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Research Article

# COMMON FIXED POINT THEOREM IN INTUTIONISTIC FUZZY METRIC SPACE UNDER STRICT CONTRACTIVE CONDITIONS

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ABSTRACT

In this paper, using the idea of intuitionistic fuzzy sets, we define the notion of intuitionistic fuzzy metric spaces with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to George and Veeramani [3]. We introduce the notation of Cauchy sequences in an Intuitionistic fuzzy metric space and prove the common fixed point theorem in intutionistic fuzzy metric space under strict contractive conditions.

Mathematical Subject Classification: 54H25, 54E50

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## INTRODUCTION

The theory of fuzzy sets was initiated by Zadeh [24]. In the last four decades, like all other aspects of Mathematics, various authors have introduced the concept of fuzzy metric in several ways; for example George and Veeramani [7] modified the concept of fuzzy metric space introduced by Kramosil and Michalek [13] and defined a Hausdorff topology on such fuzzy metric space which are often used in current researches. Grabiec [8] extended classical fixed point theorems of Banach and Edelstein to complete and compact fuzzy metric spaces respectively.

Atanassov [3] introduced and studied the concept of Intuitionistic fuzzy sets as a noted generalization of fuzzy sets which has inspired intense research progress around this new notion of Intuitionistic fuzzy set. Park [17] using the idea of Intuitionistic fuzzy sets, defined Intuitionistic fuzzy metric space (employing the notions of continuous t-norm and continuous t-conorm) as a generalization of fuzzy metric spaces (due to George and Veeramani [7]) and also proved some basic results which include Baire's theorem (a necessary and sufficient condition for completeness of the space), separability of the space, second countability of the space and it's relation with separability, uniform limit theorem besides other core results. Presently, it remains an important problem in fuzzy topology to obtain an appropriate concept of Intuitionistic fuzzy metric spaces. This problem has been investigated by Saadati and Park [19] wherein they defined precompact sets in Intuitionistic fuzzy metric spaces and proved that any subset of an Intuitionistic fuzzy metric space is compact if and only if it is precompact and complete. Also they defined topologically complete Intuitionistic fuzzy metrizable spaces.

The study of common fixed points of mappings satisfying certain contractive conditions has been at the center of strong research activity and, being the area of the fixed point theory, has very important application in applied mathematics and sciences. It was the turning point in the "fixed point arena" when the notion of commutativity was used by Jungck [11] to obtain a generalization of Banach's fixed point theorem for a pair of mappings. This theorem has had many applications, but

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suffers from one drawback-the definition requires that $T$ be continuous throughout $X$. This result was further generalized and extended in various ways by many authors.

On the other hand, Sessa [21] coined the notion of weak commutativity and proved a common fixed point theorem for these mappings. Sessa [21] introduced a weaker version of commutativity for a pair of selfmaps, and it is shown that weakly commuting pair of maps in metric pace is commuting, but the converse may not be true.Later, Jungck [11] introduced the notion of compatible mappings in order to generalize the concepts of weak commutativity and showed that weak commuting map is compatible, but the reverse implication may not hold.

It is well known that in the setting of metric space, strict contractive condition do not ensure the existence of common fixed point unless the Space is assumed to be compact or the strict conditions are replaced by stronger conditions. In 1986, Jungck [12] introduced the notion of compatible mappings. This concept was frequently used to prove existence theorems in common fixed point theory. However, the study of common fixed points of non-compatible mappings is also very interesting. More recentely Aamri and El Moutawakil [1] generalized the concept of non compatibility in metric spaces by defining the notion property (E.A.) and proved common fixed point theorems under strict contractive conditions. It was pointed out in [1] that property ( E. A.) buys containment of ranges without any continuity requirements besides minimizes the commutativity conditions of the maps to the commutativity at their points of coincidence. Moreover, property (E. A.) allows replacing the completeness requirement of the space with a more natural condition of closeness of the range. A major benefit of property (E.A.) is that it ensures. Sharma and Bamoria [22] elucidated a property (SB) in fuzzy metric spaces for self maps.

