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Research Article

COMMON FIXED POINT THEOREMS USING COMPATIBLE MAPS IN D-METRIC SPACES OVER TOPOLOGICAL SEMI FIELD

Rajesh Tokse[1]., Sanjay Choudhari[2]., Kamal Wadhwa[3] and Vineet Kumar Agarawal[4]

[1]Department of Mathematics, Corporate Institute of Research and Technology, Bhopal (India)-462021
[2]Department of Mathematics, Govt. P.G. Collage, Hoshangabad M.P. India
[3]Department of Mathematics, Govt. P.G. College, Pipariya (M.P.), India
[4]Department of Mathematics, Govt. Home science P.G. College Hosangabad M.P., India,

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ABSTRACT

In this paper, we introduce the concepts of compatible mappings in D- metric spaces over Topological semi field and prove the common fixed point theorem.

INTRODUCTION

The notion of 2-metric space was investigated by Gähler [6],[7],[8]. The notion of generalized metric space (D-metric space) is introduced by Dhage [5]. He proved some results on fixed point theorems for a self mapping of a complete and bounded D-metric space. A number of fixed point theorems have been proved for 2-metric spaces. However Hsiao showed that all such theorems are trivial in the sense that the iterations of f are collinear.

Fortunately, in the case of D-metric spaces the situation in not alike sees Ahmad *et al*. [1]. Here it should be noted that Singh, Kumar and Ganguly [9] in their paper explained their differences with the argument of Hsiao with example. Probably because the paper was published in Hindi, it could not find much publicity in the international area.

Antonovskii *et al*. [2] defined topological semi field in 1960. Sharma and Sharma [11] proved a fixed point theorem in 2-metric space over topological semi field. Recently the concept of compatible maps in D-metric space introduced by Sing and Sharma [10] and they proved common fixed point theorems for compatible maps in generalized metric spaces.

In this paper we wish to present the result on common fixed point theorems using compatible maps in D-metric space over topological semi field.

Preliminaries

We recall some definitions

Definition 2.1: We shall call a commutative associative topological ring E, a topological semi field if there is isolated in some set K satisfying:

1. $K + K \subset K$; $KK \subset K$,
2. $K - K = E$
3. The least upper bound and greatest lower bound exists,
4. For a, b $\in$ K the equation $ax = b$ has at least one solution in K,
5. The intersection $K \cap (-K)$ contains only the zero element of the ring.

***Corresponding author:* Rajesh Tokse**
Research Scholar, MGCGVV, Chitrakoot (MP) India

Remark 2.1: The axioms for a topological semi field are so chosen that its properties recall those of the fields of real numbers. In fact, it was proved by Antonovskii *et al*. [10], that any topological semi field contains topological fields isomorphic with the real line.

Remark 2.2: We shall call elements of the set K positive elements of the semi field E and elements of the set K – K will be called boundary elements of the semi field E.

Remark 2.3: We agree to write the relation $x - y \in K$, $x - y \in K$ also in the form $x > y$, $x \geq y$ (or in the form $y < x$, $y \leq x$). In particular, the inequality $x > 0$ means $x \in K$ and $x \geq 0$ means that $x \in K$.

Remark 2.4: The set K contains elements, which are different from zero.

Definition 2.2: Let E be a semi field and K be the set of all its positive elements. The non-empty set X is called a D-metric space over the topological semi field E if there exists a real function $D: X\times X\times X \rightarrow K$ that satisfies the following conditions:

(D-1) $D(x, y, z) \geq 0$ for all $x,y,z \in X$ (non-negativity) and the equality holds if and only if $x = y = z$,
(D-2) $D(x,y,z) = D(y,x,z) = D(y,z,x)$ … (Symmetry),
(D-3) $D(x,y,z) \leq D(x,y,a) + D(x,a,z) + D(a,y,z)$ for all x,y,z,a in X.
(Rectangle inequality)

Definition 2.3: A sequence $\{x_n\}$ of points in a D-metric space (X,D) over a topological semi field E is said to be D-convergent and converges to a point $x \in X$ if , $\lim_{n,m\to\infty} D(x_n , x_m , x) \in U$ for all $x \in X$ where $U \in E$ is the neighborhood of the origin.

