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COMPLEMENTED ELEMENTS IN TERNARYSEMIRINGS

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ABSTRACT

Since we introduced the term complemented element in a Ternarysemiring and it is proved that (1) if $p, q \in$ U such that $p \lhd q$, then $ppq = pqp = qpp = pp1$. Further, if U is simple $\Rightarrow p + q = q$. (2) If U is a zero sum free Ternarysemiring and if $l, g, h \in$ comp(U) then, (i) $lgl^{\perp}gl = 0$ (ii) llg and $l \sqsubset g \in$ comp(U) (iii) $llg = lgl = gll$. (3) Let U be the zero sum free, then (i) If $l, h \in$ comp(U) then $l + h \in$ comp(U); (ii) 1+ 1 $\in$ comp(U); (iii) comp(U) $\subseteq \Gamma^{+}$(U) ; (iv) (comp(U) , + , [ ]) is a ternary sub semi ring of U are equivalent.

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INTRODUCTION

Complemented elements play vital role in lattices, as well as frames. Here, frames are illustrations of semi rings, as it turns out, such elements play a special role in the semi ring expression of the semantics of computer programs.

Preliminaries

Def 2.1: U a Ternarysemiring, $l \in$ U, l is known as an +ve zero provided $l + x = x + l = x$ for all $x \in$ U.

Def 2.2: U a Ternarysemiring, $l \in$ U, l is known as a ***lelt*(resp. *lateral, right*) *zero*** of U provided $lbc = l$ (resp. $blc = l$, $bcl = l$) $\forall\, b,c \in$ U. $l \in$ U known as a ***two sided zero*** of U if $lbc = bcl = l\, \forall\, b, c \in$ U. $l \in$ U known as ***zero*** of U if $lbc = blc = bcl = l\, \forall\, b, c \in$ T.

Def 2.3: U a Ternarysemiring, $l \in$ U, l is known as an ***absorbing*** w. r. t addition if $l + x = x + l = l\ \forall\, x \in$ T. $0 \in$ U, 0 is known as an ***absorbing zero*** of U if $0 + x = x = x + 0$and $0ab = a0b = ab0 = 0\ \forall\, l, b, x \in$ U.

Ex 2.4: Consider the set Z^{+} with $a + b = lcm(a,b)$. Then Z^{+} is a Ternarysemiring with zero element 1, but 1 is not an absorbing zero since $1.1.a = a.1.1 = a \neq 1$ for any $a \in Z^{+}$ and $a \neq 1$.

Ex 2.5: In the power set P(X), define the addition and multiplication such that for any F, G $\in$P(X) as K + M = K$\cap$M and K.M = (K$\cup$M)\(K$\cap$M). Then P(X) is a Ternarysemiring with zero X, since K$\cap$X = K, and the unity is φ. But for any nonempty proper subset K of X we have X.K = (K $\cup$ X)\(K $\cap$ X) = X \K $\neq$ X. So X is not absorbing zero.

Def 2.6: A Ternarysemiring in which every element is a left (resp. lateral, right) zero is called a ***lelt* (resp. *lateral, right*) *zero Ternarysemiring***. A Ternarysemiring with 0 in which the product of any three elements equal to 0 is called ***a zero Ternarysemiring*** (or) ***null Ternarysemiring***.

Ex 2.7: Let $0 \in$ T$\subseteq$ R and $|T| > 2$ and Γ be the any non-empty set. Then T with the usual addition and the ternary operation defined by $xyz = x$ if $x = y = z$ and $xyz = 0$ otherwise is a Ternarysemiring with 0.

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Def 2.8: A Ternarysemiring U is known as a ***strict Ternarysemiring*** or ***zero sum lree*** provided $a + b = 0$ implies $a = 0$ and $b = 0$.

Ex 2.9: The set Z^0 is a strict Ternarysemiring.

Def 2.10: A Ternarysemiring T is said to be ***zero divisor lree*** (ZDF) if for $a, b, c \in T$, $[abc] = 0$ implies that $a = 0$ or $b = 0$ or $c = 0$.

Def 2.11: A ternarysemiring U is said to be ***semi-subtractive*** if for any elements $a; b \in$ T; there is always some $x \in$ T for some $y \in$ U such that $a + y = b$ or $b + x = a$.

Def 2.12: A Ternarysemiring U is called a half ring if the ***additive cancellation law*** holds on T, i.e. if $l + y = y + u \Rightarrow l = u$ for all $y, l, u \in$ U. The set of all cancellable elements are denoted by $K^+(U)$.