***Preliminaries***

***Dentition 2.1 (Schweizer and Sklar) [20]:*** "A binary operation $*:[0,1]\times[0,1]\rightarrow[0,1]$ is a continuous *t-norms* if "*" is satisfying conditions:

1. * is an commutative and associative
2. * is continuous
3. $a * 1 = a$ for all $a\in [0, 1]$
4. $a * b \le c*d$ whenever $a \le c$ and $b \le d$, and $a, b, c, d \in [0, 1]$.

Basic example of $t$ - norm are the Lukasiewicz $t$ - norm $T_l$, $T_l(a, b) = \max(a+b-1, 0)$,

$t$ -norm $T_p$, $T_p(a,b) = ab$, and $t$ - norm $T_M$, $T_M(a,b) = \min\{a,b\}$.

***Definition 2.2 (Schweizer and Sklar) [20]:*** A binary operation $\diamond:[0,1]\times[0,1]\rightarrow[0,1]$ is a continuous ***t-co norms*** if "$\diamond$" satisfying following conditions:

1. $\diamond$ is commutative and associative;
2. $\diamond$ is continuous;
3. $a \diamond 0 = a$ for all $a\in [0, 1]$
4. $a \diamond b \le c \diamond d$ whenever $a \le c$ and $b \le d$, and $a, b, c, d \in [0, 1]$.

***Remark 2.1 (Lowen R.) [14]:*** Every fuzzy metric space ($X$, $M$, *) is an Intuitionistic fuzzy metric space of the form ($X$, $M$, $1-M$, *, $\diamond$) such that $t$-norm * and $t$-conorm $\diamond$ are associated as $x \diamond y = 1-((1-x) * (1-y))$ for all $x, y \in X$,

***Leema2. 1:*** An Intuitionistic fuzzy metric spaces with continuous $t$ -norm "*" and continuous t-conorm "$\diamond$" defined by $a*a \ge a$ and $(1-a) \diamond (1- a) \le (1-a)$ for all $a\in [0, 1]$. Then for all $x, y \in X$, $M(x, y, *)$ is non-decreasing and $N(x, y, \diamond)$ is non-increasing.

***Proof.*** Suppose $M(x, y, *)$ is non- increasing, therefore for $t \le s$,
We have $M(x, y, t) \ge M(x, y, s)$ For all $x, y,z \in X$,
We have $M(x, z, t + s) \ge M(x, y, t) * M(y, z, s)$ .
In particular for $z=y$,
We have $M(x, y, t + s) \ge M(x, y, t) * M(y, y, s)$.
$M(x, y, t + s) \ge M(x, y, t) * 1= M(x, y, t)$,
A contradiction, hence $M(x, y, *)$ is non-decreasing.
Again suppose $N(x, y, \diamond)$ is non-decreasing, therefore for $t \le s$,
we have $N(x, y,s) \ge N(x, y, t)$ For all $x, y,z \in X$,
we have $N(x, z, t + s) \le N(x, y, t) \diamond N(y, z, s)$
In particular for $z=y$,
we have $N(x, y, t + s) \le N(x, y, t) \diamond N(y, y, s)= N(x, y, t)$,
A contradiction, hence $N(x, y, \diamond)$ is non-increasing.

***Definition -2.3 (Alaca et. al.)[2]:*** A 5-tuple ($X$, $M$, $N$, *, $\diamond$) is said to be an intuitionistic fuzzy metric space if $X$ is an arbitrary set, "*" is a continuous t-norm, $\diamond$ is a continuous t-conorm and $M$, $N$ are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions:

for all $x, y, z \in X$, $s, t > 0$,
(IFM-1) $M(x, y, t) + N(x, y, t) \le 1$
(IFM-2) $M(x, y, t) > 0$
(IFM-3) $M(x, y, t) = 1$ if and only if $x = y$
(IFM-4) $M(x, y, t) = M(y, x, t)$
(IFM-5) $M(x, y, t) *M(y, z, s) \le M(x, z, t + s)$
(IFM-6) $M(x, y, .) : (0, \infty) \rightarrow [0, 1]$ is continuous
(IFM-7) $N(x, y, t) < 1$
(IFM-8) $N(x, y, t) = 0$ if and only if $x = y$
(IFM-9) $N(x, y, t) = N(y, x, t)$
(IFM-10) $N(x, y, t) \diamond N(y, z, s) \ge N(x, z, t + s)$
(*IFM*-11) $N(x, y,.): (0,\infty) \rightarrow [0, 1]$ is continuous

Then ($X$, $M$, $N$, *, $\diamond$) or simply ($M$, $N$) is called an *intuitionistic fuzzy metric* on $X$.

