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RESEARCH ARTICLE

ON THE METRIC DIMENSION OF GRAPH P_n,1,2

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ABSTRACT

In this paper we have found the metric dimension of dodecahedral other embedding (also called $P_{n,1,2}$) for inner cycle, outer cycle and its extensions for pen- dent and prism graphs. We have proved that metric dimension of $P_{n,1,2}$ is bounded and only three vertices chosen appropriately suffice to resolve all the vertices of these graphs for n = 0, (mod 4), n ≥ 16, n = 2 (mod 4), n ≥ 18 and n = 3 (mod 4), n ≥ 11 for inner cycle, outer cycle, pendent and prism graphs respectively and only four vertices chosen appropriately suffice to resolve all the vertices of these graphs for n = 1, (mod 4), n ≥ 17, key concepts:Metric dimension, basis, resolving set, dodecahedral other embedding called $P_{n,1,2}$ open problem:further it can be proved that the metric dimensions of inner cy-cle, outer cycle and its extensions for pendent and prism graphs may be constant.

INTRODUCTION

Notations and preliminary results

Let G(V, E) be a connected graph where V and E represents the vertex and edge sets of G respectively. If x_1, x_2 V (G) are the two vertices of connected graph G, if there is an edge between x_1, and x_2 then distance of these two vertices i.e x_1, x_2 V (G) described as d(x_1, x_2) and it would be the shortest length or smallest $x_1 - x_2$ path in the connected graph G. Let w = {$w_1, w_2, w_3, ...,$ w_m} be the set of vertices of G which must be an ordered set i.e while x V (G). Then r(x/W) will be the representation of x with respect to w and it is called m−tuple and is denoted by (d(x/w_1), d(x/w_2), ..., d(x/w_m)).[12, 27] Then W is a "resolving set" for G, if vertices of G which are distinct have distinct representation with respect to W .[8] A "Basis" for G is actually a set of minimum cardinality and when we take cardinality of the basis of G then it would be the metric dimension of G written as dim(G).

For an order set of vertices W = {$w_1, w_2, w_3, ..., w_m$} of a graph G, the ith component of r(x/W) is 0 if and only if x = w_i, thus to show that W is resolving set it suffices to verify that r(x_1/W) = r(x_2/W) for each pair of distinct points x_1, x_2 V (G)

A useful property in finding dim(G) is the following

Lemma 1. [27] Let W be the resolving set for a connected graph G and x_1, x_2 V (G).if r(x_1 /w) =r(x_2/w) for all vertices w V (G) {x_1, x_2} then {x_1, x_2} T W =φ[24 − 25] Slater (1975) was the first mathematician who introduces the idea or concept of metric dimension of graphs and after this the number of researchers in graph theory have been projected their work on the problem of metric dimension of different types of graphs. By denoting G + H the join of G and H a wheel W_n is defined as $W_n = K_1 + C_n$ for n ≥ 3,a fan is $F_n = K_1 + P_n$ for n ≥ 1 and Jahangir graph J_2n, (n ≥ 2) which is obtained from wheel graph W_2n by alternating deleting n-spokes.[8] Buczkowski et al. determine the dimension of wheel W_n,[10−11] Caceres et al. the dimension of fan graph F_n and [17] Tomesko and Javed the dimension of Jahangir graph J_2n, (n ≥ 2). [14] In P (m, n) when we take m = 1, P (n, 1) is called prism and n−prism graph has 2n−nodes and 3n−edges it is denoted by D_n and Caceres et al. (2005) while working with metric dimension of some families of graphs it is shown that

$$dim(P_m \times C_n) = \begin{cases} 3 & if\ n = even \\ 2 & if\ n = odd \end{cases}$$

Since Prism is in fact the cross product ($P_2 \times C_n$) and this suggest that

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$$dim(D_n)=\begin{cases} 3 & if\ n = even \\ 2 & if\ n = odd \end{cases}$$ thus it is obvious that prism contains a class of

Regular graphs with bonded metric dimension

The generalized Peterson graph P (n, 2) becomes a useful example for many prob- lems in the field of graph theory. We consider the metric dimension of generalized Peterson graph P (n, m), for m = 2, $\{x_1, x_2, ..., x_n\}$ induces a cycle in P (n, 2) with $x_i x_{i+1}$ $(1 \le i \le n)$ as edges. When n is odd then $\{y_1, y_2, ..., y_m\}$ induce a cycle of length n with $y_i y_{i+2}$ $(1 \le i \le n)$ as edges, with indices taken modulo n, and when n is even i.e n = 2k, (k ≥ 3), $\{y_1, y_2, ..., y_n\}$ generate two cycle of length k with $y_i y_{i+2}$ $(1 \le i \le n)$ as edges. [17, 20] Javaid et al. (2008). proved that some regular graphs namely generalized Peterson graph P (n, 2), antiprism A_n and Harary graph H_4; n are families of graph with constant metric dimension and it is shown that dim(P (n, 2)) = 3, for every n ≥ 5.

