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MATHEMATICAL MODEL OF A FOUR-BEAM TURBIDITY SENSOR

Bozhilov, R. M

Department of Electronics, Technical University Sofia, Branch Plovdiv, Bulgaria

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ABSTRACT

Fresh water importance is increasing worldwide. The only way to provide the demanded amounts of fresh water is by means of increasing the amounts of processed and purified water. Turbidity measurements are accomplished at all stages of water processing. This paper proposes a mathematical model of a four-beam turbidity sensor. The transmission functions of both the sensor and the measured medium have been analytically described and evaluation of the processes of light transmission and light scattering has been carried out. The availability of a mathematical model of a four-beam turbidity sensor would facilitate its analysis and its construction improvement.

INTRODUCTION

The most specific part of the four-beam turbidity sensor is its optical sensor. For each measurement four light paths are formed in the four-beam sensor and four photo-currents are generated respectively. Two of them are defined by the intensity of the transmitted light and the other two - by the intensity of the scattered by the sample light. The electronic block of the device measures the four photo-currents, forms a ratio of the signals, related to the transmitted light, to the ones, corresponding to the scattered light, and carries out additional mathematical processing. The output variable of the device, by means of which conclusions about the measured turbidity are drawn, does not depend directly on the magnitude of the measured photo-currents but on their ratio instead (Johnson, 2003). Thus the main advantage of these devices, working by the ratio method, is achieved - reduced sensibility of the result from turbidity measurement toward the color of the sample, the contamination of the optics and the level of degradation of the optical devices (Sadar, 1998).

The mathematical model of the four-beam sensor analytically describes the relationship between the generated photo-currents and the magnitude of the measured turbidity, depending on the parameters of the used optical devices the geometric dimensions of the sensor. For the analytical description it is necessary to derive the transmission function for each of the four light paths. In case of a symmetric 90^0 construction of the sensor, Fig. 1, and uniformity of the corresponding LEDs - D1, D2 – and photodiodes - D_A, D_B, the transmission functions coincide two in two. Therefore it is only necessary to define one transmission function, describing the process of light transmission and one function, evaluating the process of light scattering in the measured sample.

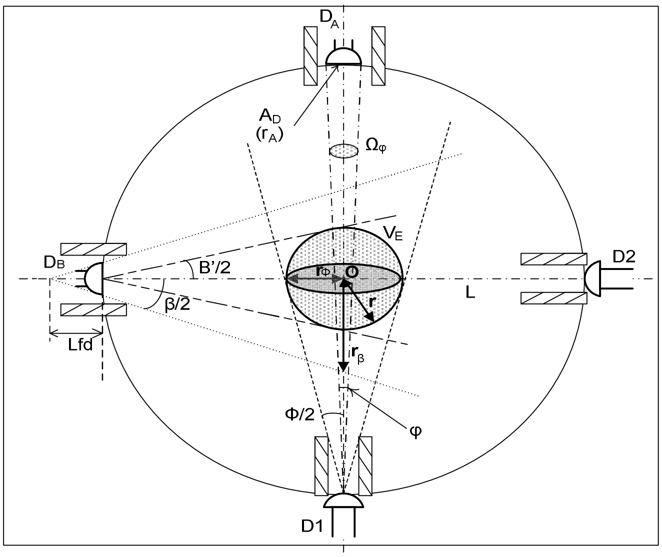

Fig 1 Four-beam turbidity sensor geometric parameters, defining the processes of light transmission and light scattering in a solution

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*Corresponding author: **Bozhilov, R. M**
Department of Electronics, Technical University Sofia, Branch Plovdiv, Bulgaria

In order to obtain the analytical description of the transmission functions, realized by the four-beam sensor, the transmission functions of the following elements, defining them, have been sequentially determined: of the infrared light emitting diode; the silicon photodiode; the basic geometric parameters of the sensor and the measured liquid medium.

Defining the transmission function of an infrared light emitting diode

The transmission function of a light emitting diode (LED) is an analytical expression, describing the output signal of the device by the factors, defining it. The average luminous intensity Iav at a given spatial solid angle is taken as an output signal. The average luminous intensity identically illuminates every point of the spatial solid angle by which it is defined. The average value of the luminous intensity Iav of a LED depends on the current, running through the forward-biased junction I_F, on the ambient temperature t^o, on the magnitude of the spatial solid angle Ω under which the studied sample is illuminated, on the intensity of the output radiation along the optical centerline Io_o, and on the distribution of this radiation in space. Except for the given spatial solid angle, the other parameters, defining the transmission function of the LED, are typically provided by its manufacturer in tabular or in graphical form, e.g., for LED L-53SF7C (KINGBRIGHT datasheet). The device complies with the requirements of ISO7027 toward a light source for measuring turbidity, as to maximum wavelength, width of the radiation spectrum, and narrow angle of radiation. In order to use and process analytically the graphical and tabulated data of a given device, the data should be converted into corresponding mathematical expressions.

