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CODEN: IJRSFP (USA)

International Journal of Recent Scientific Research
Vol. 9, Issue, 10(C), pp. 29222-29228, October 2018

ISSN: 0976-3031

International Journal of Recent Scientific Research

DOI: 10.24327/IJRSR

Research Article

MHD FLOW AND HEAT TRANSFER THROUGH A CIRCULAR ANNULUS OF NON-DARCY POROUS MEDIA SATURATED WITH VISCOUS INCOMPRESSIBLE FLUID

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DOI: http://dx.doi.org/10.24327/ijrsr.2018.0910.2816

ARTICLE INFO

Article History:
Received 06th July, 2018
Received in revised form 14th August, 2018
Accepted 23rd September, 2018
Published online 28th October, 2018

Key Words:

MHD, non-Darcy, Partial filled circular pipe, Joule heating.

ABSTRACT

A steady incompressible axisymmetric flow in a circular annulus filled with non-Darcy porous medium is studied in the influence of a static magnetic field applied in radial direction. The Joule heating effect produced by the magnetic field is also included to analyze effect of magnetic field and fluid flow field on heat convection process. The governing equations of flow and heat transfer are non-linear coupled differential equations, are solved with Quasi-numerical method – the Differential Transform method (DTM). The velocity and temperature profiles for the porous region and clear fluid central region are derived and computed with the use of Matlab at various physical parameters and there effects are discussed through graphs. The skin-friction coefficient and Nusselt number at the surfaces of the porous annulus are computed and discussed through graphs.

INTRODUCTION

The flow through a semi porous medium is of great interest in oil refineries, chemical sciences, life sciences and medical sciences. In agriculture sector, the proper distribution of fertilizers and pesticides is insured using the cylindrical semi porous medium. The flow through semi porous cylindrical type configurations are encountered in many industries in one or other ways for cooling purposes or for heat connection processes. Loganathan *et al*. [7] studied MHD effects on free convective flow over moving semi-infinite vertical cylinder with temperature oscillation. Ziya Uddin *et al*. [12] studied heat and mass transfer characteristics and the flow behavior on MHD flow near the lower stagnation point of a porous isothermal horizontal circular cylinder. Chamkha A.J [4] investigated steady, laminar, hydromagnetic flow and heat and mass transfer over a permeable cylinder moving with a linear velocity in the presence of heat generation/absorption, chemical reaction, suction /injection effects developing a uniform transverse magnetic field. Abbas *et al*. [1] dealt with laminar flow and heat transfer of an electrically conducting viscous fluid over a stretching cylinder in the presence of thermal radiations through a porous medium. Nagaraju, *et al*. [8] investigated the steady flow of an electrically conducting, incompressible micropolar fluid in a narrow gap between two concentric rotating vertical cylinders with porous lining on inside of outer cylinder under an imposed axial magnetic field. Yadav *et al*. [11] found out numerical solution of MHD fluid flow and heat transfer characteristics of a viscous incompressible fluid along a continuously stretching horizontal cylinder embedded in a porous medium in presence of internal heat generation or absorption. Aldoss [2] studied the MHD mixed convection flow about a vertical cylinder embedded in a non-Darcian porous medium with variable heat transfer boundary. Suneetha *et al*. [10] analyzed the interaction of free convection with thermal radiation of a viscous incompressible unsteady MHD flow past a moving vertical cylinder with heat and mass transfer in a porous medium. Shihhao *et al*. [9] derived analytical solution for MHD flow of a magnetic fluid within a thick porous annulus. In present study, the effect of magnetic field and Joule heating in the flow and heat transfer in a circular tube having a concentric circular porous cylinder of non-Darcy behavior are investigated.

Formulation of the problem

Steady incompressible, axisymmetric flow of an electrically conducting viscous fluid through a circular cylinder with an annulus region of non-Darcy porous medium saturated with the fluid is undertaken. The radius of the outer cylinder is d and the radius of the inner cylinder is di (di< d) therefore the width of

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the annulus porous circular region is (d-di)>0. In the cylindrical coordinates (r, θ, z) the axis of cylinder coincides with the z-axis. A static magnetic field of strength (B_0, 0, 0) is applied on the cylinder. Thus the flow in 0 ≤ r≤ di is tube flow and in the region di ≤ r≤ d, the flow in the non-Darcy medium.

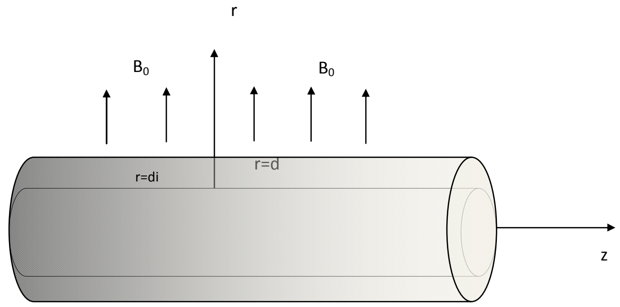

Figure 1 Physical model of the problem

The equation of continuity is defined as

$$\nabla . \vec{q} = 0 \qquad \ldots (1)$$

The equations of motion are

$$\rho(\vec{q}.\nabla\vec{q})\vec{q} = -\nabla p + \mu\nabla^2\vec{q} - \frac{\mu}{K}\vec{q} - \frac{\mu c_d}{\sqrt{K\upsilon}}|\vec{q}|\vec{q} + \vec{J}\times\vec{B} \qquad \ldots (2)$$

