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RESEARCH ARTICLE

INDEPENDENT MIDDLE DOMINATION NUMBER IN GRAPHS

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ABSTRACT

The Middle graph of a graphG, denoted by M(G) ,is a graph whose vertex set is V(G)∪E(G),and two vertices are adjacent if they are adjacent edges of G or one is a vertex and other is a edge incident with it . A set S of vertices of graph M(G) is an independent dominating set of M(G) if S is an independent set and every vertex not in S is adjacent to a vertex inS. The independent middle domination number of G , denoted by iM(G) is the minimum cardinality of an independent dominating set ofM(G) .

In this paper many bounds on iM(G) were obtained in terms of the vertices ,edges and other different parameters of G and not in terms of the elements of M(G).Further its relation with other different domination parameters are also obtained.

INTRODUCTION

In this paper, all the graph considered here are simple, finite, nontrivial, undirected and connected. The vertex set and edge set of graph G are denoted by $V(G) = p$ and $E(G) = q$ respectively. Terms not defined here are used in the sense of Harary [1].

The degree, neighbourhood and closed neighbourhood of a vertex v in a graph G are denoted by $deg(v), N(v), and\ N[v] = N(v) \cup \{v\}$ respectively. For a subset S of V, the graph induced by $S \subseteq V$ is denoted by $\langle S \rangle$.

As usual, the maximum degree of a vertex (edge) in G is denoted by $\Delta(G)\left(\Delta'(G)\right)$. For any real number x, $\lceil x \rceil$denotes the smallest integer not less than x and $\lfloor x \rfloor$denotes the greater integer not greater than x .

A vertex cover in a graph G is a set of vertices that covers all the edges of G.The vertex covering number $\alpha_o(G)$is the minimum cardinality of a vertex cover in G.A set of vertices/edges in a graph G is called independent set if no two vertices/edges in the set are adjacent. The vertex independence number $\beta_0(G)$ is the maximum cardinality of an independent set of vertices. The edge independence number $\beta_1(G)$of a graph G is the maximum cardinality of an independent set of edges.

A Line graph $L(G)$is a graph whose vertices corresponds to the edges of G and two vertices in $L(G)$ are adjacent if and only if the corresponding edges in G are adjacent.

A subdivision of an edge $e = uv$ of a graph G is the replacement of the edge e by a path (u, v, w). The graph obtained from a graph G by subdividing each edge of G exactly once is called the subdivision graph of G and is denoted by $S(G)$.

A set D of graph $G = (V, E)$ is called a dominating set if every vertex in $V - D$ is adjacent to some vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality taken over all dominating set of G.

A set F of edges in a graph G is called an edge dominating set of G if every edge in $E - F$ is adjacent to at least one edge in F. The edge domination number$\gamma'(G)$ of a graph G is the minimum cardinality of an edge dominating set of G . Edge domination number was studied by S.L.Mitchell and Hedetniemi in [2].

A dominating set D of a graph G is a strong Split dominating set if the induced subgraph $\langle V - D \rangle$ is totally disconnected with only two vertices. The Strong Split domination number $\gamma_{ss}(G)$ of a graph G is the minimum cardinality of a strong split dominating set of G. See [4].

A dominating set D of a graph $G = (V, E)$ is a paired dominating set if the induced subgraph $\langle D \rangle$ contains at least one perfect matching. The paired domination number $\gamma_p(G)$of a graph G is the minimum cardinality of the paired dominating set.[4].

A dominating set D of a graph $G = (V, E)$ is an independent dominating set if the induced subgraph $\langle D \rangle$ has no edges. The independent domination number $i(G)$ of a graph G is the minimum cardinality of an independent dominating set.

Analogously, we define Independent Middle DominationNumber In Graphs as follows.
A set S of vertices of graph $M(G)$ is an independent dominating set of $M(G)$ if S is an independent set and every vertex not in S is adjacent to a vertex in S. The independent domination number of $M(G)$, denoted by $i_M(G)$ is the minimum cardinality of an independent dominating set of$M(G)$.

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In this paper many bounds on $i_M(G)$ were obtained and expressed in terms of the vertices ,edges and other different parameters of G but not in terms of members of $M(G)$. Also we establish Independent Middle Domination number in graphs and express the results with other different domination parameters of G.