Definition 2.4: A sequence $\{x_n\}$ of points in (X,D) over a topological semi field E is said to be D-Cauchy if $\lim_{m,n,p \to \infty} D(x_n , x_m , x_p) \in U$.

Definition 2.5: A D-metric space over topological semi field E is called D-bounded if there exists a constant M such that $D(x,y,z) \leq M$ for all $x,y,z \in X$.

Definition 2.6: A self-mapping T of a D-metric space (X,D) over a topological semi field E is said to be continuous at $x \in X$ if $Tx_n \to Tx$, whenever $x_n \to x$.

Definition 2.7: Two self mappings A and B of a D-metric space (X,D) over a topological semi field E are said to be D-weakly commuting if $D(ABx, BAx, z) \leq D(Ax, Bx, z)$ Where $y = ABx$ (or BAx) and $z = Ax$ (or Bx) for all $x \in X$.

Definition 2.8 : Two self mappings A and B of a D-metric space (X,D) over a topological semi field E are said to be D-compatible if

$\lim_{n\to\infty} D(ABx, Bax, z) = 0$ where $z = ABx_n$ (or BAx_n) whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = y$, for some y in X.

Clearly, commutativity implies D-weak commutativity and D-weak commutativity implies D-compatibility; but neither implication is reversible always as this can be seen in Singh and Sharma [11].

Remark 2.5: If E = R be the field of real numbers, we arrive at the definition of D-metric space. (See Dhage [12], [13]).

Further, Singh and Sharma [14] proved the following.

Proposition 2.1: Let A and B be D-compatible self mappings of D-metric space X.

1. If $Ay = By$, then $ABy = BAy$,
2. If $Ax_n, Bx_n \rightarrow y$, for some y in X then.

a. $BAx_n \rightarrow Ay$, if A is continuous,
b. If A and B are continuous at y , then $Ax = By$ and $ABy = BAy$.

Singh and Sharma [15] proved the following.

Theorem 2.1: Let A, B, F and G be self mappings of a complete bounded D-metric space (X,D) satisfying

1. $A(X) \subseteq G(X)$ and $B(X) \subseteq F(X)$
2. One of A, B, F or G is continuous
3. Pairs of mappings {A,F} and {B,G} are D-compatible
4. $D(Ax, By, z) \leq \phi[\max\{D(Fx, Gy, z), \alpha D(Fx, By,z), \alpha D(Gy, Ax, z)\}]$

for all $x,y,z \in X$ where ϕ is non-decreasing function and $0 < \alpha \leq 1/3$.

Then A, B, F and G have a unique common fixed point in X.

Inspired by the above result, we prove the following, using proposition 1.

MAIN RESULTS

Theorem 3.1: Let A, B, S and T be self mappings of a complete and bounded

D-metric space over a topological semi field E satisfying:

1. $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$,
2. One of A, B, S or T is continuous,
3. Pair of mappings {A, S} and {B, T} are D-compatible,
4. $D(Ax, By, z) \leq \alpha D(Sx, Ty, z) + \beta\{D(Ax, Sx, z) + D(By, Ty, z)\}$

$+\gamma \{D(Ax, Ty, z) + D(By, Sx, z)\}$,
for all $x,y,z \in X$ and non negative reals α, β, γ such that $½ < k^{l+1} < 1$ where $k = (\alpha + \beta + 2\gamma)/(1 - \beta - \gamma)$
Then A, B, S and T have a unique common fixed point in X.

Proof: Let x_0 be an arbitrary element in X. Then from (1.1), there exists x_1, x_2 in X such that

$y_1 = Ax_0 = Tx_1$ and $y_2 = Bx_1 = Sx_2$ **.**
Inductively construct a sequence $\{y_n\}$ in X such that $y_{2n}= Sx_{2n} = Bx_{2n-1}$ and
$y_{2n+1} = Tx_{2n+1}= Ax_{2n}$,n =1,2,..
Using (1.4) for $m \geq 2$, we have