The equation of energy

$$\rho c_p(\vec{q}.\nabla\vec{q}) = \nabla^2 T + \frac{\vec{J}^2}{\sigma} \qquad \ldots (3)$$

Under the above assumptions the equations of motion and energy for the inner circular clear fluid region 0 ≤ r≤ di are given by

$$\mu\left(\frac{\partial^2 u_f}{\partial r^2} + \frac{1}{r}\frac{\partial u_f}{\partial r}\right) - \frac{\partial p}{\partial x} - \sigma B_0{}^2 u_f = 0 \qquad \ldots (4)$$

$$\kappa\left(\frac{\partial^2 T_f}{\partial r^2} + \frac{1}{r}\frac{\partial T_f}{\partial r}\right) + \sigma B_0{}^2 u_f{}^2 = 0 \qquad \ldots (5)$$

The equation of motion and energy for the annulus non-Darcy porous region di ≤ r≤ d are given by

$$\mu\left(\frac{\partial^2 u_p}{\partial r^2} + \frac{1}{r}\frac{\partial u_p}{\partial r}\right) - \frac{\partial p}{\partial x} - \sigma B_0{}^2 u_p - \frac{\mu}{K}u_p - \frac{\mu}{\sqrt{K}}\frac{c_d}{\upsilon}u_p{}^2 = 0 \qquad \ldots (6)$$

$$\kappa\left(\frac{\partial^2 T_p}{\partial r^2} + \frac{1}{r}\frac{\partial T_p}{\partial r}\right) + \sigma B_0{}^2 u_p{}^2 = 0 \qquad \ldots (7)$$

The corresponding boundary conditions are

$$r = 0: \quad \frac{\partial u_f}{\partial r} = 0, \quad \frac{\partial T_f}{\partial r} = 0$$
$$r = d: \quad u_p = 0, \quad T_f = T_w \qquad \ldots (8)$$

$$r = d_i: \quad u_f = u_p, \quad T_f = T_p \qquad \ldots (9)$$

Where, K the permeability of the porous medium, u_f and T_f the velocity and temperature of the fluid in the central clear region, u_p and T_p the velocity and temperature of the fluid in the annulus porous region, T_w temperature of the wall of outer cylinder, T_{pw} temperature at the inner surface of porous annulus, μ viscosity of the fluid, σ electrical conductivity, c_d drag-force constant.

Method of solution

To make the differential equations (4) to (7) dimensionless, introducing the following non-dimensional quantities

$$r^* = \frac{r}{d}, x^* = \frac{x}{d}, di^* = \frac{di}{d}, u_f{}^* = \frac{u_f}{V_0}. \qquad u_p{}^* = \frac{u_p}{V_0},$$

$$G^* = \frac{\partial p}{\partial x}\bigg/\frac{\rho V_0{}^2}{d}, \ \mathrm{Re} = \frac{\rho V_0 d}{\mu}, M = \sqrt{\frac{\sigma B_0{}^2 d^2}{\mu}}$$

$$Da = \frac{K}{d^2}, \qquad F = \frac{c_d \rho V_0{}^2 d^3}{\sqrt{K}\nu\mu}, \qquad \theta_f = \frac{T_f - T_w}{T_{pw} - T_w},$$

$$\theta_f = \frac{T_f - T_w}{T_{pw} - T_w}, \ \theta_p = \frac{T_p - T_w}{T_{pw} - T_w}, \ Br = \frac{\mu V_0{}^2}{\kappa(T_{pw} - T_w)}$$

Where G the constant dimensionless pressure gradient, Da the Darcy number, Re the Reynolds number, M the Hartmann number, F the For chheimer number, Br the Brinkman number. There is no loss of generality if the asterisks are dropped from the dimensionless form of the equations (4) to (7) the respective equations are given by

$$\frac{d^2 u_f}{dr^2} + \frac{1}{r}\frac{du_f}{dr} = \mathrm{Re}\,G + M^2 u_f \qquad \ldots (10)$$

$$\frac{d^2\theta_f}{dr^2} + \frac{1}{r}\frac{d\theta_f}{dr} + M^2 Br\, u_f{}^2 = 0 \qquad \ldots (11)$$

$$\frac{d^2 u_p}{dr^2} + \frac{1}{r}\frac{du_p}{dr} = \mathrm{Re}\,G + \left(\frac{1}{Da} + M^2\right)u_p + \frac{F}{\mathrm{Re}}u_p{}^2 \ldots (12)$$

$$\frac{d^2\theta_p}{dr^2} + \frac{1}{r}\frac{d\theta_p}{dr} + M^2 Br\, u_p{}^2 = 0 \qquad \ldots (13)$$

The corresponding boundary conditions are

$$r = 0: \qquad \frac{\partial u_f}{\partial r} = 0, \quad \frac{\partial\theta_f}{\partial r} = 0$$

$$r = 1: \qquad u_p = 0 \qquad \theta_p = 0$$

At the interface

$$r = di: \qquad u_f = u_p, \qquad \theta_f = \theta_p \qquad \ldots (15)$$

Solution of the coupled momentum and energy equations, are obtained by the Differential Transform Method (DTM).The efficiency of method can be seen in literature [3] ,[5], [6].