Def 2.13: A ternarysemiring U is known as ***multiplicatively lelt* (resp. *lateral, right*) *cancellative*** (MLC) (resp. MLLC, MRC) if $abl = abu$ (resp. $alb = aub$, $lab = uab$) implies that $l = u$ for all $a, b, l, u \in$ U. A Ternarysemiring U is known as be ***multiplicatively cancellative*** (MC) if it is (MLC), (MRC) & (MLLC).

Complemented Elements

Def 3.1: A lrame as frame is a complete lattice in which meets distribute over arbitrary loins

Example 3.2: If $\mathbb{B}$={O , I} . Note that the algebraic structure of $\mathbb{B}$ is not the same as that of the field $\mathbb{Z}/(2)$ since 1+1 = 1 in $\mathbb{B}$, whereas 1+ 1 = 0 in $\mathbb{Z}/(2)$. The Ternarysemiring $\mathbb{B}$ is called the ***Boolean Ternarysemiring***.

***Th 3.3[ ]:* The following conditions on a Ternarysemiring U are equivalent.**

1. U is simple
2. $l = lur + lu1 + lr1 + l \;\forall\; l, u, r \in R$
3. $l = url + ul1 + rl1 + l \;\forall\; l, u, r \in R$
4. $lur = lur + lsutr \;\; \forall\; l, u, r, s, r \in R$

Def 3.4: If $p, q \in U$ then p is ***well inside*** q, denoted by $p \lhd q$, iff $\exists\; r \in U \ni ppr = prp = rpp = 0$ & $r + q = 1$.

In any Ternarysemiring U we have $0 \lhd 0$ & $r \lhd 1 \;\forall\; r \in U$. If U is a simple Ternarysemiring then we observe that $0 \lhd q$ for any element $q \in U$. If $r \in C(U)$ then $r \lhd q$ implies that $ssr \lhd q \;\forall\; s \in U$.

Th 3.5: If $p, q \in$ U such that $p \lhd q$, then $ppq = pqp = qpp = pp1$. Further, if U is simple $\Rightarrow p + q = q$.

***Prool*:** Here, $p \lhd q$, $\exists\; r \in$ U $\ni ppr = prp = rpp = 0$ & $r + q = 1$. Hence $pp1 = pp(r + q) = ppr + ppq = ppq$. Similarly $pqp = pp1$ and $qpp = pp1$. Now can consider that U is simple. Then by the Th 3.3, we get $p + q = p(r + q)(r + q) + q = prr + prq + pqr + pqq + q = prq + pqr + (pq + 1)q = prq + prq + q = prq + (pr + 1)q = prq + q = (pr + 1)q = q$.

Def 3.6: An element $r \in$ U is said to be ***complemented*** iff $r \lhd r$. That is r is complemented iff there exist an element p such that $rrp = rpr = prr = 0$, $r + p = 1$. The element p of U is complement of r of U. Suppose r has complement implies it is unique. Suppose r is a complement element then $r^{\perp}$ is the complement of r. The complemented elements in Ternarysemiring U is denoted by *comp*(U). Comp(U) $\neq \emptyset$ because $0 \in$ comp(U) and $0^{\perp} = 1$. If comp(U) = {0, 1} the comp(U) is known as ***integral.***

Example 3.7: The ternary semi-ring ($\mathbb{I}$, max, min) is an integral. Here, comp(U) $\subseteq I^{\times}(U)$ because $l = l11 = l(l + l^{\perp})(l + l^{\perp}) = l^3 + ll l^{\perp} + l l^{\perp} l + l l^{\perp} l^{\perp} = l^3$. If $l \in$ comp (U) and construct $l \sqsubset g = l + l^{\perp}lg + l^{\perp}gl + l^{\perp}gg$. Observe that $l \sqsubset l^{\perp} = l + l^{\perp}l\, l^{\perp} + l^{\perp} l^{\perp}l + l^{\perp} l^{\perp} l^{\perp} = l + l^{\perp}l\, l^{\perp} + l^{\perp} l^{\perp}l + l^{\perp} = (l + l^{\perp}) +(l^{\perp}l\, l^{\perp} + l^{\perp} l^{\perp}l) = 1 +(l^{\perp}l\, l^{\perp} + l^{\perp} l^{\perp}l) = 1 \;\forall\; l \in$ comp(U). Further $l + g = 1$, then $l^{\perp} = l^{\perp}(l + g)(l + g) = l^{\perp}ll + l^{\perp}lg + l^{\perp}gl + l^{\perp}gg = l^{\perp}lg + l^{\perp}gl + l^{\perp}gg$, so $l \sqsubset g = l + l^{\perp}lg + l^{\perp}gl + l^{\perp}gg = l + l^{\perp} = 1$.