$D(y_1, y_2 , y_m) = D(Ax_0 , Bx_1 , y_m)$
$\leq \alpha D(Sx_0 , Tx_1 , y_m)+ \beta\{D(Ax_0, Sx_0, y_m)+D(Bx_1, Tx_1, y_m)\}$
$+ \gamma \{D(Ax_0, Tx_1, y_m) +(Bx_1, Sx_0, y_m)\}$
$\leq \alpha D(y_0 , y_1 , y_m) + \beta\{D(y_1, y_0, y_m) + D(y_2, y_1, y_m)\}$
$+ \gamma \{D(y_1, y_1, y_m) + D(y_2, y_0, y_m)\}$,

Or

$(1 - \beta)D(y_1, y_2, y_m) \le (\alpha + \beta) D(y_0, y_1, y_m)+\gamma\{D(y_0, y_1, y_m)+ D(y_1, y_0, y_m) + D(y_2, y_1, y_m)+D(y_2, y_0, y_1)\}$,

$(1 - \beta - \gamma) D(y_1, y_2, y_m) \le (\alpha + \beta + 2\gamma) D(y_0, y_1, y_m) + \gamma D(y_0, y_1, y_2)$,

Or

$D(y_1, y_2, y_m) \le (\alpha + \beta + 2\gamma)/(1 - \beta - \gamma) D(y_0, y_1, y_m)+ \gamma/(1 - \beta - \gamma) D(y_0, y_1, y_2)$

1. $= k D(y_0, y_1, y_m) + k_1 D(y_0, y_1, y_2)$,

where $k = (\alpha + \beta + 2\gamma)/(1 - \beta - \gamma)$ and $k_1 = \gamma/(1 - \beta - \gamma)$. Similarly, using (1.4) for any $m \ge 3$, we have

$D(y_2, y_3, y_m) = D(Bx_1, Ax_2, y_m) = D(Ax_2, Bx_1, y_m)$
$\le \alpha D(Sx_2, Tx_1, y_m) + \beta\{D(Ax_2, Sx_1, y_m) + D(Bx_1, Tx_1, y_m)\} +\gamma\{D(Ax_2, Tx_1, y_m) + D(Bx_1, Sx_2, y_m)\}$,
$\le \alpha D(y_2, y_1, y_m) + \beta\{D(y_3, y_2, y_m) + D(y_2, y_1, y_m)\} + \gamma \{D(y_3, y_1, y_m) + D(y_2, y_2, y_m)\}$,

$\le \alpha D(y_2, y_1, y_m) + \beta\{D(y_3, y_2, y_m) + D(y_2, y_1, y_m)\} + \gamma \{D(y_2, y_1, y_m) + D(y_3, y_2, y_m) + D(y_3, y_1, y_2) + D(y_1, y_2, y_m)\}$,

or

$(1 - \beta - \gamma) D(y_2, y_3, y_m) \le (\alpha + \beta + 2\gamma) D(y_2, y_1, y_m) + \gamma D(y_1, y_2, y_3)$,

i.e. $D(y_2, y_3, y_m) \le (\alpha + \beta + 2\gamma)/(1 - \beta - \gamma) D(y_1, y_2, y_m)+ \gamma/(1 - \beta - \gamma) D(y_1, y_2, y_3)$

$= k D(y_1, y_2, y_m) + k_1 D(y_1, y_2, y_3)$,

or

$D(y_2, y_3, y_m) \le k [k D(y_0, y_1, y_m) + k_1 D(y_0, y_1, y_2)] + k_1 D(y_1, y_2, y_3)$.
$= k^2 D(y_0, y_1, y_m) + kk_1 D(y_0, y_1, y_2)] + k_1 D(y_1, y_2, y_3)$.

i.e.

2. $D(y_2, y_3, y_m) \le k^2 D(y_0, y_1, y_m) + kk_1 D(y_0, y_1, y_2)] + k_1 D(y_1, y_2, y_3)$.