The differential transform U(k) of the derivative $\frac{d^k u(y)}{dy^k}$ is defined by $U(k)=\frac{1}{k!}\left[\frac{d^k u(y)}{dy^k}\right]_{y=y_0}$

The inverse differential transform of U(k) is defined by

$$u(y)=\sum_{k=0}^{\infty} U(k)\left(y-y_0\right)^k$$

Table1 The fundamental mathematical operations under DTM

Function	***Differential transform***
$u(y)=f(y)\pm g(y)$	$U(k)=F(k)\pm G(k)$
$u(y)=\lambda g(y)$	$U(k)=\lambda G(k)$
$u(y)=\frac{\partial g(y)}{\partial y}$	$U(k)=(k+1)G(k+1)$
$u(y)=\frac{\partial^m g(y)}{\partial y^m}$	$U(k)=(k+1)\ldots(k+m)G(k+m)$
$u(y)=y^m$	$U(k)=\delta(k-m)=\begin{cases}1 & at \quad k=m\\ 0 & otherwise\end{cases}$
$u(y)=f(y)g(y)$	$U(k)=\sum_{r=0}^{k}F(r)G(k-r)$
$u(y)=f_1(y)f_2(y)\ldots f_m(y)$	$U(k)=\sum_{k_1}^{k}\cdots\sum_{k_{m-1}=0}^{k_2}F_1(k_1)F_2(k_2-k_1)\ldots F_m(k-k_{m-1})$

Calculation for the velocity profiles

Velocity profile in clear fluid region

Applying DTM on (10) we will get the following recurrence relation

$$\sum_{h=o}^{k}\delta(h-1)(k-h+1)(k-h+2)U_f(k-h+2)+(k+1)U_f(k+1)=G\,\mathrm{Re}\,\delta(k-1)+M^2\sum_{h=0}^{k}\delta(h-1)U_f(k-h) \quad \ldots(16)$$

Where $U_f(k)$ is differential transform of $u_f(r)$. Since the value of $u_f(r)$ at $r=0$ is not known explicitly, therefore assuming $U_f(0)=a$ (constant) which will be determined later with the prescribed boundary conditions.

For k=0, 1, 2, 3, 4, 5 in (16) we get

$$U_f(1)=0,\ U_f(2)=\frac{1}{4}\left(G\,\mathrm{Re}+M^2a\right),\ U_f(3)=0,$$

$$U_f(4)=\frac{M^2}{64}\left(G\,\mathrm{Re}+M^2a\right), U_f(5)=0,$$

$$U_f(6)=\frac{M^4}{2304}\left(G\,\mathrm{Re}+M^2a\right)$$

Using the above values in the inverse differential transform of $U_f(k)$ the velocity profile is given by

$$u_f(r)=a+\frac{1}{2^2}\left(G\mathrm{Re}+M^2a\right)r^2+\frac{M^2}{2^2 4^2}\left(G\mathrm{Re}+M^2a\right)r^4+\frac{M^4}{2^2 4^2 6^2}\left(G\mathrm{Re}+M^2a\right)r^6 \quad \ldots(17)$$

Now the interface condition provides that

$$a+\frac{1}{4}\left(G\,\mathrm{Re}+M^2a\right)di^2+\frac{M^2}{2^2 4^2}\left(G\,\mathrm{Re}+M^2a\right)di^4+\frac{M^4}{2^2 4^2 6^2}\left(G\,\mathrm{Re}+M^2a\right)di^6$$
$$=b+\frac{1}{2^2}\left(\mathrm{Re}G+\left(\frac{1}{Da}+M^2\right)b+\frac{F}{\mathrm{Re}}b^2\right)di^2+\frac{1}{2^2 4^2}\left(\frac{1}{Da}+M^2+2\frac{F}{\mathrm{Re}}b\right)\left(\mathrm{Re}G+\left(\frac{1}{Da}+M^2\right)b+\frac{F}{\mathrm{Re}}b^2\right)di^4$$
$$+\frac{1}{2^2 4^2 6^2}\left(\frac{1}{Da}+M^2+2\frac{F}{\mathrm{Re}}b\right)^2\left(\mathrm{Re}G+\left(\frac{1}{Da}+M^2\right)b+\frac{F}{\mathrm{Re}}b^2\right)+\frac{1}{4^2 6^2}\left(\mathrm{Re}G+\left(\frac{1}{Da}+M^2\right)b+\frac{F}{\mathrm{Re}}b^2\right)^2 d$$
$$=H$$

The arbitrary constant '*a*' is obtained and given by

$$a=\frac{H-\left(\frac{G\,\mathrm{Re}\,di^2}{4}+\frac{M^2G\,\mathrm{Re}\,di^4}{64}+\frac{M^4G\,\mathrm{Re}\,di^6}{2304}\right)}{1+\frac{M^2di^2}{4}+\frac{M^4di^4}{64}+\frac{M^6di^6}{2304}}$$

Using value of '*a*' in the equation (17), we get the velocity profile of the fluid in the central clear fluid region.