A methodology has been developed for practical derivation of the LED transmission function by using polynomials of first order to approximate either the graphical or the tabulated data. The dependencies between the output luminous intensity Io_o on the forward current I_F and the ambient temperature t^o are linear and, respectively, described by two coefficients each - a_1, b_1 and a_3, b_3. The LED typically radiates symmetrically in space with respect to the optical centerline, that is, it shines equally in the horizontal and vertical plane with respect to the centerline. Then the spatial distribution of a LED is only described by one graph of the dependence on a planar angle in a circular coordinate system. When moving from a circular coordinate system to a Cartesian one, it is established that the dependence of the spatial distribution on the planar angle, defining it, is close to a linear one. It is suitable to approximate with a straight line, defined by two coefficients - a_2, b_2. In order to calculate the average luminous intensity, the indicatrix of spatial distribution should be integrated in a spatial solid angle Ω, defined by the given planar angle θ. For this purpose the relationships between the planar and spatial angle are used in normal and differential form (Мирошников, 1984). Using the methodology for practical derivation of the transmission function of a light emitting diode, the following expression is obtained, showing the relationship between the average luminous intensity and the factors, defining it:

$$Iav(I_F,t^o,\theta)=\frac{(a_1.I_F+b_1).(a_3.t^o+b_3).(1-a_2.\theta.\cos\theta+a_2.\sin\theta-b_2.\cos\theta)}{2.\pi.(1-\cos(\frac{\theta}{2}))} \quad (1)$$

Using this approximation, the calculated value for the average luminous intensity will not exceed its real value, which is appropriate for practical engineering calculations.

Defining the transmission function of a photodiode

The transmission function of a photodiode expresses the relationship between the magnitude of its output electric signal and the power of the incident light radiation Pr. This relationship is conditioned by a number of factors. Depending on the working mode and on the load resistance of the photodiode, its output electric signal is either voltage or current. From a metrological point of view, the most appropriate working mode of the photodiode for carrying out photometric measurements is its short circuit operation, when the output signal of the device is current - I_p.

The mathematical expression, describing the transmission function of a photodiode, is easily obtained based on the data, published by the manufacturers, and characterizing the device:

$$I_{T/S}=S_\lambda.\mathrm{Pr} \quad (2)$$

The photo-current is denoted by I_T for the case when the transmitted light is measured, while I_S denotes the current in case of evaluating the scattered light. Responsivity S_λ in (2) is defined for a particular device from graphs, given in the technical specification of the device, for a given wavelength λ of the incident light.

Analytical description of the processes of light transmission and light scattering in liquid solutions during measurements by a four-beam sensor

The transmission function of the optically evaluated medium when it is measured by a four-beam sensor, expresses the relationship between the optical power Pr of the light flux, going out of the medium and illuminating the photodiode, and the average intensity Iav, irradiating the medium. Significant factors, influencing the transmission function of the medium are: the geometric parameters of the sensor and the specificity of the optical properties of the medium itself in the processes of light transmission and light scattering. The turbidity of the analyzed sample – M at fixed physical and chemical properties of the suspended solids in the solution is directly proportional to the concentration of the undissolved particles in the measured medium.

Defining the transmission function of the medium when the transmitted light is measured

When evaluating the passage of light through the solution, the light emitting diode D1radiates and the photodiode D_A receives (or the pair D2 – D_B, respectively), Fig. 1. The light emitting diode is a point source of light and the flux, created by it, irradiates the sensitive area of the photodiode and propagates in the spatial solid angle Ω_φ. A planar angle φ, defined by the radius of the photodiode r_A and the distance L, corresponds to the spatial solid angle Ω_φ. When going through the measured solution, the incident average intensity Iav changes only its magnitude as it is absorbed. The light flux, going out of the solution, has intensity It, which is also expressed by its average value. By definition, the direction of propagation of the intensity It is just the same as of the irradiating intensity – and it propagates in the same spatial solid angle Ω_φ. The expression, defining the light flux power Pr_t illuminating the photodiode during a process of light transmission through a solution, expressed by means of the basic geometric parameters of the four-beam turbidity sensor, is:

$$\mathrm{Pr}_t = 2.\pi.(1-\cos(arctg\frac{r_A}{L})).It \qquad (3)$$

The theoretical dependence, describing the relationship between the intensity It, going out of the solution, and the intensity of irradiation Iav in the process of light transmission through a solution, is based on Beer-Lambert-Bouguer law. The expression is a damping exponential function with an exponent coefficient, depending on: α – the extinction coefficient – defined by the measured medium; the distance L, passed by the light flux; and the turbidity M. The intensity It, going out of the solution, is determined by the formula:

$$It = Iav.e^{-\alpha.L.M} \qquad (4)$$

Defining the transmission function of the medium when the scattered light is measured