We need the following Theorems:

Theorem A [4]: For any connected graph G,with$p \geq 4$,$G \neq K_p$,$\gamma_{ss}(G) = \alpha_0(G)$.

Theorem B [4]: For any connected graph , $\left\lceil \frac{p}{1+\Delta} \right\rceil \leq \gamma(G)$.
Theorem C [5]:For any graph , $i[L(G)] \leq \beta_1(G)$.
Theorem D [4]: For any graph G with no isolated vertices $\gamma_p(G) \leq 2\beta_1(G)$.

Theorem E [3]: For any graph G ,$\gamma[M(G)] = i_M(G)$, where $M(G)$ denotes the middle graph of$G$.

RESULTS

We list out the exact values of $i_M(G)$for some standard graphs.

Theorem 1

a. For any path P_p with $p \leq 2$ vertices

$$i_M(G) = \frac{p}{2} \text{ if } p \text{ is even.}$$
$$i_M(G) = \left\lceil \frac{p}{2} \right\rceil \text{ if } p \text{ is odd .}$$

b. For any cycle C_p

$$i_M(G) = \frac{p}{2} \text{ if } p \text{ is even .}$$
$$i_M(G) = \left\lceil \frac{p}{2} \right\rceil \text{ if } p \text{ is odd .}$$

c. For any star $K_{1,p}$,

$$i_M(G) = p - 1.$$

d. For any Wheel W_p

$$i_M(G) = \frac{p}{2} \text{ if } p \text{ is even .}$$
$$i_M(G) = \left\lceil \frac{p}{2} \right\rceil \text{ if } p \text{ is odd .}$$

Theorem 2: Let G be a connected graph of order$p \geq 2$, then$i_M(G) \geq \left\lceil \frac{p}{2} \right\rceil$.

Proof : Let $V(G) = \{v_1, v_2 \dots \dots \dots \dots . v_p\} = p$ and $E(G) = \{e_1, e_2 \dots \dots \dots \dots \dots e_q\} = q$. Now, by the definition of middle graph $V[M(G)] = p + q$ .Let D be the minimal independent dominating set of $M(G)$.Let $D_1 = D \cap V(G)$ and $D_2 = D \cap [V[M(G)] - V(G)]$.Now we consider the following cases….

Case 1: Suppose$D_2 = \emptyset$, then $D_1 = V(G)$ and hence$i_M(G) \geq \left\lceil \frac{p}{2} \right\rceil$.

Case 2: Suppose $D_1 = \emptyset$ .Since each vertex of D dominates exactly two vertices of$V[M(G)] - D$, then it follows that$|V[M(G)] - D| = 2|D|$. Hence $p + q - |D| \leq 2|D|$ so that $|D| \geq \left\lceil \frac{p}{2} \right\rceil$.

Case 3: Suppose D_1 and D_2 are non - empty, since each vertex of D_2 dominates exactly two vertices of M(G) , then it follows that $p - |D_1| \leq 2|D_2|$. Hence $p \leq |D_1| + 2|D_2| \leq 2|D|$.Thus $i_M(G) = |D| \geq \left\lceil \frac{p}{2} \right\rceil$.

The next Theorem gives the relationship between domination number of G and $i_M(G)$.

Theorem 3: For any connected graph$G$, $\gamma(G) \leq i_M(G)$.

Proof: Let $V_1 = \{v_1, v_2 \dots \dots \dots \dots \dots v_n\} \subseteq V(G)$ be the set of all the vertices with $deg(v_i) \geq 2, \forall\, v_i \in G, 1 \leq i \leq n$. Then there exist a minimal set$S \subseteq V_1$, such that $[S] = V(G)$. Clearly S forms a minimal dominating set of G with $|S| = \gamma(G)$. Now by the definition of M(G) , $V[M(G)] = V(G) \cup E(G)$. Consider a minimal set of vertices $D_1 \subseteq V[M(G)]$ such that $|D_1| = \gamma[M(G)]$. If $\langle D_1 \rangle$ is totally disconnected,then D_1 itself forms the independent dominating set of $M(G)$.Otherwise ,we consider a set $D_2 = D_1' \cup D_2'$,where $D_1' \subseteq D_1$ and $D_2' \subseteq V[M(G)] - D_1$,such that for all $v_i \in \langle D_1' \cup D_2' \rangle$, $\deg v_i = \mathbf{0}$. Thus $D_1' \cup D_2'$ is the minimal independent dominating set of (G) . Since$V(G) \subset V[M(G)]$, we have$|D_1' \cup D_2'| \geq |S|$. Hence $\gamma(G) \leq i_M(G)$.