**Note:** $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non nearness between $x$ and $y$ with respect to '$t$' respectively.

***Definition2.4.*** Let ($X$, $M$, $N$, *, $\diamond$) be an intuitionistic fuzzy metric space. Then

1. A sequence $\{x_n\}$ in $X$ is said to be *convergent to a point* $x\in X$ if, for all $t > 0$ $\lim_{n\to\infty} M(x_n, x, t) = 1$ and $\lim_{n\to\infty} N(x_n, x, t) = 0.$
2. A sequence $\{x_n\}$ in $X$ is said to be *Cauchy sequence* if, for all $t > 0$ and $p > 0$, $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$

and $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0.$

3. Intuitionistic fuzzy metric space (*X, M, N,* *, ◇) is said to be *complete* if and only if every Cauchy sequence in $X$ is convergent.

***Remark -2.1:*** Every fuzzy metric space (*X, M,* *) is an intuitionistic fuzzy metric space of the form (*X,M*, 1−*M*, *,◇) such that t-norm "*" and t-conorm "◇" are associated
*i.e.* $x◇y = 1 - ((1 - x) * (1 - y))$ for any $x, y \in [0, 1]$.

***Remark -2.2:*** An intuitionistic fuzzy metric space *X*, *M*(*x, y,*.) is non-decreasing and *N* (*x, y,* .) is non-increasing for all *x, y*∈*X.*

***Example- 2.1***: Let (*X, d*) be a metric space. Denote $a * b = a.b$ and $a◇b = \min\{1, a + b\}$ for all
$a, b \in [0, 1]$. Let *M* and *N* be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$M(x,y,t) = \frac{t}{t + md(x,y)} \quad , \quad N(x,y,t) = \frac{d(x,y)}{t + d(x,y)}$$

in which $m > 1$ . Then (*X, M, N,* *, ◇) is an intuitionistic fuzzy metric space.

***Lemma 2.2: (Rodriguez)[18]:*** Let (*X, M, N,* *, ◇) is an intuitionistic fuzzy metric space, then *M* and *N* are continuous function on $X^2 \times (0, \infty)$.

***Lemma 2.3 (Alaca et. al.) [2]:*** In IFM space *X,* For all $x, y \in X$, *M* (*x, y,* .) is nondecreasing and *N* (*x, y,* .) is nonincreasing.

***Lemma 2.4 (Mishra et. al.) [16].*** If for all $x, y \in X$, $t > 0$ and for a number $k \in (0, 1)$

$M(x, y, kt) > M(x, y, t)$ and $N(x, y, kt) < N(x, y, t)$
then $x = y$.

***Definition 2.5 (Park) [17]:*** Let (*X, M, N,* *, ◇) is an intuitionistic fuzzy metric space:
A sequence $\{x_n\}$ in *X* is said to be convergent to the a point $x \in X$ (denoted by $\lim_{n\to\infty} x_n = x$ ), if

$\lim_{n\to\infty} M(x_n, x, t) = 1$, and $\lim_{n\to\infty} N(x_n, x, t) = 0$ for all $t > 0$.

***Definition 2.6:*** Let *A* and *T* be self-maps of a set *X*. If $Ax = Tx = t$ (say), $t \in X$, for some *x* in *X*, then *x* is called a ***coincidence point*** of *A* and *T*. The set of coincidence points of *A* and *T* in *X* is denoted by Ψ (*A, T*) and *t* is called a point of coincidence of *A* and *T*.