Further we have

$D(y_3, y_4, y_m) = D(Ax_2, Bx_3, y_m)$
$\le \alpha D(Sx_2, Tx_3, y_m) + \beta\{D(Ax_2, Sx_2, y_m) + D(Bx_3, Tx_3, y_m)\}$
$+ \gamma \{D(Ax_2, Tx_3, y_m) + D(Bx_3, Sx_2, y_m)\}$,

or

$D(y_3, y_4, y_m) \le \alpha D(y_2, y_3, y_m) + \beta\{D(y_3, y_2, y_m) + D(y_4, y_3, y_m)\}$
$+ \gamma \{D(y_3, y_3, y_m) + D(y_4, y_2, y_m)\}$,

or

$D(y_3, y_4, y_m) \le \alpha D(y_2, y_3, y_m) + \beta\{D(y_3, y_2, y_m) + D(y_3, y_4, y_m)\}$
$+ \gamma \{D(y_2, y_3, y_m) + D(y_3, y_2, y_m) + D(y_4, y_3, y_2) + D(y_4, y_2, y_m)\}$,

i.e.

$(1 - \beta - \gamma) D(y_3, y_4, y_m) \le (\alpha + \beta + 2\gamma) D(y_2, y_3, y_m) + \gamma D(y_2, y_3, y_4)$,

$D(y_3, y_4, y_m) \le (\alpha + \beta + 2\gamma)/(1 - \beta - \gamma) D(y_2, y_3, y_m) + \gamma/(1 - \beta - \gamma) D(y_2, y_3, y_4), = k D(y_2, y_3, y_m) + k_1 D(y_2, y_3, y_4)$,

or

$D(y_3, y_4, y_m) \le k [k^2 D(y_0, y_1, y_m) + kk_1 D(y_0, y_1, y_2)] + k_1 D(y_1, y_2, y_3) + k_1 D(y_2, y_3, y_4)$,

or

$D(y_3, y_4, y_m) \le k^3 D(y_0, y_1, y_m) + k^2k_1 D(y_0, y_1, y_2)] + kk_1 D(y_1, y_2, y_3) + k_1 D(y_2, y_3, y_4)$,

Similarly, using (1.4), we have

$D(y_4, y_5, y_m) \le k^4 D(y_0, y_1, y_m)+ k^3k_1 D(y_0, y_1, y_2)]+k^2 k_1 D(y_1, y_2, y_3) + kk_1 D(y_2, y_3, y_4)+ k_1 D(y_3, y_4, y_5)$

Proceeding in this way, we have

$D(y_n, y_{n+1}, y_m) \le k^n D(y_0, y_1, y_m) + k_1 [k^{n+!} D(y_0, y_1, y_m)] + k^{n+2} D(y_1, y_2, y_3) + k^{n+3} D(y_2, y_3, y_4) + \ldots k D(y_{n-1}, y_n, y_{n+1})$

$= k^n D(y_0, y_1, y_m) + k_1 k^{n-1} D(y_0, y_1, y_m) + k_1 \sum_{r=0}^{n-1} k^{n-r-1} D(y_{n-1}, y_n, y_{n+1})$

or

$D(y_n, y_{n+1}, y_m) \le k^n M + k_1 k^{n+1} M + k_1 \sum_{r=0}^{n-1} k^{n-r-1} M$

$D(y_n, y_{n+1}, y_m) \le k^{n-1}(k + k_1) M + k_1 k^{n+1} M + k_1 \Sigma\sum_{r=0}^{n-1} k^{n-r-1} M$

or

$D(y_n, y_{n+1}, y_m) \le k^{n-1}k_2 M + k_1 \sum_{r=0}^{n-1} k^{n-r-1} M$, where $k + k_1 = k_2$, now for $p,t \in N$, we have

$D(y_n, y_{n+p}, y_{n+p+t}) \le D(y_n, y_{n+1}, y_{n+p+t}) + D(y_n, y_{n+p}, y_{n+1}) + D(y_{n+1}, y_{n+p}, y_{n+p+t})$,

or

$D(y_n, y_{n+p}, y_{n+p+t}) \le 2k, k^{n-1}M + 2 k_1 \sum_{r=0}^{n-1} k^{n-r-1} M + D(y_{n+1}, y_{n+2}, y_{n+p+t}) + D(y_{n+1}, y_{n+p}, y_{n+2}) + D(y_{n+2}, y_{n+p}, y_{n+p+t})$

or

$D(y_n, y_{n+p}, y_{n+p+t}) \le 2k_1 k^{n-1}M + 2 k_1 \sum_{r=0}^{n-1} k^{n-r-1} M + 2 k_2 k^n M + 2 k_1 \sum_{r=0}^{n-1} k^{n-r} M$