Velocity profile in annulus porous region

Applying DTM on (12), we get the recurrence relation

$$\sum_{h=o}^{k}\delta(h-1)(k-h+1)(k-h+2)U_p(k-h+2)+(k+1)U_p(k+1)$$
$$=G\,\mathrm{Re}\,\delta(k-1)+\left(\frac{1}{Da}+M^2\right)\sum_{h=0}^{k}\delta(h-1)U_p(k-h)+\frac{F}{\mathrm{Re}}\sum_{h=0}^{k}\sum_{i=0}^{h}\delta(i-1)U_p(h-i)U_p(k-i) \quad \ldots(18)$$

Where $U_p(k)$ is differential transform of $u_p(r)$. Since value of $u_p(r)$ is not known at $r=0$ explicitly therefore assuming $U_p(0)=b$ (constant), which will be determined later with the aid of interface conditions.

Corresponding to k=0, 1, 2, 3, 4, 5 the recurrence relation (18) gives

$$U_p(1)=0,$$

$$U_p(2)=\frac{1}{2^2}\left(\mathrm{Re}G+\left(\frac{1}{Da}+M^2\right)b+\frac{F}{\mathrm{Re}}b^2\right),$$

$$U_p(3)=0,$$

$$U_p(4)=\frac{1}{2^2 4^2}\left(\frac{1}{Da}+M^2+2\frac{F}{\mathrm{Re}}b\right)\left(\mathrm{Re}G+\left(\frac{1}{Da}+M^2\right)b+\frac{F}{\mathrm{Re}}b^2\right),$$

$$U_p(5)=0,$$

$$U_p(6)=\frac{1}{2^2 4^2 6^2}\left(\frac{1}{Da}+M^2+2\frac{F}{\mathrm{Re}}b\right)^2\left(\mathrm{Re}G+\left(\frac{1}{Da}+M^2\right)b+\frac{F}{\mathrm{Re}}b^2\right)+\frac{1}{4^2 6^2}\left(\mathrm{Re}G+\left(\frac{1}{Da}+M^2\right)b+\frac{F}{\mathrm{Re}}b^2\right)^2$$

Using these values in the inversion of $U_p(k)$, we have

$$u_p(r)=b+\frac{1}{2^2}\left(\mathrm{Re}G+\left(\frac{1}{Da}+M^2\right)b+\frac{F}{\mathrm{Re}}b^2\right)r^2$$
$$+\frac{1}{2^2 4^2}\left(\frac{1}{Da}+M^2+2\frac{F}{\mathrm{Re}}b\right)\left(\mathrm{Re}G+\left(\frac{1}{Da}+M^2\right)b+\frac{F}{\mathrm{Re}}b^2\right)r^4$$
$$+\left(\begin{array}{l}\dfrac{1}{2^2 4^2 6^2}\left(\dfrac{1}{Da}+M^2+2\dfrac{F}{\mathrm{Re}}b\right)^2\left(\mathrm{Re}G+\left(\dfrac{1}{Da}+M^2\right)b+\dfrac{F}{\mathrm{Re}}b^2\right)\\ +\dfrac{1}{4^2 6^2}\left(\mathrm{Re}G+\left(\dfrac{1}{Da}+M^2\right)b+\dfrac{F}{\mathrm{Re}}b^2\right)^2\end{array}\right)r^6 \quad \ldots(19)$$

Using the boundary condition $r=1$; $u_p(r)=0$ gives

$$b+\frac{1}{2^2}\left(\mathrm{Re}G+\left(\frac{1}{Da}+M^2\right)b+\frac{F}{\mathrm{Re}}b^2\right)+\frac{1}{2^2 4^2}\left(\frac{1}{Da}+M^2+2\frac{F}{\mathrm{Re}}b\right)\left(\mathrm{Re}G+\left(\frac{1}{Da}+M^2\right)b+\frac{F}{\mathrm{Re}}b^2\right)$$
$$+\frac{1}{2^2 4^2 6^2}\left(\frac{1}{Da}+M^2+2\frac{F}{\mathrm{Re}}b\right)^2\left(\mathrm{Re}G+\left(\frac{1}{Da}+M^2\right)b+\frac{F}{\mathrm{Re}}b^2\right)+\frac{1}{4^2 6^2}\left(\mathrm{Re}G+\left(\frac{1}{Da}+M^2\right)b+\frac{F}{\mathrm{Re}}b^2\right)^2=0 \quad \ldots(20)$$

Now (20) is a polynomial of 4th degree in b. A MATLAB code has been generated for the computation of unknown constant b and using this value, the velocity profile in the porous region is known, computed and presented through graphs.