***Definition 2.7 (Jungck)[11]:*** The pair of mappings (*A, T*) is said to be ***compatible*** if $\lim_{n\to\infty} d(ATx_n, TAx_n) = 0$, whenever $\{x_n\}$ is a sequence in *X* such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Tx_n = t$, for some $t \in X$.

***Definition 2.8 (Jungck)[11]:*** The pair of mappings (*A, T*) be ***noncompatible*** if there is at least one sequence $\{x_n\}$ in *X* such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Tx_n = x$, for some $x \in X$.

but $\lim_{n\to\infty} d(ATxn, TAxn)$ is either non-zero or non-existent.

***Definition 2.9 (Aamri and Moutawakil ) [1]:*** The pair of mappings (*A, T*) said to ***satisfy property (E.A.)*** , if there exists a sequence $\{x_n\}$ in *X* such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Tx_n = x$ for some $x \in X$.

***Definition 2.10 (Al-Thagafi, and Shahzad) [4]:*** Two self mappings *S* and *T* are said to be ***weakly compatible*** if they commute at their coincidence points; i.e., if $Tx = Sx$ for some $x \in X$, then $TSx = STx$.

***Definition 2.11 (Al-Thagafi, and Shahzad) [4]:*** Two self mappings *S* and *T* are said to be ***occasionally weakly compatible*** (owc) if $TAx = ATx$ for some $x \in \Psi(A, T)$.

***Remark 2.3 (Al-Thagafi and Shahzad) [4]:*** Al-Thagafi and Naseer Shahzad shown that occasionally weakly is weakly compatible but converse is not true.

***Example 1.3. :*** Let *R* be the usual metric space. Define $S, T : R \to R$ by $Sx = 2x$ and $Tx = x^2$ for all $x \in R$. Then $Sx = Tx$ for $x = 0, 2$ but $ST0 = TS0$, and $ST2 \neq TS2$. The pair (*S,T*) is occasionally weakly compatible but not weakly compatible.

***Remark 2.4. (Babu and Alenmayehu)[5]:*** Every pair of non compatible self maps of a metric space (X, d) satisfies property (E.A.), but converse need not be true.

***Remark 2.5:*** Weak compatibility and property (E.A.) are independent of each other.

***Remark 2.6(Jungck)[11]:*** Every compatible pair is weakly compatible but its converse need not be true.

***Remark 2.7 (Al-Thagafi and Shahzad) [4]:*** Every weakly compatible pair is occasionally weakly compatible but its converse need not be true.

***Remark 2.8(Jungck)[ 11].*** Occasionally weak compatibility and property (E.A.) are independent of each other.

***Definition 2.12.(Liu et. al.)[13]***: Let (*X, d*) be a metric space and *A, B, S* and *T* be four self maps on *X*. The pairs (*A, S*) and (*B, T*) are said to satisfy ***common property (E.A.),*** if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in *X* such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = t$, for some *t* in *X*.

***Definition 2.13.( Sharma and Bamboria)[22]:*** Let *S* and *T* be two self mappings of an Intuitionistic fuzzy metric space (*X, M, N,* *, ◇) . We say that *S* and *T* satisfy the ***property (SB)*** if there exists a sequence $\{x_n\}$ in *X* such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in X$.

***Example 2.2***: Let $X = [0, +\infty)$. Define $S, T: X \to X$ by $Tx = \frac{x}{5}$ and $Sx = \frac{3x}{5}$, for all *x* in *X*. Consider the sequence $\{x_n\} = \{1/n\}$. Clearly $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = 0$ . Then *S* and *T* satisfy the property (*SB*).

***Example 2.3***: Let $X = [2, +\infty)$. Define $S, T : X \to X$ by $Tx = x + 1/2$ and $Sx = 2x + 1/2$, $\forall x \in X$.

Suppose property (*SB*) holds; then there exists in $X$ a sequence $\{x_n\}$ satisfying $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$.

*Therefore* $\lim_{n\to\infty} x_n = z - \frac{1}{2}$ *and*

$$\lim_{n\to\infty} x_n = \frac{(2z-1)}{4}.$$

Then $z = 1/2$, which is a contradiction since $1/2 \notin X$. Hence $S$ and $T$ do not satisfy the property (SB).