The process of light scattering in a solution occurs when the LED D1 emits radiation and irradiates the sample under an aperture angle Φ, and the photodiode D_B is illuminated at an angle β. The pair D2 – D_A interacts analogically. The planar aperture angle Φ defines a corresponding to itself spatial angle of irradiation, while the planar angle β defines a corresponding spatial angle of illumination. The real dimensions of the planar angles Φ and β are defined by the diameter of the additionally placed in front of the optical devices apertures.

The total spatial cross-section of the two volumetric angles with mutually perpendicular axes forms a volume V, where the phenomenon of light scattering in a solution occurs. The volume V is a closed space formed by the intersection of two different cones. Volume V has a qualitative rather than a quantitative role in turbidity measurement. The accurate determination of the volume as a section of two different cones is a difficult and optional task. So it is assumed that the volume V is represented by an equivalent volume – a simple geometric shape – a sphere, denoted by V_E. The sphere V_E has a central point O and a radius r. The volume of the sphere V_E is part of the volume V, proportional to its size and defined by the same geometrical parameters. The actual radius of the sphere V_E is equal to the smaller of the two radii - r_Φ or r_β. If the four-beam sensor has a typical construction, compliant with ISO 7027 (International standard, 1999), the planar aperture angle of irradiation Φ is considerably smaller than the planar angle of illumination β, the radius r_Φ is smaller than the radius r_β, so r_Φ defines V_E. The sphere V_E, in which the process of light scattering occurs, determines a new resultant angle of illuminating the photodiode - $\beta' = \Phi$. The geometric center (point O) of the sensor is the representative point of the process of light scattering by the whole volume V_E.

The expression, defining the light flux power Pr_s, incident on the photodiode in the process of light scattering in a solution, by means of the irradiating intensity $\mathrm{Iav}(\Omega_\Phi)$, solution turbidity M and the basic geometrical parameters of a four-beam turbidity sensor, is:

$$\mathrm{Pr}_s = \frac{\pi^3}{3.L}.k_s.r_A^2.[1-\cos(\frac{\Phi}{2})].[tg(\frac{\Phi}{2})]^3.Iav(\Omega_\Phi).M \qquad (5),$$

where k_s is a non-dimensional constant, characterizing the process of scattering.

The standard theoretical model, describing the phenomenon of light scattering in a solution is based on Rayleigh law, according to which each particle from the interacting volume participates only once in the process of scattering. In result, the intensity of the light, scattered by a sample, is always proportional to the turbidity in the solution. This standard theoretical model contradicts to previous practical experience, as well as to the published in (Sadar, 1998) dependencies of the intensity of the scattered light on turbidity at various angles of detection. The results, obtained practically, show that a strictly linear relationship between the scattered light intensity and turbidity exists only in diluted solutions. When measuring turbidity in a wide range of turbidity, the relationship is strongly non-linear, with an explicit maximum at a definite value of turbidity - Mmax.

In order to carry out correct measurements of turbidity in a solution in a wide range of variation, an improved theoretical model is proposed, giving mathematical description of the process of light scattering. According to the proposed improved theoretical model, each particle from the solution participates in two single interactions. The first interaction is the light scattering by the particle on Is, and the second, subsequent one, is the absorption of the intensity of the scattered light flux. The process of light scattering is accomplished only by the particles, located in the defined volume V from the whole solution, represented by the sphere V_E. The subsequent absorption of the scattered already light flux occurs along the whole remaining path till reaching the photo-receiver. The improved theoretical model, describing light scattering by double interaction, is analytically presented by an expression, containing two factors: the first shows the proportionality of the scattered light flux intensity from the turbidity; the second expresses the exponential reduction of the output intensity by the turbidity of the sample. The proposed analytical expression, describing the dependence of the output luminous intensity Is on turbidity M of the sample when measuring the scattered light by a four-beam sensor, is:

$$Is = k_s.\frac{V_E}{L^4}.e^{-\alpha.\frac{L}{2}.M}.Iav(\Omega_\Phi).M \qquad (6)$$