$+ D(y_{n+2}, y_{n+p}, y_{n+p+t})$
$= 2 k_2 M (k^{n-1} + k^2) + 2 k_1 M[\sum_{r=0}^{n-1} k^{n-r-1} + \sum_{r=0}^{n-1} k^{n-r}]$
$+ D(y_{n+2}, y_{n+p}, y_{n+p+t})$,

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or

$D(y_n, y_{n+p}, y_{n+p+t}) \le 2 M k_2 [k^{n-1} + k^n + \ldots + k^{n+p-3}]$

$+ 2 M k\sum_{r=1}^{n-1} k^{n-r-1}+ \sum_{r=1}^{n} k^{n-r} + \sum_{r=1}^{n+p-3} k^{n+p-r-3}]$
$+ D(y_{n+p-1}, y_{n+p}, y_{n+p+t})$

$D(y_n, y_{n+p}, y_{n+p+t}) \le 2 M k_2 [k^{n-1} + k^n + k^{n+1} + \ldots + (1/2) k^{n+p-2}]$
$+ 2 M k_1[\sum_{r=1}^{n-1} k^{n-r-1} + \sum_{r=1}^{n} k^{n-r} + \sum_{r=1}^{n+1} k^{n-r+1} + \ldots + (1/2) \sum_{r=1}^{n+p+1} k^{n+p-r-2}]$

Now if $(1/2) k^{l+1} < 1$, then $1 + k + k^2 + \ldots + k^l < k^{l+1} + k^{l+1} + k^{l+1} + \ldots$

i.e.
$\sum_{r=0}^{l} k^r < \sum_{r=l+1}^{l} k^r$.

Therefore

$D(y_n, y_{n+p}, y_{n+p+t}) \leq 2 M k_2 [k^{n-1} + k^n + k^{n+1} + \ldots + (1/2) k^{n+p-2}]$
$+ 2 M k_1 [\sum_{r=n-1}^{r} k^r + \sum_{r=n}^{r} k^r + \sum_{r=n+1}^{r} k^r + \ldots + (1/2) + \sum_{r=n+p-2}^{r} k^r]$
$\to 0$ as $n \to \infty$, since $k < 1$.

This shows that $\{y_n\}$ is a D-Cauchy sequence in X, by completeness of X, $\{y_n\}$ converges to some point u in X and also its subsequences $\{Ax_{2n}\},\{Bx_{2n-1}\},\{Sx_{2n}\}$ and $\{Tx_{2n+1}\}$ also converges to some u in X. Now by (1.2) one of the mapping A, B, S or T is continuous. Assume this true for S (The other alternatives will be treated in similar way). From the conclusion we just derived $SAx_{2n}, SSx_{2n} \to Su$. By D-compatibility of (A,S), we have

$\lim_{n\to\infty} D(SAx_{2n}, ASx_{2n}, SAx_{2n}) = 0$
which yields $ASx_{2n} \to Su$. On the other hand by (1.4), we have

$D(ASx_{2n}, Bx_{2n+1}, u) \leq \alpha D(SSx_{2n}, Tx_{2n+1}, u) + \beta\{D(ASx_{2n}, SSx_{2n}, u) + D(Bx_{2n+1}, Tx_{2n+1}, u)\} + \gamma \{D(ASx_{2n}, Tx_{2n+1}, u) + D(Bx_{2n+1}, SSx_{2n}, u)\}$.
Letting $n \to \infty$, we have

$D(Su,u,u) \leq \alpha D(Su,u,u) + \beta \{D(Su,u,u) + D(u,u,u)\} + \gamma \{ D(Su,u,u) + D(u,Su,u)\}$,
or $D(Su,u,u) \leq (\alpha + \beta + 2\gamma) D(Su,u,u)$,

which yields $Su = u$. Further,

$D(Au, Bx_{2n+1}, u) \leq \alpha D(Su, Tx_{2n+1}, u) + \beta\{D(Au, Su, u) + D(Bx_{2n+1}, Tx_{2n+1}, u)\} + \gamma \{D(Au, Tx_{2n+1}, u) + D(Bx_{2n+1}, Su, u)\}$.
Letting $n \to \infty$, we have
$D(Au, u, u) \leq \alpha D(Su, u, u) + \beta\{D(Au, Su, u) + D(u, u, u)\} + \gamma \{D(Au, u, u) + D(u, Su, u)\}$,
or $D(Au, u, u) \leq (\beta + \gamma) D(Au, u, u)$,
and this yields $Au = u$.

