



RESEARCH ARTICLE

**SELECTION OF BAYESIAN MULTIPLE DEFERRED STATE (BMDS-1) SAMPLING PLAN
BASED ON QUALITY REGIONS**

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INTRODUCTION

To ensure a good quality of the final product from the factory, inspection of the raw material and the product material should be done. The acceptance sampling plans are well known to reduce the above risks. In all the type of sampling plans namely single sampling plan, double sampling, multiple sampling plan and chain sampling plan, the basic assumption is the lot or fraction of defective goods is constant. Even though the process is stable, in practical situations, the goods produced from a process will slightly differ in their quality due to random fluctuations. Calvin (1984) has introduced the procedures and tables for using Bayesian sampling plans.

Comparison of traditional theory and Bayesian theory was done in excellent manner by Hald (1981). Bayesian theory is more compact and scientific technique, this idea was given by Wetherill and chiu (1975). Suresh and Latha (2001) have designed procedure for average probability of acceptance function for single sampling plan with Gamma prior

ABSTRACT

Our goal in this article is to briefly present the theory and technique of Bayesian Multiple Deferred State (BMDS) using Gamma prior distribution and demonstrate how it can facilitate to find Average Quality Level (AQL) and Limiting Quality Level (LQL) and bring about the result to reduce the producers risk (α) and consumer risk (β). In this article, we further introduce probabilistic Quality Region (PQR), and Indifference Quality Region (IQR) which gives potential application to improve the quality level in industry products. This paper proposes Bayesian Multiple Deferred State sampling plan to improve the quality of any product and service.

distribution. Latha and Jeyabharathi have studied the performance measures for Bayesian chain sampling plan using Beta Binomial distribution. Latha and Arivazhagan (2015) have studied the Bayesian Chain sampling plan using Beta prior distribution. Latha and Subbiah (2014) have given Bayesian Multiple Deferred State (BMDS-1) sampling plan with weighted Poisson distribution.

Latha and Rajeswari (2013) have discussed the asymptotic property for Bayesian Chain sampling plan. Suresh and Sangeetha (2010) have given a procedure for selection of Repetitive Deferred Sampling plan through quality region.

The main aim of this paper is to account for determining Acceptance Quality Level (AQL), Limiting Quality Level (LQL), Producer's Risk (α), Consumer's Risk (β) and Indifference Quality Level (IQL).

This article also leads to final the Probabilistic Quality Region (PQR) and Indifference Quality Region (IQR) for specified

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parameters Procedures and tables given in this article provides applications in industrial ground.

MDS - I(c₁,c₂) Plan

Rembert Vaerst (1980) has developed Multiple Deferred State MDS-I(c₁, c₂) Sampling Plans in which the acceptance or rejection of a lot is based in not only on the results from the current lot but also on sample results of the past or future lots.

Condition for Application of MDS-I(c₁,c₂)

1. Interest centers on an individual quality characteristic that involves destructive or costly tests such that normally only a small number of tests per lot can be justified.
2. The product to be inspected comprises a series of successive lots or batches (or material or of individual units) produced by an essentially continuing process.
3. Under normal conditions the lots are expected to be essentially of the same quality.
4. The product comes from a source in which the consumer has confidence.

Operating Procedure of MDS-I(c₁, c₂)

Step 1: For each lot, Select a Sample of n units and test each unit for conformance to the specified requirements.

Step 2: Accept the lot if d (the observed number of defectives) is less than or equal to c₁; reject the lot if d is greater than c₂.

Step 3: If c₁< d < c₂, accept the lot provided in each of the samples taken from the preceding or succeeding i lots, the number of defectives found is less than or equal to c₁; otherwise reject the lot.

The OC function of MDS-I(c₁, c₂) is given by,

$$P_a(p) = P_a(p, n, c_1) + [P_a(p, n, c_2) - P_a(p, n, c_1)] [P_a(p, n, c_1)]^i$$

Rembert Vaerst has presented certain tables giving minimum MDS-I(c₁,c₂) plans indexed by AQL and LQL and observes the following properties.

1. MDS-I(c₁,c₂) Plans are natural extension of ChSP-1 Plans of Dodge (1955).
2. MDS-1 (c₁,c₂) plans allows significant reduction in sample size as compared to single sampling plans.
3. The use of acceptance number c₂ increases the chances of acceptance in the region of principal interest. Where the product percent defective is very low.
4. When i= 0, the plan becomes a single sampling plan with sample size n, and acceptance number c₂.
5. When i= ∞, the plan becomes a single sampling plan with sample size n, and acceptance number c₁.