Transmission function of a four-beam sensor and the medium in the process of evaluating the transmitted light through a solution

The transmission function of a four-beam turbidity sensor when evaluating the process of light transmission is obtained by consecutive substitution of: - expression (1) of the average luminous intensity $\mathrm{Iav}(\Omega_\varphi)$ of the irradiating light flux (in this case φ corresponds to angle θ) into expression (4); - the obtained result for It into (3) and the value of the optical power Pr_t into expression (2). After processing it is obtained:

$$I_T = S_{\lambda}.e^{-\alpha.L.M}.(a_1.I_F + b_1).(a_3.t^o + b_3).[1 - 2.a_2.(arctg\frac{r_A}{L}).\cos(2.arctg\frac{r_A}{L}) \quad (7)$$
$$+ a_2.\sin(2arctg\frac{r_A}{L}) - b_2.\cos(2.arctg\frac{r_A}{L})]$$

The expression represents the detailed relationship between the magnitude I_T of the current, generated by a photodiode, and the turbidity of the solution M, in dependence on the parameters of a four-beam sensor. The values of the parameters $S_{(\lambda)}$, the constants a_1, b_1, a_2, b_2, a_3, b_3, r_A, and L are either known or preliminary determined for a specific turbidity sensor. In order to determine uniquely the relationship between the photodiode current I_T and the turbidity of the solution M, the value of the parameter α should be known as well. There is no theoretical prediction of the value of the extinction coefficient α for a given solution at a predetermined sensor. The only way for defining the value of coefficient α is by practical measuring the current I_T, corresponding to the intensity of the light, transmitted through the solution by means of the specific four-beam sensor for at least one known value of the sample turbidity M. The measured turbidity value should be in the middle of the range of measuring turbidity for the specific sensor. The value of α is calculated by using the known value of turbidity M, the corresponding result for the current I_T and by corresponding substitution in the rearranged expression (7). After substituting the obtained value of the extinction coefficient α into expression (7), the transmission function of the specific sensor is obtained when evaluating the transmitted light.

Transmission function of a four-beam sensor and the medium in the process of evaluating the light , scattered by a solution

The transmission function of a four-beam turbidity sensor when evaluating the process of light scattering is obtained by consecutive substitution of: - the expression (1), describing the average intensity $Iav(\Omega_{\Phi})$ of the irradiating light flux (in this case Φ corresponds to angle θ) into expression (6); - the obtained result for Is into (3) and the value of the optical power Pr_s into expression (2). After processing it is obtained:

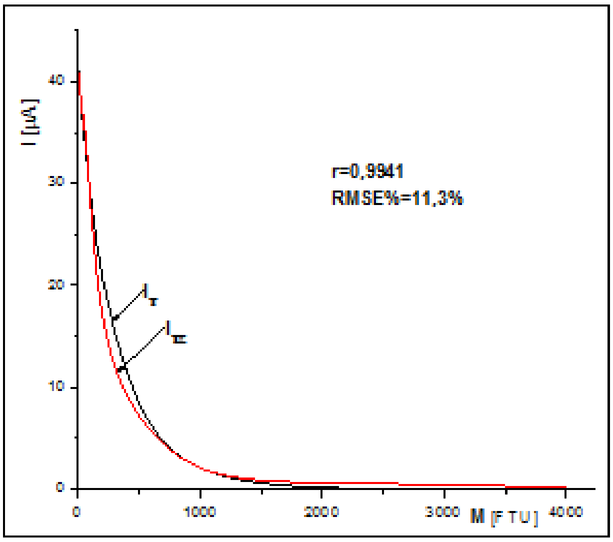
$$I_S = \frac{\pi^2}{6.L}.k_s.r_A^2.S_{\lambda}.[tg(\frac{\Phi}{2})]^3.e^{-\alpha.\frac{L}{2}.M}.(a_1.I_F + b_1).(a_3.t^o + b_3). \quad (8)$$
$$(1 - a_2.\Phi.cos\Phi + a_2.\sin\Phi - b_2.\cos\Phi).M$$

Analogically to the situation, concerning expression (7), the constants, describing the construction of the particular four-beam sensor are known. The value of the extinction coefficient α is defined on the basis of expression (7) during experimental measurement of the transmitted through the sample light. The problem with determining the value of the coefficient of proportionality k_s is similar. Its value cannot be predicted theoretically. To define k_s at least one practical measurement by means of the specific four-beam sensor for finding out the value of the intensity Is is needed at preliminary known value of turbidity M for the evaluated solution. The value of k_s is calculated based on the revised expression (8). An estimate of the reliability of the obtained transmission function has been made by the turbidity value, at which the coefficient of proportionality k_s has been practically defined. The lowest values of RMSE for the defined transmission function with respect to the experimental results are obtained when the coefficient k_s is defined at turbidity value M, corresponding to the middle point of the range for the particular four-beam sensor, between zero and Mmax.