Theorem 4:For any non-trivial connected graph$G$,$i_M(G) + \gamma_{ss}[M(G)] \leq p + q$.

Proof: Since $i_M(G) \leq \beta_0[M(G)]$
Also from Theorem A$\gamma_{ss}[M(G)] = \alpha_0[M(G)]$
Further $i_M(G) + \gamma_{ss}[M(G)] \leq \beta_0[M(G)] + \alpha_0[M(G)]$

$$= V[M(G)]$$
$$= V(G) \cup E(G)$$
$$= p + q$$

Hence$i_M(G) + \gamma_{ss}[M(G)] \leq p + q$.

Theorem 5: Let G be any connected graph, then$i_M(G) = \alpha_1(G)$.

Proof: Let $E_1 = \{e_1, e_2 \dots \dots \dots \dots \dots \dots \dots \dots e_n\} \subseteq E(G)$ be the minimal set of edges in G such that$|E_1| = \alpha_1(G)$. Since$V[M(G)] = V(G) \cup E(G)$,let $S = \{s_1, s_2, s_3 \dots \dots \dots \dots \dots \dots s_k\}$ be the set of vertices subdividing the edges of G in $M(G)$.Now let $S_1 = \{s_1, s_2, s_3 \dots \dots \dots \dots \dots s_i\} \subseteq S, 1 \leq i \leq k$ be the vertices subdividing each edge $e_i \in E_1(G)$,$1 \leq i \leq n$.Thus $N(S_1) = V(G)$ and also in $M(G)$, $N(S_1) = V(S - S_1)$.Thus $N[S_1] = V(G) \cup V(S - S_1) = V[M(G)]$.Hence clearly $\langle S_1 \rangle$ forms the minimal dominating set of $M(G)$,such that $|S_1| = \gamma[M(G)]$.Further ,by the Theorem E ,we have $\gamma[M(G)] = i_M(G)$.Therefore $i_M(G) = |S_1| = |E_1|$,which gives $i_M(G) = \alpha_1(G)$.

Theorem 6: For any complete bipartite graph$K_{m,n}$, $i_M(K_{m,n}) = n$, $for\ n \geq m$.

Proof: Let (X, Y) be a bipartition of $K_{m,n}$,$n \geq m$ with $|X| = m$and$|Y| = n$. Let $X = \{x_1, x_2, x_3 \dots \dots \dots \dots \dots \dots x_m\}$and $Y = \{y_1, y_2, y_3 \dots \dots \dots \dots \dots y_n\}$.Let $E_1 = \{x_i y_j / 1 \leq i \leq m , 1 \leq j \leq n\}$ be the independent edges in$K_{m,n}$.Clearly $|E_1| = min(m, n) = m$ .In $M(G)$, let $S = \{v_1, v_2, v_3 \dots \dots \dots \dots \dots \dots v_k\}$ be the vertices subdividing each edge of G in $M(G)$.Consider a set $S_1 = \{v_i / 1 \leq i \leq k\} \subseteq S$ be the vertices subdividing the edges of E_1.Clearly S_1 is an independent set of vertices in $M(G)$.Now,let $Y_1 = \{y_j / y_j = N(v_i), for\ each\ v_i \in S_1\}$.Clearly $|Y_1| = m$. Without loss of generality,$Y_2 = Y - Y_1$ is an independent set of vertices in $M(G)$.Now , $N(S_1) = X \cup V(S - S_1) \cup Y_1$ and hence $N[S_1 \cup Y_2] = V[M(G)]$.Since $\langle S_1 \cup Y_2 \rangle$ is independent ,thus the induced sub graph $\langle S_1 \cup Y_2 \rangle$ is a minimal independent dominating set in $M(G)$.Clearly $|S_1| = |E_1| = m$ and $|Y - Y_1| = n - m$.Therefore $|S_1 \cup Y_2| = |S_1| + |Y_2| = m + n - m = n$.Hence $i_M(K_{m,n}) = n$ *where* $n \geq m$.