Th 3.8: If U is a zero sum free Ternarysemiring and if $l, g, h \in$ comp(U) then,

(i) **$lgl^{\perp}gl = 0$ (ii) llg and $l \sqsubset g \in$ comp(U) (iii) $llg = lgl = gll$.**

***Prool*:** (i) $lgl^{\perp}gl + lg^{\perp}l^{\perp} g^{\perp}l = l(g + g^{\perp})l^{\perp}(g + g^{\perp})l = ll^{\perp}l = 0$. Since U is zero sum free Ternarysemiring and hence $lgl^{\perp}gl = 0$.

(ii) First we prove that $(l \sqsubset g)^{\perp} = l^{\perp}l^{\perp}g^{\perp} + l^{\perp}g^{\perp}l^{\perp} + l^{\perp}g^{\perp}g^{\perp}$.
For this $(l \sqsubset g) + l^{\perp}l^{\perp}g^{\perp} + l^{\perp}g^{\perp}l^{\perp} + l^{\perp}g^{\perp}g^{\perp}$
$= l + l^{\perp}lg + l^{\perp}gl + l^{\perp}gg + l^{\perp}l^{\perp}g^{\perp} + l^{\perp}g^{\perp}l^{\perp} + l^{\perp}g^{\perp}g^{\perp}$
$= l + l^{\perp}[(lg + l^{\perp}g^{\perp}) + (gl + g^{\perp}l^{\perp}) + (gg + g^{\perp}g^{\perp})]$
$= l + l^{\perp}[1 + 1 + 1] = l + l^{\perp} + l^{\perp} + l^{\perp} = 1 + l^{\perp} + l^{\perp} = 1 + l^{\perp} = 1$.

Also, by condition (i), $(l \sqsubset g)\, (l \sqsubset g)\, l^{\perp}l^{\perp}g^{\perp} + l^{\perp}g^{\perp}l^{\perp} + l^{\perp}g^{\perp}g^{\perp}$
$= (l + l^{\perp}lg + l^{\perp}gl + l^{\perp}gg)(l + l^{\perp}lg + l^{\perp}gl + l^{\perp}gg)(l^{\perp}l^{\perp}g^{\perp} + l^{\perp}g^{\perp}l^{\perp} + l^{\perp}g^{\perp}g^{\perp})=0$.
Similarly, $(l \sqsubset g)(l^{\perp}l^{\perp}g^{\perp} + l^{\perp}g^{\perp}l^{\perp} + l^{\perp}g^{\perp}g^{\perp})\, (l \sqsubset g) = (l^{\perp}l^{\perp}g^{\perp} + l^{\perp}g^{\perp}l^{\perp} + l^{\perp}g^{\perp}g^{\perp})\, (l \sqsubset g)\, (l \sqsubset g) = 0$. Therefore, $(l \sqsubset g) \in$ comp(U).
Finally, we prove that $(llg)^{\perp} = l^{\perp} \sqsubset g^{\perp} = l^{\perp} + ll^{\perp}g^{\perp} + lg^{\perp}l^{\perp} + lg^{\perp}g^{\perp}$.

Here, $llg + l^{\perp} \sqsubset g^{\perp} = llg + l^{\perp} + ll^{\perp}g^{\perp} + lg^{\perp}l^{\perp} + lg^{\perp}g^{\perp} = l\,(lg + l^{\perp}g^{\perp} + l^{\perp}g^{\perp} + g^{\perp}g^{\perp}) + l^{\perp}$
$= l + l^{\perp} = 1$.
And $(llg)\, (llg)(l^{\perp} \sqsubset g^{\perp}) = (llg)(llg)(l^{\perp} + ll^{\perp}g^{\perp} + lg^{\perp}l^{\perp} + lg^{\perp}g^{\perp}) = 0$ by condition (i).
Similarly, we can show that $(llg)(l^{\perp} \sqsubset g^{\perp})\, (llg) = (l^{\perp} \sqsubset g^{\perp})\, (llg)\, (llg) = 0$.

(iii) By condition (i), $lgl^{\perp}gl = 0 = l^{\perp}glgl^{\perp}$ hence, $llg = llg(l + l^{\perp})(l + l^{\perp}) = llgll + llgll^{\perp} + llg\, l^{\perp}l + llg\, l^{\perp} l^{\perp} = llgll = llgll + l^{\perp}lgll = (l + l^{\perp})\, lgll = lgll = lgll + lgll^{\perp} = lgl(l + l^{\perp}) = lgl$.

Similarly, $llg = gll$ and hence $llg = lgl = gll$.