Calculation for Temperature profiles

Temperature profile in clear fluid region

Applying DTM on (11), we get recurrence relation

$$\sum_{h=o}^{k}\delta(h-1)(k-h+1)(k-h+2)\Theta_f(k-h+2)+(k+1)\Theta_f(k+1)=$$
$$-M^2 Br\sum_{h=0}^{k}\sum_{i=0}^{h}\delta(i-1)U_f(h-i)U_f(k-h)$$

Where $\Theta_f(k)$ is differential transform of $\theta_f(r)$

Let $\Theta_f(0)=c$ (constant) will be determined with the aid of boundary condition on $\theta_f(r)$

Corresponding to k=0, 1, 2, 3, 4, 5,6,7 the recurrence relation (21) gives

$$\Theta_f(1)=0,\quad \Theta_f(2)=-\frac{M^2 Br}{4}a^2,\quad \Theta_f(3)=0,$$
$$\Theta_f(4)=-\frac{M^2 Br}{8}U_f(2),\quad \Theta_f(5)=0$$
$$\Theta_f(6)=-\frac{M^2 Br}{36}\left(2aU_f(4)+U_f^{\,2}(2)\right)$$
$$\Theta_f(7)=0$$
$$\Theta_f(8)=-\frac{M^2 Br}{32}\left(aU_f(6)+U_f(2)U_f(4)\right)$$

Using these values in the inversion of differential transform of $\Theta_f(k)$ we have

$$\theta_f(r)=c+\Theta_f(2)r^2+\Theta_f(4)r^4+\Theta_f(6)r^6+\Theta_f(8)r^8$$

On applying the interface condition the equation, gives the value of unknown constant α .

Invoking value of c in (23), Furthermore, the temperature profile in the annulus region is given by

$$\theta_f(r)=\frac{M^2 Br\, a^2}{4}\left(1-r^2\right)+\frac{M^2 Br}{32}\left(G\mathrm{Re}+aM^2\right)\left(1-r^4\right)$$
$$+\frac{M^2 Br}{36}\left(\frac{aM^2}{32}\left(G\mathrm{Re}+aM^2\right)+\frac{1}{16}\left(G\mathrm{Re}+aM^2\right)\right)\left(1-r^6\right)$$
$$-\frac{M^2 Br}{32}\left(a\left(\frac{M^4}{2304}\left(G\mathrm{Re}+M^2 a\right)\right)+\frac{M^2}{625}\left(G\mathrm{Re}+M^2 a\right)^2\right)r^8$$

Temperature profile for annulus porous region

Applying DTM on (13), we get recurrence relation

$$\sum_{h=o}^{k}\delta(h-1)(k-h+1)(k-h+2)\Theta_p(k-h+2)+(k+1)\Theta_p(k+1)=-M^2 Br\sum_{h=0}^{k}\sum_{i=0}^{h}\delta(i-1)U_p(h-i)U_p(k-h) \quad \ldots(24)$$

Where $\Theta_p(k)$ is differential transform of $\theta_p(r)$. Since the initial value $\Theta_p(0)$ is not knwown, therefore assuming its differential transform $\Theta_p(0)=\alpha$ (constant) will be determined from interface condition.

Corresponding to k=0, 1, 2, 3, 4, 5 the recurrence relation (23) gives

$$\Theta_p(1)=0$$
$$\Theta_p(2)=-\frac{M^2 Br}{4}a^2 \quad \ldots(21)$$
$$\Theta_p(3)=0$$
$$\Theta_p(4)=-\frac{M^2 Br}{8}U_p(2)$$
$$\Theta_p(5)=0$$
$$\Theta_p(6)=-\frac{M^2 Br}{36}\left(2aU_p(4)+U_p^{\,2}(2)\right)$$
$$\Theta_p(8)=-\frac{M^2 Br}{32}\left(bU_p(4)+U_p(2)U_p(4)\right)$$
$$\theta_p(r)=\alpha+\Theta_p(2)r^2+\Theta_p(4)r^4+\Theta_p(6)r^6+\Theta_p(8)r^8 \quad \ldots(25)$$

With the use of boundary condition $\theta_p(1)=0$, the constant α is determined and given by

$$\alpha=-\Theta_p(2)-\Theta_p(4)-\Theta_p(6)-\Theta_p(8) \quad \ldots(26)$$

The temperature profiles for annulus porous region is computed from (24) with help of Matlab programming and presented through graphs.

Skin friction coefficient

The non-dimensional shearing stress at the outer and inner wall of porous annulus in terms of the local skin-friction coefficient is derived as follows and computed values are given in table 2 & 3.

(22)

$$c=\alpha+\left(\Theta_p(2)-\Theta_f(2)\right)di^2+\left(\Theta_p(4)-\Theta_f(4)\right)di^4+\left(\Theta_p(6)-\Theta_f(6)\right)di^6+\left(\Theta_p(8)-\Theta_f(8)\right)di^8$$

$$C_f = \left(\frac{\mu \frac{\partial u}{\partial r}}{\rho V_0^{\ 2}} \right)_{r=d}$$

The skin friction at the outer wall of the cylinder

$$C_f = \frac{1}{\mathrm{Re}} \left(\frac{\partial u_f}{\partial r} \right)_{r=1}$$

The skin friction at the inner surface of the porous annulus

$$C_f = \frac{1}{\mathrm{Re}} \left(\frac{\partial u_p}{\partial r} \right)_{r=di}$$

Nusselt number

The non-dimensional coefficient of heat transfer at the outer and inner wall of the porous annulus is derived as follows and computed values are given in table 4 & 5.