When we take for m = 3, $\{x_1, x_2, ..., x_n\}$ generate a cycle in P (n, 3) with $x_i x_{i+1}$ $(1 \le i \le n)$ as edges, If n = 3k (k ≥ 3) then $\{y_1, y_2, ..., y_m\}$ generate three cycles of length k or else generate cycle of length "n" with $y_i y_{i+3}$ $(1 \le i \le n)$ as edges. Generalized Peterson graph produce an important class of 3−regular graph with

2n−vertices and 3n−edges so it is necessary to determine their metric dimension.

[14] M. Imran found that generalized Peterson graph consists a family of 3−regular graph having bonded metric dimension, and for n = 0, 3, 4, 5 (mod 6) resolving sets consisting of only four vertices are chosen that resolves all the vertices of gener- alized Peterson graph P (n, 3), except n = 2 (mod 6). For n = 1 (mod 6) resolving set consisting of three vertices is taken. In graph P (n, 3) all the indices "i" which do not satisfy the inequality 1 ≤ i ≤ n will be taken modulo n. [14] Upper bounds for metric dimension of generalized Peterson graphs P (n, 3) as proved by M. Imran are given below,

For generalized Peterson graph P (n, 3) we have

1. dim(P (n, 3) ≤ 4, for n = 0, 3, 4, 5 (mod 6) and n ≥ 17
2. dim(P (n, 3) ≤ 3, for n = 1 (mod 6) for n ≥ 13
3. dim(P (n, 3) ≤ 5, for n = 2 (mod 6) for n ≥ 8

The graph of Pn,1,2

In this paper we have found and studied the metric dimension of the graph $P_{n,1,2}$. This graph has the following set of vertices and the set of edges denoted by $V(P_{n,1,2})$ and $E(P_{n,1,2})$ for the inner cycle and the outer cycle are as under:

$$V(P_{n,1,2}) = \{u_1, u_2, ..., u_n, v_1, v_2, ..., v_n, \}$$

and the edge set

$$E(P_{n,1,2}) = \{u_i u_{i+2}, u_i v_i\} \quad \{v_i v_{i+1}\}$$

for 1 ≤ i ≤ n, where the indices n + 1 and n + 2 must be replaced by 1 and 2 respectively.

For our convenience, we represent the cycle induced by $\{u_i : 1 \le i \le n\}$ the inner cy- cle, the cycle induced by $\{v_i : 1 \le i \le n\}$ the outer cycle and the set of outer vertices by $\{w_i : 1 \le i \le n\}$. Again the vertices choice chosen is crucial for the basis. Note that throughout our discussion remember that $ext^{(1)}(P_{n,1,2})$, $ext^{(2)}(P_{n,1,2})$,stands for the pendent and prism graphs respectively.

Case 1 n = 0 (mod 4), n ≥ 16, In this case it can be written as n = 4k, k ≥ 4, and k Z^+, The resolving set in general form is $W = \{u_1, u_7, v_{2k+3}\}$ $V(P_{n,1,2})$, k ≥ 4.

Theorem1 We have the metric dimension of $P_{n,1,2}$ denoted by $dim(P_{n,1,2}) \le 3$ for inner and outer cycles for n ≥ 16.

Proof. In this case it can be written as n = 4k, k ≥ 4, and k Z^+, The resolving set in general form is $W = \{u_1, u_7, v_{2k+3}\}$ $V(P_{n,1,2})$, k ≥ 4.

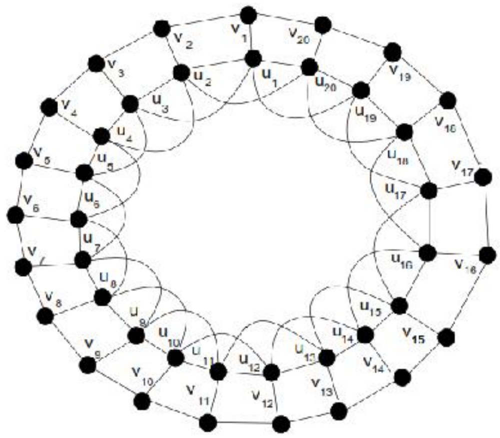

Figure 1 The graph of $P_{20,1,2}$

Representations of vertices w.r.t w in general form are Representation of the vertices of inner cycle.

$$r(u_{2i}/w) = \begin{cases} (1,3,k+1), & if \quad i=1; \\ (2,2,k+1), & if \quad i=2; \\ (3,1,k), & if \quad i=3; \\ (i,i-3,k-i+3), & if \quad 4 \le i \le k; \\ (k,k-2,2), & if \quad i=k+1; \\ (k-1,k-1,2), & if \quad i=k+2; \\ (k-2,k,3), & if \quad i=k+3; \\ (2k-i+1,2k-i+4,i-k), & if \quad k+4 \le i \le 2k; \end{cases}$$