Theorem 7: For any connected (p,q)graph,$\left\lceil \frac{p}{1+\Delta(G)} \right\rceil \leq i_M(G)$.

Proof: By Theorem B and also by the Theorem 3,we have the required result.

The following Theorem relates $i_M(G)$ and the independent domination number in line graph $i[L(G)]$ in terms of the vertices of G

Theorem 8: For any connected non trivial graph $G, i_M(G) + i[L(G)] \leq p$.

Proof: Let G be a connected graph.

By the Theorem C, we have $i[L(G)] \leq \beta_1(G)$
Also by the Theorem 5, $i_M(G) = \alpha_1(G)$
Hence $i_M(G) + i[L(G)] \leq \alpha_1(G) + \beta_1(G)$

$$= V(G)$$
$$= p$$

Therefore $i_M(G) + i[L(G)] \leq p$.

Theorem 9: For any connected (p,q)graph G,$i[L(G)] \leq i_M(G)$.

Proof: Suppose $D = \{v_1, v_2 \dots \dots \dots \dots \dots \dots v_n\} \subseteq V[L(G)]$ be the minimal set of vertices such that $N[D] = V[L(G)]$.Then D is the minimal dominating set of $L(G)$.Further if $\langle D \rangle$ contains the set of vertices v_i ,$1 \leq i \leq n$,such that $deg v_i = 0$,then D itself forms the independent dominating set of $L(G)$.Otherwise let $S = D' \cup I$ where $D' \subseteq D$ and $I \subseteq V[L(G)] - D$,such that for all $v_i \in \langle D' \cup I \rangle$,$deg v_i = 0$,then S forms an independent dominating set of $L(G)$.Since $V[M(G)] = V(G) \cup E(G)$,then clearly $L(G)$ is an induced subgraph of $M(G)$.Hence $S \subseteq V[M(G)]$.If $N[S] = V[M(G)]$ then $\langle S \rangle$ itself forms an independent dominating set of $M(G)$.Otherwise ,we consider a set $D_2 = V[M(G)] \cap V[L(G)]$ and $D_2' = V[M(G)] \cap V(G)$,such that $N[D_2 \cup D_2'] = V[M(G)]$ and $\langle D_2 \cup D_2' \rangle$ is totally disconnected. Thus clearly $|S| \leq |D_2 \cup D_2'|$ which gives $i[L(G)] \leq i_M(G)$.

The following Theorem relates $i_M(G)$ *and* $\gamma'(G)$ in terms of the vertices of G.

Theorem 10: For any connected graph G,$i_M(G) \leq p - \gamma'(G)$.

Proof: Let $E_1 = \{e_1, e_2, e_3, \dots \dots \dots \dots \dots \dots e_q\} \subseteq E(G)$ be the minimal set of edges, such that for each $e_i \in E_1, i = 1,2,3, \dots \dots \dots \dots \dots q$,$N(e_i) \cap E_1 = \phi$.Then $|E_1| = \gamma'(G)$.In $M(G)$,$V[M(G)] = V(G) \cup E(G)$.Let

$D = \{v_1, v_2, v_3, \dots \dots \dots \dots \dots \dots v_i\}$ be the set of vertices subdividing the edges of G in $M(G)$.Let $D_1 \subseteq D$,such that each $v_i \in D_1$ subdivides the edges $e_i \in E_1$ in $M(G)$.Thus $|E_1| = |D_1|$.Now ,if $N[D_1] = V[M(G)]$ and for each $v_i \in D_1$ is an isolate ,then D_1 forms an independent dominating set of $M(G)$.Otherwise consider a set $D_1' = D_2 \cup D_2'$,where $D_2 \subseteq D_1$ and $D_2' \subseteq V[M(G)] - D_1$ such that $\forall v \in V[M(G)] - \langle D_2 \cup D_2' \rangle, N(v) \cap \langle D_2 \cup D_2' \rangle \neq \phi$ and also $\langle D_2 \cup D_2' \rangle$ is totally disconnected .Then $\langle D_2 \cup D_2' \rangle$ forms a minimal independent dominating set of $M(G)$.Hence clearly $|\langle D_2 \cup D_2' \rangle| \leq |V(G)| - \gamma'(G)$ which gives $i_M(G) \leq p - \gamma'(G)$.