Nusselt number at the outer wall of the cylinder

$$Nu = -\left(\frac{\partial \theta_f}{\partial r} \right)_{r=1}$$

Nusselt number at the inner surface of the porous annulus

$$Nu = -\left(\frac{\partial \theta_p}{\partial r} \right)_{r=di}$$

RESULTS AND DISCUSSION

Fluid velocity is well controlled by the magnetic field as demonstrated in figure 2. The fluid velocity decreases with increase in Hartmann number in both the inner clear fluid region and annulus porous region.

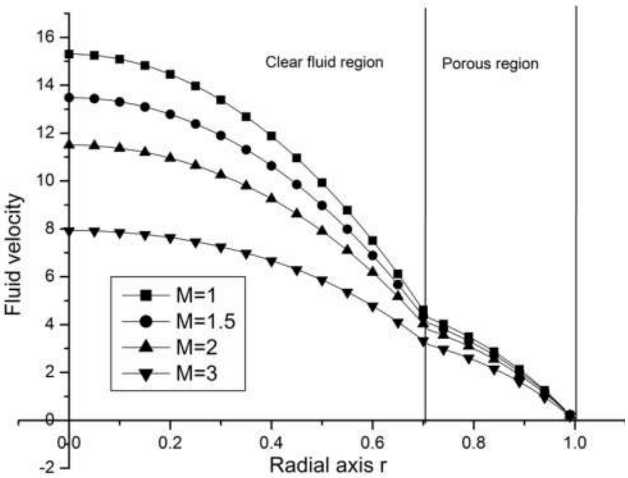

Figure 2. The effect of Hartmann number on fluid flow profile at Re=10, F=5, Da=0.1, G=-10

In figure 3, it is observed that the fluid velocity is augmented with the increase in Reynolds number for both the region, the central region of clear fluid and the annulus region of the fluid saturated non-Darcy porous medium but the magnitude of the change in velocity is large in clear fluid region as compare to the porous region which is in good agreement with the physical law. Figure 4 demonstrated that the effect of Forchheimer number on the flow profile is relatively small. The velocity of fluid decreases with increasing value of Forchheimer number for clear fluid region and porous region. The effect of permeability of the porous medium on the flow velocity is analyzed with the Darcy number. It is observed in figure 5 that the fluid velocity enhanced with the rise in Darcy number.

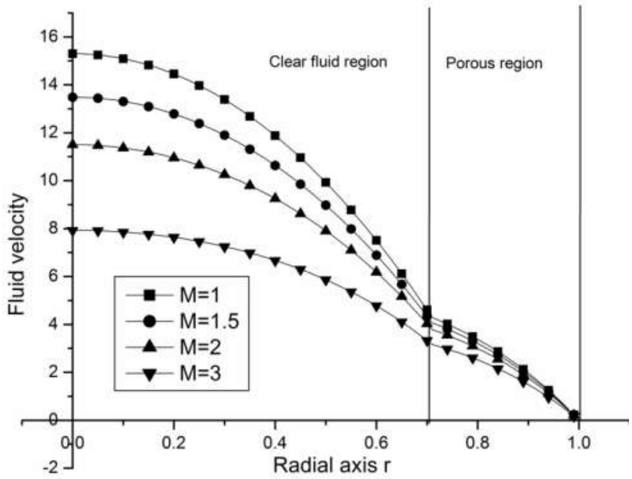

Figure 2. The effect of Hartmann number on fluid flow profile at Re=10, F=5, Da=0.1, G=-10

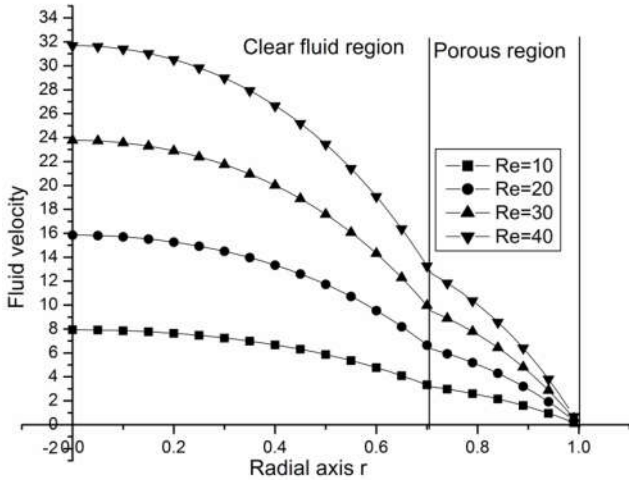

Figure 3. The effect of Reynolds number on fluid flow profile at M=5, F=5, Da=0.1, G=-10

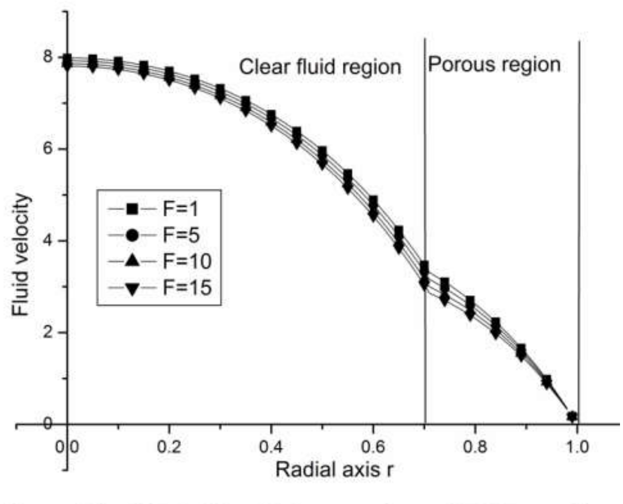

Figure 4.The Effect of Forchheimer number on fluid flow profile at M=3, Re=10, Da=0.1, G=-10

The effect of porous annulus is also contributed in the clear fluid region confined by the porous region and therefore the fluid velocity in the central region is also varying in the same pattern with the Darcy number.