And

$$r(u_{2i-1}/w)=\begin{cases}(1,2,k+1), & if \quad i=2;\\ (2,1,k), & if \quad i=3;\\ (i,i-3,k-i+2), & if \quad 4\le i\le k;\\ (k-1,k-2,1), & if \quad i=k+1;\\ (k-2,k-1,2), & if \quad i=k+2;\\ (2k-i,2k-i+3,i-k), & if \quad k+3\le i\le 2k+1;\end{cases}$$

Representation of the vertices of Outer cycle

$$r(v_{2i}/w)=\begin{cases}(3,3,k+2), & if \quad i=2;\\ (4,2,k+1), & if \quad i=3;\\ (i+1,i,i-2,k-i+4), & if \quad 4\le i\le k-1;\\ (k+1,k-2,3), & if \quad i=k;\\ (k+1,k-1,1), & if \quad i=k+1;\\ (k,k,2), & if \quad i=k+2;\\ (k-1,k+1,4), & if \quad i=k+3;\\ (2k-i+2,2k-i+5,i-k+1), & if \quad k+4\le i\le 2k-1;\\ (2,2,5), & if \quad i=2k;\end{cases}$$

And

$$r(v_{2i-1}/w)=\begin{cases}(1,4,k+1), & if \quad i=1;\\ (2,3,k+2), & if \quad i=2;\\ (3,2,k+1), & if \quad i=3;\\ (i,i,i-3,k-i+4), & if \quad 4\le i\le k+1;\\ (k,k,1), & if \quad i=k+2;\\ (k-1,k,2), & if \quad i=k+3;\\ (2k-i+2,2k-i+5,i-k), & if \quad k+5\le i\le 2k;\end{cases}$$

Theorem 2 We have the metric dimension $\dim(ext^{(1)}(P_{n,1,2})) \le 3$ and $\dim(ext^{(2)}(P_{n,1,2})) \le 3$ for which $n \ge 16$.

Proof

In this case it can be written as $n = 4k$, $k \ge 4$, and k Z^+, The resolving set in general form is $W = \{u_1, u_7, v_{2k+3}\}$ $V(P_{n,1,2})$, $k \ge 4$.

For the $ext^{(1)}P_{n,1,2}$, The vertex and the edge sets are as under:

$V(P_{n,1,2}) = \{u_1, u_2, ..., u_n, v_1, v_2, ..., v_n, w_1, w_2, ..., w_n\}$

and
$E(P_{n,1,2}) = \{u_i u_{i+2}, u_i v_i\}$ $\{v_i v_{i+1}\}$ $\{v_i w_i\}$

for $1 \le i \le n$, respectively, where the indices $n+1$ and $n+2$ must be replaced by 1 and 2 respectively.

Foe $ext^{(2)}(P_{n,1,2})$, Following are the set of vertices and the set of edges denoted by
$V(P_{n,1,2})$ and $E(P_{n,1,2})$ as under:

$V(P_{n,1,2}) = \{u_1, u_2, ..., u_n, v_1, v_2, ..., v_n, w_1, w_2, ..., w_n\}$

And

$E(P_{n,1,2}) = \{u_i u_{i+2}, u_i v_i\}$ $\{v_i v_{i+1}\}$ $\{v_i w_i\}$ $\{w_i w_{i+1}\}$

for $1 \le i \le n$, respectively, where the indices $n+1$ and $n+2$ must be replaced by
1 and 2 respectively.

The graph $P_{n,1,2}$ for particular value of $n = 20$ for pendent and prism graph is shown in figure,

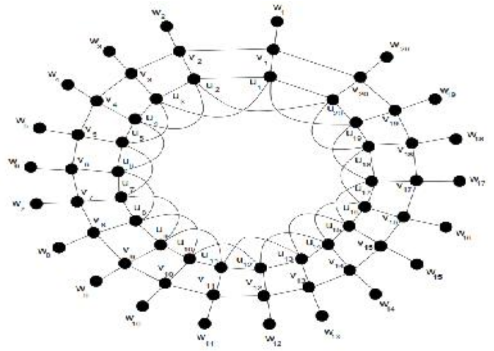

Figure 2 The pendent graph of $P_{20,1,2}$

Representation of vertices of pendent and prism graphs.