Theorem 11: Let$G$ be a connected graph ,then$i_M(G) + \gamma_p(G) \leq p + \beta_1(G)$,where $\gamma_p(G)$ is the paired domination in G.

Proof: By the Theorem 5 ,$i_M(G) = \alpha_1(G)$.

Also by the Theorem D:$\gamma_p(G) \leq 2\beta_1(G)$.
Further ,$i_M(G) + \gamma_p(G) \leq \alpha_1(G) + 2\beta_1(G)$

$$= p - \beta_1(G) + 2\beta_1(G)$$
$$= p + \beta_1(G)$$

Hence $i_M(G) + \gamma_p(G) \leq p + \beta_1(G)$.
The following theorem relates $i_M(G)$ and the independent domination number of the subdivision graph $[S(G)]$.

Theorem 12:For any connected graph $G, i_M(G) \leq i[S(G)]$.
Proof : Let $V(G) = \{v_1, v_2, v_3 \dots \dots \dots \dots \dots \dots v_i\}$ and $E(G) = \{e_1, e_2, \dots \dots \dots \dots \dots \dots \dots \dots e_j\}$.Let

$S = \{u_1, u_2, u_3 \dots \dots \dots \dots \dots \dots \dots u_k\} \sqsubset V[S(G)]$ be the minimum number of vertices subdividing the edges of G in $S(G)$.If $N[S] = V[S(G)]$,then $\langle S \rangle$ forms a minimal dominating set of $S(G)$.Also, since in $S(G)$ $\forall u_i, u_j \in S, 1 \leq i,j \leq k, N(u_i) \cap (u_j) = \phi$,hence $\langle S \rangle$ itself forms the independent dominating set of $S(G)$.In case ,if $N[S] \neq V[S(G)]$, we consider a set $I = D_1 \cup D_2$,where $D_1 \subseteq S$ and $D_2 = V[S(G)] - D_1$,such that $N[I] = V[S(G)]$ and for each $u_i \in \langle I \rangle$,since $\deg u_i = 0$,then $\langle I \rangle$ forms a minimal independent dominating set of $S(G)$.Further ,without loss of generality $S \subseteq V[M(G)]$.Since for each $u_i, u_j \in S, N(u_i) \cap (u_j) \neq \phi$ in $M(G)$.Now consider a set $S_1 \subseteq S$,such that $\forall u_i \in V[M(G)] - S_1$,$N(u_i) \cap S_1 \neq \phi$ and $u_i, u_j \in S_1, N(u_i) \cap (u_j) = \phi$,then $\langle S_1 \rangle$ forms a minimal independent dominating set of $M(G)$.Otherwise, let $S_2 \subseteq S_1' \cup S_2'$,where $S_1' \subseteq V[M(G)] \cap S$ and $S_2' \subseteq V[M(G)] \cap V(G)$,and also no two vertices in $\langle S_2 \rangle$ are adjacent .Hence $\langle S_2 \rangle$ forms the minimal independent dominating set of $M(G)$.Clearly $|I| \geq |S_2|$ which gives $i_M(G) \leq i[S(G)]$.

The following theorem gives Nordan-Gaudass type of result.
Theorem 13: Let G be a graph such that both G and $\bar{G}$ have no isolated edges then,

$i_M(G) + i_M(\bar{G}) \leq 2\left\lceil \frac{p}{2} \right\rceil$.

$i_M(G) \,.\, i_M(\bar{G}) \leq \left\lceil \frac{p}{2} \right\rceil^2$.

CONCLUSION

In this paper we established selected results on independent dominating sets in middle graphs. These results established key relationships between the independent middle domination number and other parameters including the domination number. Further, these results established optimal upper bounds on the independent domination number in terms of the order itself and the order and the maximum degree

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