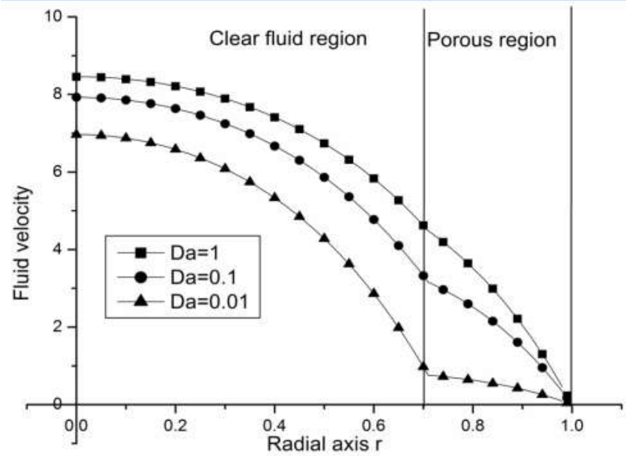

Figure 5. The effect of Darcy number on fluid flow proflie at M=3, Re=10, F=5, G=-10

The effect of pressure gradient through the cylinder is demonstrated in figure 6 and it is observed that with the increase of pressure gradient along the cylinder the velocity of the fluid enhanced significantly in both the region; the clear fluid region and the annulus porous region. This outcome is in good agreement with the experimental results validated the modeled problem.

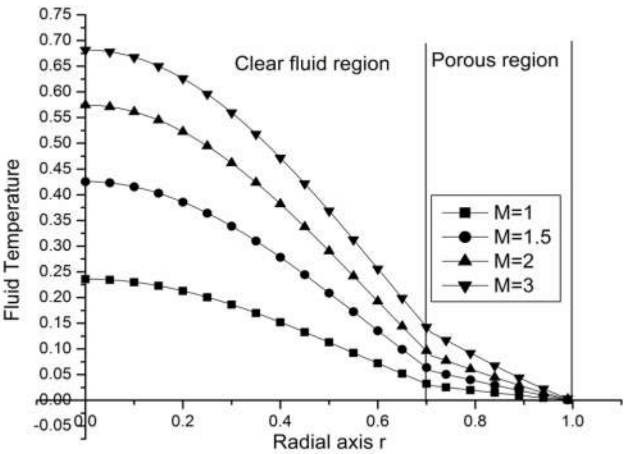

Figure 7. The effect of Hartmann number on fluid temperature at Re=10, F=5, Da=0.01, Br=0.01

The figure 7 shows that in the increase of magnetic field strength the fluid temperature increases more profoundly in the clear fluid region.

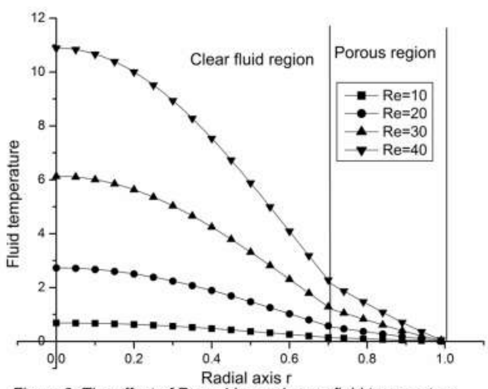

Figure 8. The effect of Reynolds number on fluid temperature at M=3, F=5, Da=0.1, G=-10, Br=0.01

The fluid temperature increases with the increase of Reynolds number, Darcy number, pressure gradient along the cylinder and Brinkman number as observed in the figure 8,10,11 and 12 respectively. The temperature of the fluid flowing through porous annulus and in the core region is decreases with the increase of Forchheimer number as observed in figure 9.

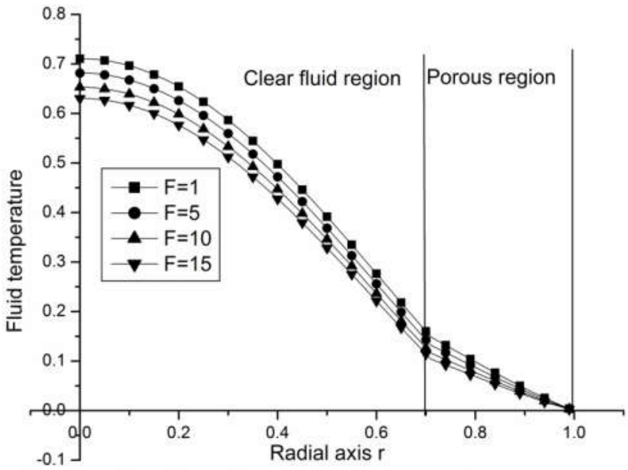

Figure 9. The effect of Forchheimer number on fluid temperature at M=3, Re=10, Da=0.1, G=-10, Br=0.01

Table 2 and 3 demonstrates that with the increase in Hartmann number the magnitude of skin friction reduces at both inner and outer wall of the porous annulus. The magnitude of skin friction increases at both inner and outer wall of the annulus with the increase in Reynolds number and pressure gradient along the cylinder. The skin friction at the outer wall increases with increase in Forchheimer number and Darcy number while there is a reverse effect of these physical parameters on the skin friction at the inner wall which is interacting with the clear fluid region.