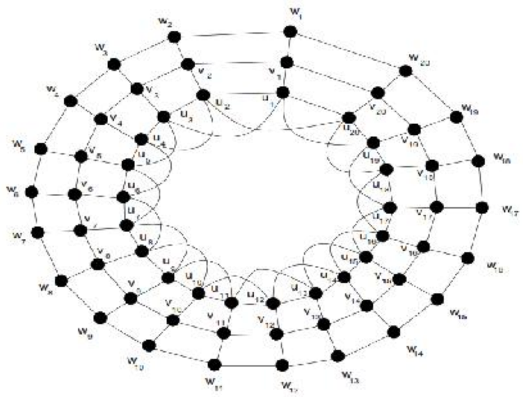

Figure 3 The prism graph of $P_{20,1,2}$

$$r(w_{2i}/w)=\begin{cases}(3,5,k+3), & if \quad i=1;\\ (4,4,k+3), & if \quad i=2;\\ (5,3,k+2), & if \quad i=3;\\ (i+2,i-1,k-i+5), & if \quad 4\le i\le k-1;\\ (k+2,k-1,4), & if \quad i=k;\\ (k+2,k,2), & if \quad i=k+1;\\ (k+1,k+1,2), & if \quad i=k+2;\\ (k,k+2,4), & if \quad i=k+3;\\ (2k-i+3,2k-i+6,i-k+2), & if \quad k+4\le i\le 2k;\end{cases}$$

And

$$r(w_{2i-1}/w)=\begin{cases}(2,5,k+2), & if \quad i=1;\\ (3,4,k+3), & if \quad i=2;\\ (4,3,k+2), & if \quad i=3;\\ (i+1,i-2,k-i+5), & if \quad 4\le i\le k;\\ (k+2,k-1,3), & if \quad i=k+1;\\ (k+1,k,1), & if \quad i=k+2;\\ (k,k+1,3), & if \quad i=k+3;\\ (2k-i+3,2k-i+6,i-k+1), & if \quad k+4\le i\le 2k;\end{cases}$$

Case 2: $n = 1 \pmod 4$, $n \ge 17$, in general form it can be written as $n = 4k+1$, $k \ge 4$, and k Z^+ and the resolving set in general form is $W = \{u_1, u_2, u_7, v_{2k+2}\}$ $V(P_{n,1,2})$, $k \ge 4$, and k Z^+ .

The graph $P_{n,1,2}$ for particular value of n for inner and outer cycle is shown in figure for $n = 21$.

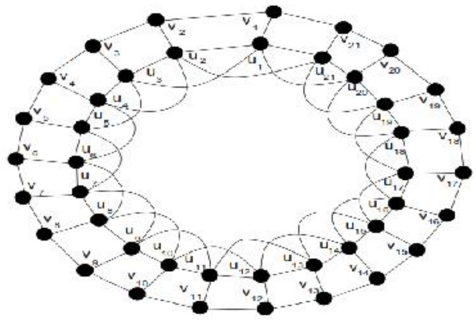

Figure 4 The graph of $P_{21,1,2}$

For every $n \geq 17$, $k \geq 4$ and $k \quad Z^{+}$ the representations of vertices w.r.t W are Theorem 3. We have the metric dimension $\dim(P_{n,1,2}) \leq 4$ for inner cycle and outer cycles for $n \geq 17$.

Proof In this case it can be written as $n = 4k + 1$, $k \geq 4$, and $k \quad Z^{+}$,The resolving set in general form is $W = \{u_1, u_7, u_7, u_{2k+2}\} \quad V(P_{n,1,2})$, $k \geq 4$.

This graph has the following set of vertices and the set of edges denoted by $V(P_{n,1,2})$

and $E(P_{n,1,2})$ for the inner cycle and the outer cycle are as under:

$V(P_{n,1,2}) = \{u_1, u_2, ..., u_n, v_1, v_2, ..., v_n,\}$

and the edge set

$E(P_{n,1,2}) = \{u_i u_{i+2}, u_i v_i\} \quad \{v_i v_{i+1}\}$
for $1 \leq i \leq n$, where the indices $n + 1$ and $n + 2$ must be replaced by 1 and 2 respectively.

Representation of the vertices of inner cycle.

$$r(u_{2i}/w) = \begin{cases} (2,1,2,k), & if \quad i = 1; \\ (3,2,1,k-1), & if \quad i = 2; \\ (i, i-1, k-i+2), & if \quad 3 \leq i \leq k; \\ (k,k,k-2,1), & if \quad i = k+1; \\ (k-1,k,k-1,2), & if \quad i = k+2; \\ (k-2,k-1,k,3), & if \quad i = k+3; \\ (2k-i+1, 2k-i+2, 2k-i+4, i-k), & if \quad k+4 \leq i \leq 2k; \end{cases}$$

And

$$r(u_{2i+1}/w) = \begin{cases} (1,1,2,k-1), & if \quad i - 1; \\ (2,2,1,k), & if \quad i = 2; \\ (i,i,i-3,k-i+2), & if \quad 4 \leq i \leq k; \\ (k,k,k-2,2), & if \quad i = k+1; \\ (k-1,k-1,k-1,3), & if \quad i - k + 2; \\ (k-2,k-2,k,4), & if \quad i = k+3; \\ (2k-i+1, 2k-i+1, 2k-i+4, i-k+1), & if \quad k-4 \leq i \leq 2k; \end{cases}$$

