(4–4) = 8–0 = 8$
$L(v_{8-3})$ = $v_5 = 8–(5–3) = 8–2 = 6$

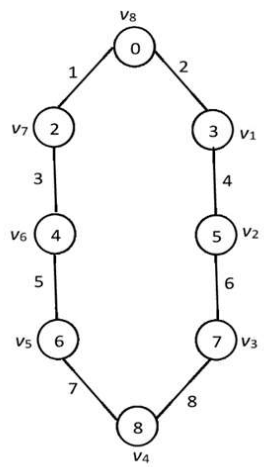

Figure 3

$L(v_{8-2})$ = $v_6 = 8– (6–2)= 8–4 = 4$
$L(v_{8-1})$ = $v_7 = 8–(7–1)= 8–6 = 2$
$L(v_{8-0})$ = $v_8 = 8–(8–0)= 8–8 = 0$

Theorem

Let G = [P_m; C_n] is m copies of C_n which are connected by a unique path P_m is a mean graph.

Proof

The graph G is given in (Figure: 4) as below.

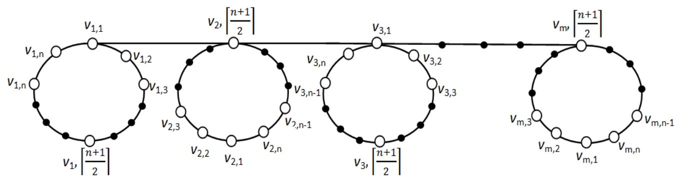

Figure 4

The vertex set of G is denoted by

$V(G) = \{v_{i,j} \ / \ i = 1,2,3,\ldots,m; \ j = 1,2,3,\ldots,n\}$

The vertices of G can be divided in to two sets V_1 and V_2 such that

$V_1(G) = \{v_{i,j} \ / \ i = 1,2,3,\ldots,m; \ j = 1,2,3,\ldots,n\}$ if i is odd

$V_2(G) = \{v_{i,j} \ / \ i = 1,2,3,\ldots,m; \ j = 1,2,3,\ldots,n\}$ if i is even

Now, we label the vertices of G as below

$L(v_{2r+1,\,j}) = 2r(n+1) + v_{1,j}$ where $r = 1,2,3,\ldots,\lceil m/2 \rceil$

$L(v_{2r+2,\,j}) = 2r(n+1) + v_{2,j}$ where $r = 1,2,3,\ldots,\lceil m/2 \rceil$ …..(B)

Hence, $G = [P_m; C_n]$ is mean graph.

Example

$G = [P_4; C_5]$ is labelled in (Figure: 5) as below by

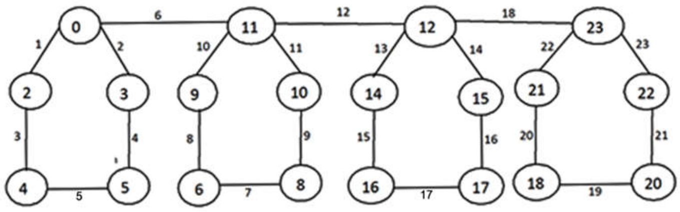

Figure 5

labeled by using (A), $L(v_{1,1}) = 0$; $L(v_{1,2}) = 3$; $L(v_{1,3}) = 5$; $L(v_{1,4}) = 4$; $L(v_{1,5}) = 2$;

$L(v_{2,1}) = 6$; $L(v_{2,2}) = 9$; $L(v_{2,3}) = 11$; $L(v_{2,4}) = 10$; $L(v_{2,5}) = 8$;

labeled by using (B), $L(v_{3,1}) = 2r(n+1)+v_{1,j} = 2\times1(5+1)+0 = 12+0 = 12$;

$L(v_{3,2}) = 2r(n+1)+v_{1,j} = 2\times1(5+1)+3 = 12+3 = 15$;

$L(v_{3,3}) = 2r(n+1)+v_{1,j} = 2\times1(5+1)+5 = 12+5 = 17$;

$L(v_{3,4}) = 2r(n+1)+v_{1,j} = 2\times1(5+1)+4 = 12+4 = 16$;

$L(v_{3,5}) = 2r(n+1)+v_{1,j} = 2\times1(5+1)+2 = 12+2 = 14$;

$L(v_{4,1}) = 2r(n+1)+v_{2,j} = 2\times1(5+1)+6 = 12+6 = 18$;

$L(v_{4,2}) = 2r(n+1)+v_{2,j} = 2\times1(5+1)+9 = 12+9 = 21$;

$L(v_{4,3}) = 2r(n+1)+v_{2,j} = 2\times1(5+1)+11 = 12+11 = 23$;

$L(v_{4,4}) = 2r(n+1)+v_{2,j} = 2\times1(5+1)+10 = 12+10 = 22$;

$L(v_{4,5}) = 2r(n+1)+v_{2,j} = 2\times1(5+1)+8 = 12+8 = 20$;

Theorem

$K_{2,n}$ is a mean graph for all $n \geq 1$.