***Remark 2.9:*** In view of definition given by Mishra et al. [16] and Sharma and Deshpande [23] , that two self-mappings $S$ and $T$ of a Intuitionistic fuzzy metric space ($X, M, N, *, \diamond$) will be ***non-compatible*** if there exists at least one sequence $\{x_n\}$ in $X$ such that

$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$,

But $\lim_{n\to\infty} M(STx_n, TSx_n, t)$ is either not equal to 1 or non-existent.

**Therefore two non-compatible self-mappings of a *Intuitionistic fuzzy metric space* ($X, M, N, *, \diamond$) satisfy the property (*SB*).**

## MAIN RESULTS

***Theorem 2.1:*** Let ($X, M, N, *, \diamond$) be an intuitionistic fuzzy metric space with $t*t \geq t$ for some $t \in [0,1]$ and the condition (FM-6). Let $S$ and $T$ be weakly compatible mappings of $X$ into itself such that

(2.1) $S$ and $T$ satisfy the property (*S-B*),
(2.2) there exist a number $k \in (0,1)$ such that
$M(Tx, Ty, kt) > M(Sx, Sy, t) * M(Sx, Tx, t) * M(Sy, Ty, t) * M(Sy, Tx, t) * M(Sx, Ty, t)$
and $N(Tx, Ty, kt) < N(Sx, Sy, t) \diamond N(Sx, Tx, t) \diamond N(Sy, Ty, t) \diamond N(Sy, Tx, t) \diamond N(Sx, Ty, t)$

for all $x \neq y \in X$,
(2.3) $TX \subset SX$.
If $SX$ or $TX$ be a closed subset of $X$, then $S$ and $T$ have a unique common fixed point.

**Proof** . Since $S$ and $T$ satisfy the property (SB), there exists in $X$ a sequence $\{x_n\}$ satisfying

$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$.

Suppose $SX$ is closed, we have $\lim_{n\to\infty} S x_n = Sa$ for some $a \in X$.
Also $\lim_{n\to\infty} Tx_n = Sa$.
We show that $Ta = Sa$.
Suppose that $Ta \neq Sa$. By condition (2.2), we have

$M(Tx_n, Ta, kt) \geq M(Sx_n, Sa, t) * M(Sx_n, Tx_n, t) * M(Sa, Ta, t) * M(Sa, Tx_n, t) * M(Sx_n, Ta, t)$.
and $N(Tx_n, Ta, kt) \leq N(Sx_n, Sa, t) \diamond N(Sx_n, Tx_n, t) \diamond N(Sa, Ta, t) \diamond N(Sa, Tx_n, t) \diamond N(Sx_n, Ta, t)$.
Letting $n \to \infty$ , yields
$M(Sa, Ta, kt) \geq M(Sa, Ta, t)$ and $N(Sa, Ta, kt) \leq N(Sa, Ta, t)$

Therefore by Lemma 2.2, we have $Sa = Ta$.
Since $T$ and $S$ are weakly compatible $STa = TSa$ and therefore, $TTa = TSa = STa = SSa$.

Finally, we show that $Ta$ is a common fixed point of $T$ and $S$. Suppose that
$Ta \neq TTa$. Then by (2.2), we have

$M(Ta, TTa, kt) \geq M(Sa, STa, t) * M(Sa, Ta, t) * M(STa, TTa, t) * M(STa, Ta, t) * M(Sa, TTa, t) \geq M(Ta, TTa, t)$ $N(Ta, TTa, kt) \leq N(Sa, STa, t) \diamond N(Sa, Ta, t) \diamond N(STa, TTa, t) \diamond N(STa, Ta, t) \diamond N(Sa, TTa, t) \leq N(Ta, TTa, t)$

So by Lemma 2.2, we have $Ta = TTa$ and therefore , $STa = TTa = Ta$. The proof is similar when $TX$ is assumed to be closed subset of $X$ since $TX \subset SX$. Uniqueness of the common fixed point follows easily.