Representation of the vertices of Outer cycle

$$r(v_{2i}/w) = \begin{cases} (2,1,4,k+2), & if \quad i = 1; \\ (3,2,3,k+1), & if \quad i = 2; \\ (4,3,2,k+1), & if \quad i = 3; \\ (i+1, i, i-2, k-i+3), & if \quad 4 \leq i \leq k-1; \\ (k,k,k-2,2), & if \quad i = k; \\ (k,k+1,k,2), & if \quad i = k+2; \\ (k-1,k,k+1,4), & if \quad i = k+3; \\ (2k-i+2, 2k-i+3, i-k+1), & if \quad k+4 \leq i \leq 2k; \end{cases}$$

And

$$r(v_{2i-1}/w) = \begin{cases} (1,2,4,k+2), & if \quad i = 1; \\ (2,2,3,k+2), & if \quad i = 2; \\ (3,3,2,k+1), & if \quad i = 3; \\ (i,i,i-3,k-i+4), & if \quad 4 \leq i \leq k-1; \\ (k,k,k-3,3), & if \quad i = k; \\ (k+1,k+1,k-2,3), & if \quad i = k+1; \\ (k+1,k+1,k-1,1), & if \quad i = k+2; \\ (k,k,k,3), & if \quad i - k + 3; \\ (k-1,k-1,k+1,5), & if \quad i = k+4; \\ (2k-i+3, 2k-i+3, 2k-i+6, i-k), & if \quad k+5 \leq i \leq 2k+1; \end{cases}$$

The graph $P_{n,1,2}$ for particular value of $n = 21$ for pendent and prism graph is shown in figure,

Theorem 4 We have the metric dimension $\dim(ext^{(1)}(P_{n,1,2})) \leq 4$ and

$\dim(ext^{(2)}(P_{n,1,2})) \leq 4$ for which $n \geq 16$.

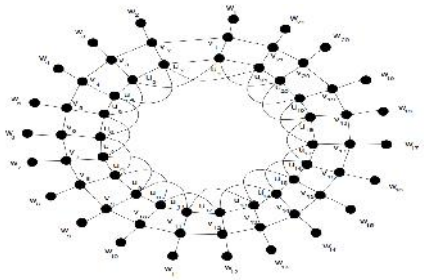

Figure 5 The pendent graph $P_{21,1,2}$

Proof

In this case it can be written as $n = 4k + 1$, $k \geq 4$, and $k \quad Z^{+}$,The resolving set in general form is $W = \{u_1, u_7, u_7, u_{2k+2}\} \quad V(P_{n,1,2})$, $k \geq 4$.

The graph $ext^{(1)}(P_{n,1,2})$ has the following set of vertices and the set of edges denoted by $V(P_{n,1,2})$ and $E(P_{n,1,2})$ for Pendent graph are as under:

$V(P_{n,1,2}) = \{u_1, u_2, ..., u_n, v_1, v_2, ..., v_n,\}$

and the edge set

$E(P_{n,1,2}) = \{u_i u_{i+2}, u_i v_i\} \quad \{v_i v_{i+1}\} \quad \{v_i w_i\}$

for $1 \leq i \leq n$, where the indices $n + 1$ and $n + 2$ must be replaced by 1 and 2 respectively.

The graph $ext^{(2)}(P_{n,1,2})$ has the following set of vertices and the set of edges denoted by $V(P_{n,1,2})$ and $E(P_{n,1,2})$ for Prism graph are as under:

$V(P_{n,1,2}) = \{u_1, u_2, ..., u_n, v_1, v_2, ..., v_n,\}$

and the edge set

$E(P_{n,1,2}) = \{u_i u_{i+2}, u_i v_i\} \quad \{v_i v_{i+1}\} \quad \{v_i w_i\} \quad \{w_i w_{i+1}\}$

for $1 \leq i \leq n$, where the indices $n + 1$ and $n + 2$ must be replaced by 1 and 2 respectively.