Proof

Let $G = K_{2,n}$ and $V(G) = \{v_i \ / \ i = 1,2,3,\ldots,n+2\}$

Now V(G) can be partitioned into two sets V_1 and V_2 such that

$V_1 = \{v_1, v_{n+2}\}$; $V_2 = \{v_i \ / \ i = 2,3,\ldots,n+1\}$

Now, the vertices of V(G) are labelled as

$$L(v_i) = \begin{cases} 2i-2, & i \leq n+1 \\ 2n-1, & i = n+2 \end{cases} \quad \text{…..(C)}$$

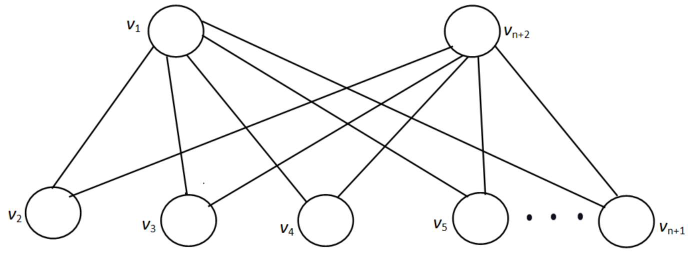

Figure 6

Hence, $K_{2,n}$ is a mean graph.

Example

Let $G = K_{2,4}$ is represented in (Figure: 7) and L(V(G)) is labelled by using (C) as below,

$V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$

$L(v_1) = 2i-2 = 2\times1-2 = 2-2 = 0$

$L(v_2) = 2i-2 = 2\times2-2 = 4-2 = 2$

$L(v_3) = 2i-2 = 2\times3-2 = 6-2 = 4$

$L(v_4) = 2i-2 = 2\times4-2 = 8-2 = 6$

$L(v_5) = 2i-2 = 2\times5-2 = 10-2 = 8$

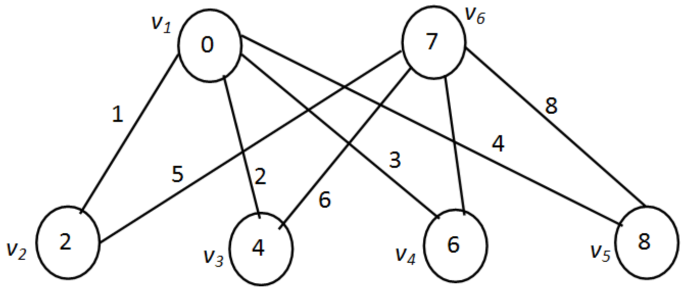

Figure 7

$L(v_6) = 2n-1 = 2\times4-1 = 8-1 = 7$

Theorem

The graph $G = C_n \otimes P_r$ is a mean graph.

Proof:

The graph $G = C_n \otimes P_r$ is given (Figure: 8) as below.

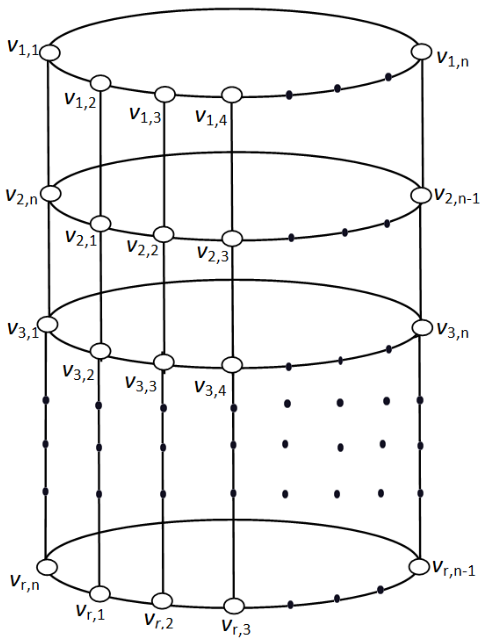

Figure 8

The vertex set of G is denoted by V(G) = $\{v_{i,j}$ / $i=1,2,3,\ldots,m;$ $j=1,2,\ldots,n\}$