Verification of the reliability of the proposed mathematical model of a four-beam turbidity sensor

In order to verify the defined functions, an assessment of the correspondence between the theoretically predicted by them data and the experimentally obtained results, defined under the same determining them conditions, has been made. For this purpose an experimental four-beam sensor with variable geometry was developed. Two light emitting diodes L-53SF7C type were used as infrared senders, and the two photodiodes were S1337-1010BR (Hamamatsu, 2016). Three values of the current I_F through the LEDs were chosen: 30mA, 20mA and 10mA. The distance L was predetermined at the following levels - 95mm, 75mm and 55mm, keeping the symmetric geometry of the sensor. The angle Φ was changed to 25°, 20^0, 15° and 10° by adding corresponding apertures to the LEDs. Reference formazine standard solutions with decreasing value of turbidity were used for the certified comparative material. The basic formazine solution of 4000FTU was prepared in compliance with ISO7027.

Comparison between the theoretically predicted and experimentally obtained results has been carried out for a number of different relationships, defining the generated photo-currents by: the turbidity; the current through the LEDs; the geometric parameters of the sensor. For the purpose of reducing the fluctuation of the experimental data, each value of the measured photo-currents $I_{T\Sigma}$ or $I_{S\Sigma}$ is the average of six individual measurements. Fig. 2(a) and 2(b) show the graphs of the theoretically predicted and the experimentally obtained dependencies for the photo-current, corresponding to the intensity of the transmitted or the scattered light, on turbidity. The practical results were obtained at "middle" parameters, describing the sensor - I_F=20mA, L=75mm and $\Phi=20^0$. The raised values of the experimental results at low levels of the measured signal are due to the influence of stray light – parasitic additional reflection within the walls of the experimental compartment.



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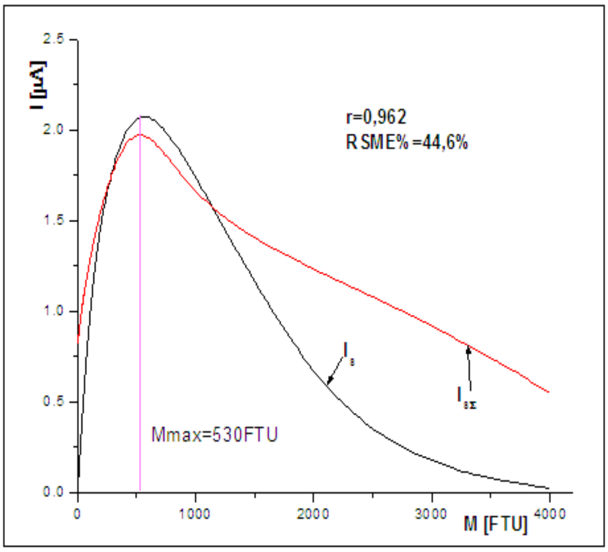

b

Fig 2 Graphs of the theoretically predicted and experimentally obtained dependencies for the photo-current, corresponding to the intensity of: - the transmitted light - (a); - the scattered light - (b), on turbidity.

The value of the correlational coefficient r – close to unity – and the acceptable for practical sizing dimensions of the relative RSME% for all drawn comparisons, defines a high level of correspondence between the theoretically predicted data and the experimentally obtained results. The high level of correspondence is a proof of the reliability of the proposed detailed theoretical model, describing the process of light scattering in a solution, as well as a confirmation of the correctness of the mathematical model of a four-beam sensor.

The mathematical model of one specific four-beam sensor is a basis for assessing unknown values of turbidity for a similar type of a solution at various geometric parameters of the sensor, different photodiodes or different infrared light emitting diodes, but with the same spectrum of radiation. The derived transmission functions, describing the mathematical model of a four-beam sensor, define in absolute units the values of the currents, generated by the photodiodes. The boundary values of the assessed turbidity, which can be measured at a given ratio of the generated information signal to the noise current (SNR) or the current in the dark for the used photodiodes and at a given temperature range, can be calculated on this basis. The mathematical model of a four-beam sensor is a basis for developing a mathematical model of a four-beam turbidity meter. The mathematical model of a turbidity meter can be used for: assessment of the contamination effect on the optical system or the influence of the ambient light; comparison or creation of new constructive solutions and methods for signal processing in four-beam structures of turbidity meters etc.

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