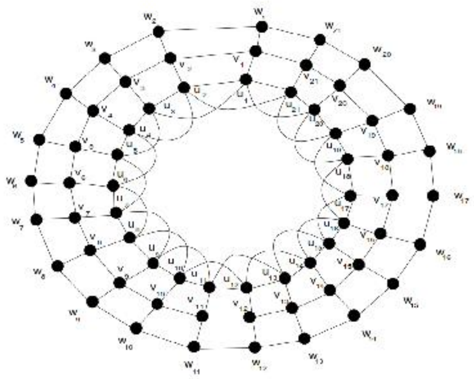

Figure 6 The prism graph $P_{21,1,2}$

Representation of vertices of $ext^{(1)}(P_{n,1,2})$ and $ext^{(2)}(P_{n,1,2})$

$$r(w_{2i}/w)=\begin{cases}(3,2,5,k+3), & if \quad i=1;\\ (4,3,4,k+2), & if \quad i=2;\\ (5,4,3,k+1), & if \quad i=3;\\ (i+2,i+1,i-1,k-i+4), & if \quad 4\le i\le k-1;\\ (k+2,k+1,k-1,3), & if \quad i=k;\\ (k+2,k+2,k,1), & if \quad i=k+1;\\ (k+1,k+2,k+1,3), & if \quad i=k+2;\\ (k,k+1,k+2,5), & if \quad i=k+3;\\ (2k-i+3,2k-i+4,2k-i+6,i-k+2), & if \quad k+4\le i\le 2k;\end{cases}$$

and

$$r(w_{2i-1}/w)=\begin{cases}(2,3,5,k+3), & if \quad i=1;\\ (3,3,4,k+3), & if \quad i=2;\\ (4,4,3,k+2), & if \quad i=3;\\ (i+1,i+1,i-2,k-i+5), & if \quad 4\le i\le k-1;\\ (k+1,k+1,k-2,4), & if \quad i=k;\\ (k+2,k+2,k-1,2), & if \quad i=k+1;\\ (k+2,k+2,k,2), & if \quad i=k+2;\\ (k+1,k+1,k-1,4), & if \quad i=k+3;\\ (k,k,k+2,6), & if \quad i=k+4;\\ (2k-i+4,2k-i+4,2k-i+7,i-k+2), & if \quad k+5\le i\le 2k+1;\end{cases}$$

Case 3: $n = 2 \pmod 4$, $n \geq 18$.

In this case it can be written as $n = 4k + 2$, $k \geq 4$, and k Z^{+},The resolving set in general form is $W = \{u_1, u_4, v_{2k+4}\}$ $V(P_{n,1,2})$, $k \geq 4$.

Theorem 5 We have the metric dimension $\dim(P_{n,1,2}) \leq 3$ for inner cycle and outer cycles for $n \geq 18$.

Proof. In this case it can be written as $n = 4k + 2$, $k \geq 4$, and k Z^{+},The resolving set in general form is $W = \{u_1, u_4, v_{2k+4}\}$ $V(P_{n,1,2})$, $k \geq 4$.

The graph $P_{n,1,2}$ has the following set of vertices and the set of edges denoted by

$V(P_{n,1,2})$ and $E(P_{n,1,2})$ for the inner cycle and the outer cycle are as under:

$V(P_{n,1,2}) = \{u_1, u_2, ..., u_n, v_1, v_2, ..., v_n,\}$

for $1 \leq i \leq n$, where the indices $n + 1$ and $n + 2$ must be replaced by 1 and 2 respectively.

The graph $P_{n,1,2}$ for particular value of n for inner and outer cycle is shown in figure for $n = 22$.

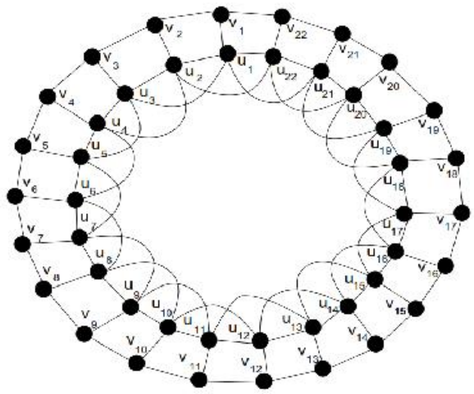

Figure 7 The graph $P_{22,1,2}$

Representation of the vertices of inner cycle

$$r(u_{2i}/w)=\begin{cases}(1,1,k+1), & if \quad i=1;\\ (i,i-2,k-i+3), & if \quad 3\le i\le k+1;\\ (k,k,1), & if \quad i=k+2;\\ (2k-i+2,2k-i+3,i-k-1), & if \quad k+3\le i\le 2k+1;\end{cases}$$

And

$$r(u_{2i+1}/w)=\begin{cases}(1,1,k+2), & if \quad i=1;\\ (i,i-1,k-i+3), & if \quad 3\le i\le k+1;\\ (k,k,2), & if \quad i=k+2;\\ (2k-i+1,2k-i+3,i-k), & if \quad k+3\le i\le 2k+1;\end{cases}$$

Representation of the vertices of Outer Cycle.

$$r(v_{2i}/w)=\begin{cases}(2,2,k+2), & if \quad i=1;\\ (3,1,k+2), & if \quad i=2;\\ (i+1,i-1,k-i+4), & if \quad 3\le i\le k;\\ (k+2,k,2), & if \quad i=k+1;\\ (k,k+1,2), & if \quad i=k+3;\\ (2k-i+3,2k-i+4,i-k), & if \quad k+4\le i\le 2k+1;\end{cases}$$