Bayesian Average Probability of Acceptance

The Poisson Model of the OC function of MDS-I(c₁,c₂) plan is given by

$$P_a(p) = \sum_{r_1=0}^{c_1} \frac{e^{-x} x^{r_1}}{r_1!} + \sum_{r_2=c_1+1}^{c_2} \frac{e^{-x} x^{r_2}}{r_2!} \left[\sum_{r_1=0}^{c_1} \frac{e^{-x} x^{r_1}}{r_1!} \right]^i$$

$$\text{for } x=np, \tag{1}$$

The past history its observe that the process average p follows Gamma prior distribution with the parameter (s, t) and density function,

$$w(p) = \frac{e^{-pt} p^{s-1} t^s}{\Gamma_s}, \quad 0 < p < 1, \quad s, t > 0, \quad q = 1 - p \tag{2}$$

Where $\mu = \frac{s}{t}$, Under the proposed Multiple Deferred State Sampling Plan, the Probability of Acceptance of Multiple Deferred State Sampling Plan of type (MDS-I(0,1)) plan based on the Gamma Poisson Distribution is given by,

$$\begin{aligned} \bar{P} &= \int_0^{\infty} P_a(p) w(p) dp \\ \bar{P} &= \int_0^{\infty} \left[\sum_{r_1=0}^{c_1} \frac{e^{-x} x^{r_1}}{r_1!} + \sum_{r_2=c_1+1}^{c_2} \frac{e^{-x} x^{r_2}}{r_2!} \left[\sum_{r_1=0}^{c_1} \frac{e^{-x} x^{r_1}}{r_1!} \right]^i \right] \left(\frac{e^{-pt} p^{s-1} t^s}{\Gamma_s} \right) dp \\ \bar{P} &= \sum_{r_1=0}^{c_1} \frac{t^s n^{r_1}}{\Gamma_s (r_1!)} * \frac{\Gamma_s + r_1}{(n + t)^{s+r_1}} + \sum_{r_2=c_1+1}^{c_2} \sum_{r_1=0}^{c_1} \frac{t^s n^{r_2+ir_1}}{\Gamma_s (r_2!)(r_1!)^i} \\ &\quad * \frac{\Gamma_s + r_2 + ir_1}{(n + in + t)^{s+r_2+r_1}} \end{aligned}$$

When c₁=0, c₂=1,

$$\bar{P} = \frac{(s)^s}{(s + n\mu)^s} + \frac{n\mu(s)^{s+1}}{(s + n\mu + in\mu)^{s+1}} \tag{3}$$

Hence the above equation is mixed distribution of Gamma Poisson distribution.

Designing Plans for given AQL, LQL, and

Tables 1 and 2 are used for selecting a Bayesian Multiple Deferred State Sampling Plan for specified AQL and LQL, and n by the following steps.

The steps utilized for selecting Bayesian Multiple Deferred State Sampling Plan (BDSP-(0,1)) are as follows

1. To design a plan for given (AQL, 1-) and (LQL,) first calculate the operating ratio μ_2/μ_1
2. For a fixed n, locate the tabular value of μ_2/μ_1 which is equal to or just less than the desired μ_2/μ_1 in the column of desired and .
3. Corresponding to the located value of μ_2/μ_1 the value of s and i, can be obtained.

Example 1 For s=2, i=1 ,n=100, and $\bar{P} = 0.50$ the corresponding IQL value $\mu_0=0.02110$ For s=1, i=3, n=100, and AQL value $\mu_1= 0.001163$ and LQL values $\mu_2=0.096554$

From Table 1 for the given variation Average Probability of Acceptance of the above equations. The average product quality level μ using Newton's approximation method is

obtained. The above examples, we can understand that when s and k are increased, the average product quality is decreased.

It is an interval of quality ($\mu_1 < \mu < \mu_2$) in which product is accepted with a minimum probability 0.10 and maximum

Table 1 Certain μ values for specified values of $P(\mu)$ BDSP-(0,1)

		probability of acceptance					
I	S	0.99	0.95	0.90	0.50	0.10	0.05
1	1	0.000645	0.001686	0.002725	0.014677	0.114957	0.239979
	2	0.000726	0.001828	0.002858	0.012110	0.051636	0.080961
	3	0.000763	0.001892	0.002923	0.011381	0.040192	0.057977
	4	0.000783	0.001929	0.002961	0.011036	0.035565	0.049355
	5	0.000797	0.001953	0.002986	0.010834	0.033079	0.044899
2	1	0.000505	0.001340	0.002192	0.012493	0.101442	0.212555
	2	0.000567	0.001445	0.002282	0.010246	0.046375	0.073483
	3	0.000591	0.001493	0.002327	0.009596	0.036417	0.053317
	4	0.000611	0.001520	0.002353	0.009285	0.032399	0.045761
	5	0.000621	0.001538	0.002371	0.009102	0.030245	0.041863
3	1	0.000432	0.001163	0.001925	0.011576	0.096554	0.202799
	2	0.000484	0.001247	0.001991	0.009460	0.044733	0.071294
	3	0.000507	0.001286	0.002023	0.008841	0.035363	0.052153
	4	0.000520	0.001308	0.002043	0.008541	0.031590	0.044973
	5	0.000529	0.001322	0.002056	0.008367	0.029571	0.041267