Th 3.9: Let U be the zero sum free, then

1. If $l, g \in$ comp(U) then $l + g \in$ comp(U);
2. $1+ 1 \in$ comp(U);
3. comp(U) $\subseteq I^{+}(U)$;
4. (comp(U), + , [ ]) is a ternary sub semi ring of U are equivalent.

***Prool*:** (1) implies (2) is obvious.

(2) implies (3): Let $l \in$ com(U), then by condition (2) $l + l \in$ com(U) and construct $g = (l + l)^{\perp}$. By Th 3.6, we get $llg + llg = l(l + l)g = (l + l)l(l + l)^{\perp}$ 0 and hence since U is zero sum free, therefore, $llg = 0$. Similarly, $lgl = gll = 0$. Therefore, $l = 1l1 =$

$(l + l + g)l(l + l + g) = l^3 + l^3 + llg + l^3 + l^3 + llg + gll + gll + glg$
$= l + l + l + l + glg = l + l + l + l + (l + l)^{\perp}l(l + l)^{\perp} = l + l + l + l$
Now if $l \notin \Gamma^+(U)$ then $l \neq l + l \neq l + l + l + l$ implies that $l \in \Gamma^+(U)$. Therefore, comp(U) $\subseteq \Gamma^+(U)$.

(1) implies (4) and (4) implies (1) is obvious by 3.7(2).

Th 3.10: Let U is zero sum free Ternarysemiring then the order relation ≤ defined as $l \leq g$ iff ∃ l, k ∈ comp(U) ∋ $l = lkg$ is a reflexive and transitive relations on U.

Prool: Obviously $l \leq l$ ∀ $l \in$ U, here $l = 11l$. Suppose $l \leq g$, $g \leq h$ implies ∃ l, k, l, m ∈ comp(U) ∋ $l = lkg$, $g = lmh$. Therefore, $l = lklmh \Rightarrow l \leq h$.

Def 3.11: Construct the set $W(U) = \{l \in U$ / if $u \in$ U then ∃ $w \in U \ni l + w = u$ or $u + w = l\}$, where U is a Ternarysemiring. If U = W(U), then the ternary semiring U is known as ***yoked*** *ternary semiring*.

Example 3.12: The sets of all natural numbers N and the set of +ve rational numbers Q^+ are yoked Ternarysemirings. The set of real numbers R with unique minimal and maximal elements 0, 1 respectively is a totally ordered, then (R, *max, min*) is a yoked ternary semiring.

Def 3.13: Let $u : X \longrightarrow U$ where $\emptyset \neq X \subseteq U$ known as the ***domain*** of u & expressed as $dom(u)$ also let V the set of all such functions, $h, l \in$ V then $h + l$ is a function domain of $dom(h) \cap dom(l)$ defined as $e \mapsto h(e) + l(e)$ and hlk defined in the same domain as $e \mapsto h(e)l(e)k(e)$ & (V, +, [ ]) is a ternary semiring. The +ve identity in which the function $e \mapsto 0$ and [ ]ve identity in which the function $e \mapsto 1$ with domain U. Further, we observe that if $h \in$ V & if $-h \in$ V is the function from $dom(h) \rightarrow$ U expressed as $e \mapsto -h(e)$ hence $-h$ is the +ve inverse of h only if $dom(h)$ = U. Therefore, M(U) = $\{h \in$ V / $dom(h)$ = U$\}$ where M(U) is the set of all +ve inverses of U.

Th 3.14: Suppose A, B & C ternary sub hemi rings of a yoked Ternarysemiring U such that ABC ⊆ M(U) then either A^3 ⊆M(U) or B^3 ⊆ M(U) or C^3 ∈ M(U).

Proof: Suppose, A^3 , $B^3 \not\subseteq$ M(U) . Then ∃ $u, v, w \in$ A, $x, y, z \in$ B ∋ $uvw \notin$ M(U) & $xyz \notin$ M(U). Let $p, q, r \in$ C, ∃ $s, t \in$ U ∋ $u + x + s = p$, $v + y + t = q$ then $uvw + xyz + vys + wzt = vyp + wzq \in$ ABC ⊆ M(U), therefore, uvw, $xyz \in$ M(U) which is a contradiction. Here, U is yoked Ternarysemiring. Therefore, there must exists $s, t \in$ U ∋ $u = x + s + p$ & $v = y + t + q$. But $uqr = xqr + sqr + pqr$ & $wvr = wyr + wtr + wqr \in$ ABC ⊆ M(U). Therefore, $pqr \in$ M(U) ⇒ C^3 ⊆ M(U).

CONCLUSION

Here, we mainly studied about complements of Ternarysemirings.

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