And

$$r(v_{2i+1}/w)=\begin{cases}(1,3,k+2), & if \quad i=1;\\ (2,2,k+3), & if \quad i=2;\\ (i,i-1,k-i+5), & if \quad 3\le i\le k;\\ (k,k,3), & if \quad i=k+1;\\ (k+1,k+1,1), & if \quad i=k+2;\\ (k,k+2,1), & if \quad i=k+3;\\ (k-1,k+1,3), & if \quad i=k+4;\\ (2k-i+3,2k-i+5,i-k), & if \quad k+5\le i\le 2k+1;\end{cases}$$

Theorem 6. We have the metric dimension $\dim(ext^{(1)}(P_{n,1,2})) \leq 3$ and $\dim(ext^{(2)}(P_{n,1,2})) \leq 3$ for which $n \geq 18$.

Proof

In this case it can be written as $n = 4k + 2$, $k \geq 4$, and k Z^{+},The resolving set in general form is $W = \{u_1, u_7, v_{2k+4}\}$ $V(P_{n,1,2})$, $k \geq 4$.

For the $ext^{(1)}(P_{n,1,2}$, The vertex and the edge sets are as under:

$V(P_{n,1,2}) = \{u_1, u_2, ..., u_n, v_1, v_2, ..., v_n, w_1, w_2, ..., w_n\}$

And

$E(P_{n,1,2}) = \{u_i u_{i+2}, u_i v_i\}$ $\{v_i v_{i+1}\}$ $\{v_i w_i\}$

for $1 \leq i \leq n$, respectively, where the indices $n + 1$ and $n + 2$ must be replaced by 1 and 2 respectively.

Foe $ext^{(2)}(P_{n,1,2})$, Following are the set of vertices and the set of edges denoted by
$V(P_{n,1,2})$ and $E(P_{n,1,2})$ as under:

$V(P_{n,1,2}) = \{u_1, u_2, ..., u_n, v_1, v_2, ..., v_n, w_1, w_2, ..., w_n\}$
and
$E(P_{n,1,2}) = \{u_i u_{i+2}, u_i v_i\}$ $\{v_i v_{i+1}\}$ $\{v_i w_i\}$ $\{w_i w_{i+1}\}$

for $1 \leq i \leq n$, respectively, where the indices $n + 1$ and $n + 2$ must be replaced by 1 and 2 respectively.

The graph $P_{n,1,2}$ for particular value of $n = 22$ for pendent and the prism graphs are shown in figure,

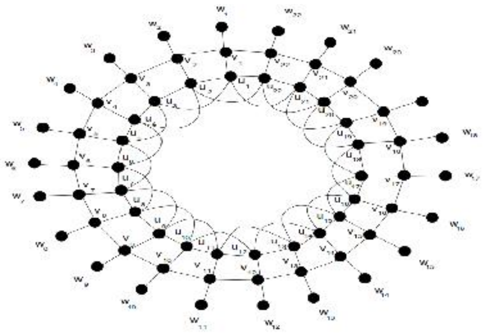

Figure 8 The pendent graph $P_{22,1,2}$

Representation of vertices of $ext^{(1)}(P_{n,1,2}))$ and $(ext^{(2)}(P_{n,1,2}))$.

$$r(w_{2i}/w) = \begin{cases} (3,3,k+3), & if \quad i=1; \\ (i+2,i,k-i+5), & if \quad 2 \leq i \leq k; \\ (k+3,k+1,3), & if \quad i=k+1; \\ (k+2,k+2,1), & if \quad i=k+2; \\ (k+1,k+1,3), & if \quad i=k+3; \\ (2k-i+4,2k-i+5,i-k+1), & if \quad k+4 \leq i \leq 2k+1; \end{cases}$$

And

$$r(w_{2i-1}/w) = \begin{cases} (2,4,k+3), & if \quad i=1; \\ (3,3,k+4), & if \quad i=2; \\ (i+1,i,k-i+6), & if \quad 3 \leq i \leq k; \\ (k+2,k+1,4), & if \quad i=k+1; \\ (k+2,k+2,2), & if \quad i=k+2; \\ (k+1,k+3,2), & if \quad i=k+3; \\ (k,k+2,4), & if \quad i=k+4; \\ (2k-i+4,2k-i+6,i-k+1), & if \quad k+5 \leq i \leq 2k; \end{cases}$$

Case 4: $n = 3 \pmod 4$, $n \geq 11$, in general form it can be written as

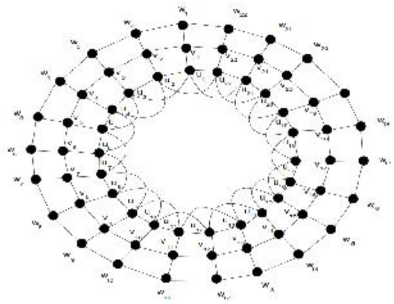

Figure 9 The prism graph $P_{22,1,2}$

$n = 4k+3$, $k \geq 2$, The Resolving set in general form is $W = \{u_1, u_4, u_{2k+3}\}$, $k \geq 2$, and k Z^+.