Since two noncompatible selfmappings of an intuitionistic fuzzy metric space ($X, M, N, *, \diamond$) satisfy the property (*S-B*), we get the following results

***Corollary*** 2.1 . Let ($X, M, N, *, \diamond$) be an intuitionistic fuzzy metric space with $t*t \geq t$ for some $t \in [0,1]$ and the condition (FM-6). Let $S$ and $T$ be two noncompatible weakly compatible mappings of $X$ into itself such that
(2.4) there exist a number $k \in (0,1)$ such that
$M(Tx, Ty, kt) \geq M(Sx, Sy, t) * M(Sx, Tx, t) * M(Sy, Ty, t) * M(Sy, Tx, t) * M(Sx, Ty, t)$ and $N(Tx, Ty, kt) \leq N(Sx, Sy, t) \diamond N(Sx, Tx, t) \diamond N(Sy, Ty, t) \diamond N(Sy, Tx, t) \diamond N(Sx, Ty, t)$
for all $x, y \in X$,
(2.5) $TX \subset SX$.

If $SX$ or $TX$ be a closed subset of $X$, then $T$ and $S$ have a unique common fixed point.

Now we prove common fixed point theorem for four discontinuous mappings in non-complete fuzzy metric space.

***Theorem*** 2.2. Let ($X, M, N, *, \diamond$) be an intuitionistic fuzzy metric space with $t*t \geq t$ for some $t \in [0,1]$ and the condition (FM-6). Let $A$, $B$, $S$ and $T$ be mappings of $X$ into itself such that
(2.6) $AX \subset TX$ and $BX \subset SX$,
(2.7) ($A$,$S$) or ($B$,$T$) satisfies the property (S-B),
(2.8) there exists a number $k \in (0,1)$ such that

$M(Ax, By, kt) \geq M(Sx, Ty, t) * M(Sx, By, t) * M(Ty, By, t)$
and $N(Ax, By, kt) \leq N(Sx, Ty, t) \diamond N(Sx, By, t) \diamond N(Ty, By, t)$ for all $x, y \in X$,
(2.9) ($A$,$S$) and ($B$,$T$) are weakly compatible,
(2.10) one of $AX$, $BX$, $SX$ or $TX$ is a closed subset of $X$.
Then $A$, $B$, $S$ and $T$ have a unique common fixed point in $X$.

***Proof.*** Suppose that ($B$,$T$) satisfies the property (*S-B*). Then there exists a sequence $\{x_n\}$ in $X$ such that $\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$.

Since $BX \subset SX$, there exists in $X$ a sequence $\{y_n\}$ such that $Bx_n = Sy_n$ . Hence $\lim_{n\to\infty} Sy_n = z$. Let us show that $\lim_{n\to\infty} Ay_n = z$.
Indeed, in view of (2.8),we have
$M(Ay_n, Bx_n, kt) \geq M(Sy_n, Tx_n, t) * M(Sy_n, Bx_n, t) * M(Tx_n, Bx_n, t) \geq M(Bx_n, Tx_n, t) * 1 * M(Tx_n, Bx_n, t) \geq M(Tx_n, Bx_n, t)$ And $N(Ay_n, Bx_n, kt) \leq N(Sy_n, Tx_n, t) \diamond N(Sy_n, B^{xn}, t) \diamond N(Tx_n, Bx_n, t) \leq N(Bx_n, Tx_n, t) \diamond 0 \diamond N(Tx_n, Bx_n, t) \leq N(Tx_n, Bx_n, t)$

it follows that

$\lim_{n\to\infty} M(Ay_n, Bx_n, kt) \geq 1$, and $\lim_{n\to\infty} N(Ay_n, Bx_n, kt) \leq 0$,

which implies that $\lim_{n\to\infty} M\left(Ay_n, Bx_n, kt\right)=1$ and $\lim_{n\to\infty} N\left(Ay_n, Bx_n, kt\right)= 0$, Hence we deduce that $\lim_{n\to\infty} Ay_n = z$. Suppose $SX$ is a closed subset of $X$. Then $z = Su$ for some $u \in X$.
Subsequently,
we have
$\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sy_n = Su$.
By (2.8), we have