Now V(G) can be divided in to two sets V_1 and V_2
$V_1(G) = \{v_{i,j}$ / $i=1,2,\ldots,m;$ $j=1,2,3,\ldots,n\}$ where i is odd
$V_2(G)=\{v_{i,j}$ / $i=1,2,3,\ldots,m$; $j=1,2,3,\ldots,n\}$ where i is even.

$$L(v_{n-i})=\begin{cases} n-(i-j) \ / \ i=n-1 \text{ to } \left\lceil \frac{n+1}{2} \right\rceil \text{ and } j=2 \text{ to } \left\lceil \frac{n+1}{2} \right\rceil \\ n-(j-i) \ / \ i=\left\lceil \frac{n-1}{2} \right\rceil \text{ to } 0 \text{ and } j=\left\lceil \frac{n+1}{2} \right\rceil \text{ to } n \end{cases} \quad \ldots..(A)$$

$L(v_{2,j}) = \{ 2n + v_{1,j} / j = 1,2,3,\ldots,n\}$ …..(D)

$$\left.\begin{aligned} L(v_{i,j}) &= 2n(i-1) + v_{1,j}, \text{ for all } v_{i,j} \in V_1(G) \text{ and} \\ L(v_{i,j}) &= 2n(i-2) + v_{2,j}, \text{ for all } v_{i,j} \in V_2(G) \end{aligned}\right\} \quad \ldots..(E)$$

Hence, $C_n \otimes P_r$ is a mean graph.

Example

G = $C_4 \otimes P_4$ is labelled using (E) is given in (figure: 9) as below
By using (A) $L(v_{1,1}) = 3$, $L(v_{1,2}) = 4$, $L(v_{1,3}) = 2$, $L(v_{1,4}) = 0$,
Using (D)$L(v_{2,1}) = 11$,$L(v_{2,2})= 12$,$L(v_{2,3})= 10$, $L(v_{2,4})= 8$,
By using (E)
$L(v_{3,1}) = 2n(i-1)+v_{1,j}= 2\times4(3-1) +3= 19$,
$L(v_{3,2}) = 2n(i-1)+v_{1,j}= 2\times4(3-1) +4 = 20$,
$L(v_{3,3}) = 2n(i-1)+v_{1,j}= 2\times4(3-1)+2= 18$,
$L(v_{3,4}) = 2n(i-1)+v_{1,j} = 2\times4(3-1) + 0= 16$,
$L(v_{4,1}) = 2n(i-2)+v_{2,j}= 2\times4(4-2)+8 =26$,
$L(v_{4,2}) = 2n(i-2) + v_{2,j}= 2\times4(4-2)+ 8 = 27$,
$L(v_{4,3}) = 2\times4(4-2)+11 = 16+12 = 28$,
$L(v_{4,4}) = 2\times4(5-1)+ 12 = 16 + 10 = 26$,
$L(v_{5,1}) = 2\times4(5-1) + 0 = 32 + 3 = 35$,
$L(v_{5,2}) = 2\times4(5-1) + 3 = 32 + 4 = 36$,
$L(v_{5,3}) = 2\times4(5 - 1) + 4 = 32 + 2 = 34$,
$L(v_{5,4}) = 2\times4(5-1) + 2 = 32 + 0 = 32$.

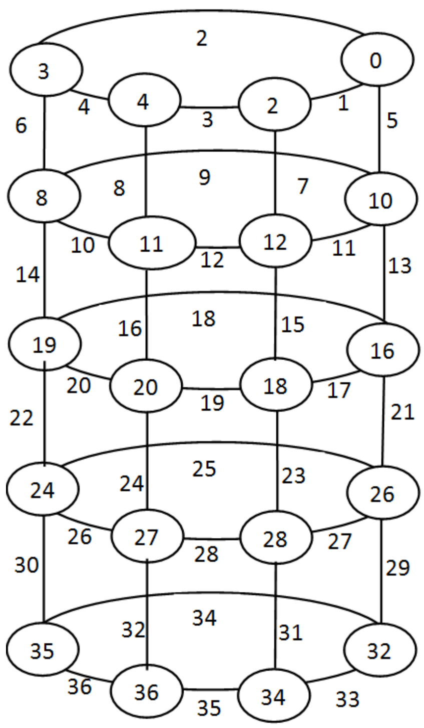

Figure 9

Hence, Theorem: 2.9 is verified**.**

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