Table 2 Values of μ_2/μ_1 tabulated against s and k for given μ_1 and μ_2 for Bayesian Double Sampling Plan

		μ_2/μ_1 for =0.05	μ_2/μ_2 for =0.05	μ_2/μ_1 for =0.01	μ_2/μ_1 for =0.01	μ_2/μ_1 for =0.10	μ_2/μ_1 for =0.10
1	1	68.18327	142.3363	178.2279	372.06050	42.18606	88.06569
	2	28.24726	44.28939	71.12397	111.51650	18.06718	28.32785
	3	21.24313	30.64323	52.67628	75.98558	13.75026	19.83476
	4	18.43701	25.58580	45.42146	63.03321	12.01114	16.66836
	5	16.93753	22.98976	41.50439	56.33501	11.07803	15.03650
2	1	75.70299	158.6231	200.8752	420.90100	46.27828	96.96852
	2	32.09343	50.85329	81.79012	129.59960	20.32209	32.20114
	3	24.39183	35.71132	61.61929	90.21489	15.64976	22.91233
	4	21.31513	30.10592	53.02619	74.89525	13.76923	19.44794
	5	19.66515	27.21912	48.70370	67.41224	12.75622	17.65626
3	1	83.02150	174.3758	223.5046	469.44210	50.15792	105.35010
	2	35.87249	57.17241	92.42355	147.30170	22.46760	35.80814
	3	27.49844	40.55443	69.74951	102.86590	17.48047	25.78003
	4	24.15138	34.38303	60.75000	86.48654	15.46256	22.01322
	5	22.36838	31.21558	55.89981	78.00945	14.38278	20.07150

Table 3 Values of PQR and IQR, μ_2/μ_1 for specified values of s and i

i	s	μ_1	μ_0	μ_2	d_2	d_0	T	μ_2/μ_1
1	1	0.001686	0.014677	0.114957	0.113271	0.012991	8.719190	68.18327
	2	0.001828	0.012110	0.051636	0.049808	0.010282	4.844194	28.24726
	3	0.001892	0.011381	0.040192	0.038300	0.009489	4.036253	21.24313
	4	0.001929	0.011036	0.035565	0.033636	0.009107	3.693423	18.43701
	5	0.001953	0.010834	0.033079	0.031126	0.008881	3.504785	16.93753
2	1	0.001340	0.012493	0.101442	0.100102	0.011153	8.975343	75.70299
	2	0.001445	0.010246	0.046375	0.044930	0.008801	5.105102	32.09343
	3	0.001493	0.009596	0.036417	0.034924	0.008103	4.310009	24.39183
	4	0.001520	0.009285	0.032399	0.030879	0.007765	3.976690	21.31513
	5	0.001538	0.009102	0.030245	0.028707	0.007564	3.795214	19.66515
3	1	0.001163	0.011576	0.096554	0.095391	0.010413	9.160761	83.02150
	2	0.001247	0.009460	0.044733	0.043486	0.008213	5.294777	35.87249
	3	0.001286	0.008841	0.035363	0.034077	0.007555	4.510523	27.49844
	4	0.001308	0.008541	0.031590	0.030282	0.007233	4.186645	24.15138
	5	0.001322	0.008367	0.029571	0.028249	0.007045	4.009794	22.36838

Example 2 Suppose the value for μ_1 is assumed as 0.002 and value for μ_2 is assumed as 0.034 then the operating ratio is calculate as 17. Now the integer approximately equal to this calculated operating ratio and their corresponding parametric values are observed from the table2. The actual values $i=3$, $s=3, \mu_1=0.002023$ and $\mu_2=0.035363$ at ($\alpha=0.10$ and $\beta=0.10$).

probability 0.95, Probability Quality Range denoted as the $d_2 = (\mu_2 - \mu_1)$ is derived from the average Probability of acceptance

$$\bar{P}(\mu_1 < \mu < \mu_2) = \frac{(s)^s}{(s + n\mu)^s} + \frac{n\mu(s)^{s+1}}{(s + n\mu + in\mu)^{s+1}}$$

Designing of Quality interval Bayesian Multiple Deferred State Sampling Plan (BMDS-(0,1))

Where $\mu = \frac{s}{t}$, is the expectation of gamma distribution and approximately the mean values of product quality.