$M(Au, Bx_n, kt) \geq M(Su, Tx_n, t) * M(Su, Bx_n, t) * M(Tx_n, Bx_n, t)$

and $N(Au, Bx_n, kt) \leq N(Su, Tx_n, t) \diamond N(Su, Bx_n, t) \diamond N(Tx_n, Bx_n, t)$

Letting $n \to \infty$, we obtain $Au = Su$. The weak compatibility of $A$ and $S$ implies that $ASu = SAu$ and then $AAu = ASu = SAu = SSu$.
On the other hand, since $AX \subset TX$, there exists a point $v \in X$ such that Au = Tv. We claim that Tv = Bv. Using (2.8), we have

$M(Au, Bv, kt) \geq M(Su, Tv, t) * M(Su, Bv, t) * M(Tv, Bv, t) \geq M(Au, Bv, t)$ And $N(Au, Bv, kt) \leq N(Su, Tv, t) \diamond N(Su, Bv, t) \diamond N(Tv, Bv, t) \leq N(Au, Bv, t)$
By Lemma 2.2, we have Au = $Bv$. Thus $Au = Su = Tv = Bv$. The weak compatibility of $B$ and $T$ implies that $BTv = TBv$ and $TTv = TBv = BTv = BBv$. Let us show that $Au$ is a common fixed point of $A$, $B$, $S$ and $T$. In view of (2.8), it follows that

$M(Au,AAu, kt) = M(AAu, Bv, kt) \geq M(SAu, Tv, t) * M(SAu, Bv, t) * M(Tv, Bv, t) \geq M(AAu, Au, t)$ and $N(Au,AAu, kt) = N(AAu, Bv, kt) \leq N(SAu, Tv, t) \diamond N(SAu, Bv, t) \diamond N(Tv, Bv, t) \leq N(AAu, Au, t)$

Therefore by Lemma 2.2, we have $Au = AAu = SAu$ and $Au$ is a common fixed point of $A$ and $S$.

Similarly, we prove that $Bv$ is a common fixed point of $B$ and $T$. Since $Au = Bv$, we conclude that $Au$ is a common fixed point of $A$, $B$, $S$ and $T$.

The proof is similar when $TX$ is assumed to be closed subset of $X$.

The cases in which $AX$ or $BX$ is closed subset of $X$ are similar to the cases in which $TX$ or $SX$, respectively, is closed since $AX \subset TX$ and $BX \subset SX$.

If $Au = Bu = Su = Tu = u$ and $Av = Bv = Sv = Tv = v$, the by (2.8), we have
$M(u,v, kt) = M(Au, Bv, kt) \geq M(Su, Tv, t) * M(Su, Bv, t) * M(Tv, Bv, t) \geq M(u, v, t)$
and $N(u,v, kt) = N(Au, Bv, kt) \leq N(Su, Tv, t) \diamond N(Su, Bv, t) \diamond N(Tv, Bv, t)$
$\leq N(u, v, t)$
By Lemma 2.2, we have u = v and the common fixed point is unique. This completes the proof of the theorem.
For three mappings, we have the following results

***Corollary 2.2*** Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space with $t*t \geq t$ for some $t \in [0,1]$ and the condition (FM-6). Let $A$, $B$ and $S$ be mappings of $X$ into itself such that

(3.11) $AX \subset SX$ and $BX \subset SX$,
(3.12) $(A,S)$ or $(B,S)$ satisfies the property (SB),
(3.13) there exist a number $k \in (0,1)$ such that

$M(Ax, By, kt) \geq M(Sx, Sy, t) * M(Sx, By, t) * M(Sy, By, t)$ for all $x, y \in X$,

and $N(Ax, By, kt) \geq N(Sx, Sy, t) \diamond N(Sx, By, t) \diamond N(Sy, By, t)$ for all $x, y \in X$,

(3.14) $(A,S)$ and $(B,S)$ are weakly compatible,
(3.15) One of $AX$, $BX$ and $SX$ is a closed subset of $X$.
Then $A$, $B$ and $S$ have a unique common fixed point in $X$.

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