Since $A(X) \subseteq T(X)$, therefore there exists $v \in X$ such that $u = Tv = Su$. Hence by (1.4), we have
$D(u, Bv, u) = D(Au, Bv, u)$
$\leq \alpha D(Su, Tv, u) + \beta\{D(Au, Su, u) + D(Bv, Tv, u)\} + \gamma \{D(Au, Tv, u) + D(Bv, Su, u)\}$,
or
$D(u, Bv, u) = D(Au, Bv, u) \leq \alpha D(u, Tv, u) + \beta\{D(u, u, u) + D(Bv, Tv, u)\} + \gamma \{D(u, Tv, u) + D(Bv, u, u)\}$.

We get $D(u, Bv, u) = (\beta + \gamma) D(u, Bv, u)$,
which gives $Bv = u = Tv$.

Hence by compatibility of (B,T) and from Proposition 1, we have $BTv = TBv$ or $Bu = Tu$.

Again by (1.4), we have
$D(u, Tu, u) = D(Au, Bu, u)$
$\leq \alpha D(Su, Tu, u) + \beta\{D(Au, Su, u) + D(Bu, Tu, u)\} + \gamma \{D(Au, Tu, u) + D(Bv, Su, u)\}$,
or
$D(u, Tu, u) \leq \alpha D(u, Tu, u) + \beta\{D(u, u, u) + D(Tu, Tu, u)\} + \gamma \{D(u, Tu, u) + D(Tu, u, u)\}$,

This follows

$D(u, Tu, u) \leq (\alpha + \beta + 2\gamma) D(u, Tu, u)$.

Which yields $Tu = u$.

Hence $Au = Bu = Su = Tu = u$. i.e. u is a common fixed point of A, B, S and T.
Finally for uniqueness let w ($w \neq u$) be another common fixed point of A, B, S and T. Then by (1.4), we have
$D(u, w, u) = D(Au, Bw, u)$
$\leq \alpha D(Su, Tw, u) + \beta\{D(Au, Su, u) + D(Bw, Tw, u)\} + \gamma \{D(Au, Tw, u) + D(Bw, Su, u)\}$,
or
$D(u, w, u) \leq \alpha D(u, w, u) + \beta\{D(u, u, u) + D(w, w, u)\} + \gamma \{D(u, w, u) + D(w, u, u)\}$.

This follows $D(u, w, u) \leq (\alpha + \beta + 2\gamma) D(u, w, u)$,
which yields that $u = v$.
This completes the proof.

Corollary 3.1: Let A and S be self mappings of a complete and bounded D-metric space (X,D) over a topological semi field E satisfying:

1. $A(X) \subseteq S(X)$,
2. one of A or S is continuous,
3. pair of mappings {A, S} is D-compatible,
4. $D(Ax, Ay, z) \leq \alpha D(Sx, Sy, z) + \beta\{D(Ax, Sx, z) + D(Ay, Sy, z)\}$

$+\gamma \{D(Ax, Sy, z) + D(Ay, Sx, z)\}$,
for all $x,y,z \in X$ and non negative reals α, β, γ such that $½ < k^{l+1} < 1$ where $k = (\alpha + \beta + 2\gamma)/(1 - \beta - \gamma)$.
Then A and S have a unique common fixed point in X.
Finally , when $S = 1$ in Corollary 1, we get.

Corollary 3.2: Let A be a self mapping of a complete and bounded D-metric space (X,D) over a topological semi field E satisfying:
(1.9) $D(Ax, Ay, z)$

$\leq \alpha D(x, y, z + \beta \{D(Ax, x, z) + D(Ay, y, z)\} + \gamma \{D(Ax, y, z) + D(Ay, x, z)\}$,
for all $x,y,z \in X$ and non negative reals α, β, γ such that $½ < k^{l+1} < 1$ where
$k = (\alpha + \beta + 2\gamma)/(1 - \beta - \gamma)$.
Then A has common fixed point in X.

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