Probabilistic Quality Region (PQR)

Indifference Quality Region (IQR)

It is an interval of quality ($\mu_1 < \mu < \mu_0$) in which product is accepted with a minimum probability 0.50 and maximum probability 0.95. Indifference Quality Range denoted as the $d_0 = (\mu_0 - \mu_1)$ is derived from the average Probability of acceptance.

$$\bar{P}(\mu_1 < \mu < \mu_0) = \frac{(s)^s}{(s + n\mu)^s} + \frac{n\mu(s)^{s+1}}{(s + n\mu + in\mu)^{s+1}}$$

Where $\mu = \frac{s}{t}$, is the expectation of gamma distribution and approximately the mean values of product quality.

Selection of the Sampling Plan

Table 3, gives unique values of T for different values of s and n. Here Operating Ratio $T = \frac{\mu_2 - \mu_1}{\mu_0 - \mu_1} = \frac{d_2}{d_0}$, Where $d_2 = (\mu_2 - \mu_1)$ and $d_0 = (\mu_0 - \mu_1)$ is used to characterize the sampling plan. For any given values of PQR(d_2) and IQR(d_0) one can find the ratio $T = \frac{d_2}{d_0}$, Find the value in the Table 3, under the column T which is equal to or just less than the specified ratio, corresponding s and i values are noted. From this ratio one can determine the parameters for the BMDS-(0,1) Plan.

Example 3 Given $\mu_1 = 0.00153$ compute the values of PQR and IQR then compute T. Select the respective values from Table 3. The nearest values of PQR and IQR corresponding to $s=5$, $i=2$ and $\mu_1=0.001538$ are $d_2=0.028707$ and $d_0=0.007564$, Then $T=3.795214$. Hence the required plan has parameters $i=2$, $s=5$, through Quality Interval.

In the similar way, the above equations are equated to the average probability of acceptance 0.95 and 0.10, AQL(μ_1) and IQL(μ_2) are obtained in Table 3.

CONCLUSION

Bayesian Acceptance Sampling is the best technique, which deals with the procedure in which decision to accept or reject lots or process based on their examination of past history or knowledge of samples. This paper deals with Bayesian Multiple Deferred State Sampling Plan based on Gamma prior distribution. However, all of them are either settled on a non-economic basis or do not take into consideration the producer's and consumer's quality and risk requirements. Using the Bayesian sampling attribute plan without a cost function for a prior distribution can reduce the sample size, while if producer's risk and consumer's risk are appropriate.

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The work presented in this paper mainly related to procedure for designing Bayesian multiple deferred state sampling plan for probability of acceptable, producer's and consumer's risk and limiting, indifference for quality levels and quality regions. The quality level and quality interval sampling plan possesses wider potential applicable in industry ensuring higher standard of quality attainment for product or process. Thus quality interval and quality level are good measure for defining and designing for acceptance sampling plan which are readymade use to industrial shop-floor situations.

Reference

1. Calvin, T.W.(1984). How and When to Perform Bayesian Acceptance Sampling, Vol.7, *American Society for Quality Control*, Milwaukee, Wisconsin, USA.
2. Dodge, H.J.(1955). Chain Sampling Inspection Plans, *Industrial Quality Control*, vol.11, No.4, pp.10-13.
3. Hald, A. (1981). *Statistical Theory of Sampling Inspection by Attributes* (New York, Academic Press).
4. Latha, M. and Jeyabharathi, S.(2012). Selection of Bayesian Chain Sampling Attributes Plans based on Geometric Distribution, *International Journal of Scientific and Research Publication*, Vol.2, issue 7.
5. Latha, M. and Arivagazahan R, S.(2015). Bayesian Chain Sampling Plan Using Binomial Distribution, *International Journal of Emerging Trends in Engineering and Development*, Vol.3, issue 5.
6. Latha, M. and Subbiah K (2014). Bayesian Multiple Deferred State (BMDS-1) Sampling Plan with weighted Poisson distribution, *International Journal of Emerging Trends in Engineering and Development*, Vol.3, issue 4, pp. 275- 282.
7. Latha, M. and Rajeswari, M.(2013). Asymptotic property for Bayesian Chain Sampling Plan-1, *International Journal of Advanced Scientific and Technical Research*, Vol 4, issue 3.
8. Suresh, K.K. and Latha, M.(2001). Bayesian Single Sampling Plans for a Gamma Prior, *Economic Quality Control*, Vol.16, No.1, pp.93-107.
9. Suresh, K. K and Sangeetha, V.(2010). Selection of Repetitive Deferred Sampling Plan Through Quality Region, *International Journal of Statistics and Systems*, Vol.5, No.3, pp.379-389.
10. Vaerst, R (1982) :A Procedure to Construct Multiple Deferred State Sampling Plan, *Method of Operation Research*, Vol.37, pp.477-485.
11. Wetherill, G.B and Chiu, W. K (1975). A Review of Acceptance sampling Schemes with emphasis on the economic aspect, *International Statistical Review*, 43, pp. 191-210.