From table 4 and 5, it is observed that the value of Nusselt number at both outer and inner surface of the annulus increases with the increase in Hartmann number, Reynolds number, Darcy number, pressure gradient along the cylinder and Brinkman number while it decreases with the increase in Forchheimer number.

CONCLUSIONS

- The magnetic field is acting as shear controlling device as with the increase in Hartmann number the shear stress at the surface of the core cylinder reduces.
- The Forchheimer number also controlling significantly the velocity gradient in the core region
- The heat convection enhanced with the increase in Brinkman number and Reynolds number.
- The heat convection decrease with the increase in Darcy number at the surface of porous cylinder.

Table 2 Skin friction at the wall of cylinder

M	Re	F	Da	G	C_f
1	10	5	0.1	-10	-4.46349
1.5	10	5	0.1	-10	-3.97079
2	10	5	0.1	-10	-3.47561
3	10	5	0.1	-10	-2.635408
3	20	5	0.1	-10	-2.6354085
3	30	5	0.1	-10	-2.63540833
3	40	5	0.1	-10	-2.63540825
3	10	1	0.1	-10	-2.635408

3	10	10	0.1	-10	-2.635408
3	10	15	0.1	-10	-2.635408
3	10	5	1	-10	-2.635408
3	10	5	0.01	-10	-2.635408
3	10	5	0.1	-1	-0.263541
3	10	5	0.1	-15	-3.953112
3	10	5	0.1	-20	-5.270817

Table 3 Skin friction at the surface of porous cylinder

M	Re	F	Da	G	C_f
1	10	5	0.1	-10	2.660946
1.5	10	5	0.1	-10	2.016756
2	10	5	0.1	-10	1.465833
3	10	5	0.1	-10	0.780006
3	20	5	0.1	-10	0.7800055
3	30	5	0.1	-10	0.78000567
3	40	5	0.1	-10	0.78000575
3	10	1	0.1	-10	0.633271
3	10	10	0.1	-10	0.974529
3	10	15	0.1	-10	1.155216
3	10	5	1	-10	0.985417
3	10	5	0.01	-10	2.91818
3	10	5	0.1	-1	0.061627
3	10	5	0.1	-15	1.31669
3	10	5	0.1	-20	1.949058

Table 4 Nusselt number at wall of the cylinder

M	Re	F	Da	G	Br	Nu
1	10	5	0.1	-10	0.01	0.69669
1.5	10	5	0.1	-10	0.01	1.03043
2	10	5	0.1	-10	0.01	1.14515
3	10	5	0.1	-10	0.01	1.07469
5	10	5	0.1	-10	0.01	0.85407
8	10	5	0.1	-10	0.01	0.5409
3	20	5	0.1	-10	0.01	4.29874
3	30	5	0.1	-10	0.01	9.67217
3	40	5	0.1	-10	0.01	17.19496
3	10	1	0.1	-10	0.01	1.07469
3	10	10	0.1	-10	0.01	1.07469
3	10	15	0.1	-10	0.01	1.07469
3	10	5	1	-10	0.01	1.07469
3	10	5	0.01	-10	0.01	1.07469
3	10	5	0.1	-15	0.01	2.41804
3	10	5	0.1	-20	0.01	4.29874
3	10	5	0.1	-10	0.05	5.37343
3	10	5	0.1	-10	0.1	10.74685
3	10	5	0.1	-10	0.2	21.4937

Table 5 Nusselt number at the surface of porous cylinder

M	Re	F	Da	G	Br	Nu
1	10	5	0.1	-10	0.01	0.24738
1.5	10	5	0.1	-10	0.01	0.45732
2	10	5	0.1	-10	0.01	1.64674
3	10	5	0.1	-10	0.01	0.80107
5	10	5	0.1	-10	0.01	0.66965
8	10	5	0.1	-10	0.01	0.38641
3	20	5	0.1	-10	0.01	3.20427
3	30	5	0.1	-10	0.01	7.20961
3	40	5	0.1	-10	0.01	12.81709
3	10	1	0.1	-10	0.01	0.88753
3	10	10	0.1	-10	0.01	0.7094
3	10	15	0.1	-10	0.01	0.63506
3	10	5	1	-10	0.01	0.9844
3	10	5	0.01	-10	0.01	0.10759
3	10	5	0.1	-15	0.01	1.69417
3	10	5	0.1	-20	0.01	2.83758
3	10	5	0.1	-10	0.05	4.00534
3	10	5	0.1	-10	0.1	8.01068
3	10	5	0.1	-10	0.2	16.02136

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