Theorem 7. We have the metric dimension $\dim(P_{n,1,2}) \leq 3$ for inner cycle and outer cycles for $n \geq 11$.

Proof. In this case it can be written as $n = 4k + 3$, $k \geq 2$, and k Z^+, The resolving set in general form is $W = \{u_1, u_4, u_{2k+3}\}$ $V(P_{n,1,2})$, $k \geq 2$.

This graph has the following set of vertices and the set of edges denoted by $V(P_{n,1,2})$
and $E(P_{n,1,2})$ for the inner cycle and the outer cycle are as under:

$V(P_{n,1,2}) = \{u_1, u_2, ..., u_n, v_1, v_2, ..., v_n, \}$

and the edge set

$E(P_{n,1,2}) = \{u_i u_{i+2}, u_i v_i\}$ $\{v_i v_{i+1}\}$

for $1 \leq i \leq n$, where the indices $n + 1$ and $n + 2$ must be replaced by 1 and 2 respectively.

The graph $P_{n,1,2}$ for particular value of n for inner and outer cycle is shown in figure for $n = 19$.

Representation of the vertices of inner cycle.

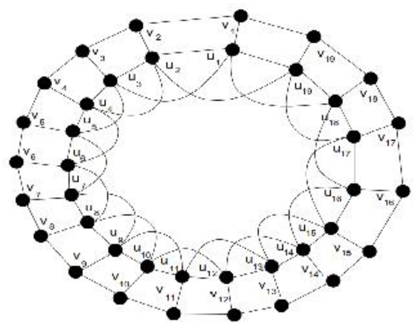

Figure 10 The graph of $P_{19,1,2}$

$$r(u_{2i}/w) = \begin{cases} (1,1,k+2), & if \quad i=1; \\ (i,i-2,k-i+3), & if \quad 3 \leq i \leq k+1; \\ (k,k,2), & if \quad i=k+2; \\ (2k-i+2,2k-i+4,i-k), & if \quad k+3 \leq i \leq 2k+1; \end{cases}$$

and

$$r(u_{2i+1}/w)=\begin{cases}(1,1,k+1), & if \quad i=1;\\ (i,i-1,k-i+2), & if \quad 2\le i\le k+1;\\ (2k-i+2,2k-i+3,i-k), & if \quad k+2\le i\le 2k+1;\end{cases}$$

Representation of the vertices of Outer cycle.

$$r(v_{2i}/w)=\begin{cases}(2,2,k+3), & if \quad i=1;\\ (i+1,i-1,k-i+4), & if \quad 2<i<k-1;\\ (k+1,k-1,3), & if \quad i=k;\\ (k+2,k,1), & if \quad i=k+1;\\ (k+1,k+1,1), & if \quad i=k+2;\\ (k,k+2,3), & if \quad i=k+3;\\ (2k-i+3,2k-i+5,i-k+1), & if \quad k+4\le i\le 2k+1;\end{cases}$$

And

$$r(v_{2i+1}/w)=\begin{cases}(1,3,k+3), & if \quad i=1;\\ (2,2,k+2), & if \quad i=2;\\ (i,i-1,k-i+4), & if \quad 3\le I\le k;\\ (k+1,k,2), & if \quad i=k+1;\\ (k+1,k+2,2), & if \quad i=k+3;\\ (2k-i+4,2k-i+5,i-k), & if \quad k+4\le i\le 2k+2;\end{cases}$$

Theorem 8 We have the metric dimension $\dim(ext^{(1)}(P_{n,1,2}))\le 3$ and
$\dim(ext^{(2)}(P_{n,1,2}))\le 3$ for which $n\ge 11$.

Proof.

In this case it can be written as $n = 4k+3$, $k\ge 2$, and $k \in Z^+$, The resolving set in general form is $W = \{u_1, u_4, u_{2k+3}\} \subset V(P_{n,1,2})$, $k\ge 2$.

The graph $ext^{(1)}(P_{n,1,2})$ has the following set of vertices and the set of edges denoted by $V(P_{n,1,2})$ and $E(P_{n,1,2})$ for Pendent graph are as under:

$V(P_{n,1,2}) = \{u_1, u_2, ..., u_n, v_1, v_2, ..., v_n,\}$

and the edge set

$E(P_{n,1,2}) = \{u_i u_{i+2}, u_i v_i\} \cup \{v_i v_{i+1}\} \cup \{v_i w_i\}$

for $1\le i\le n$, where the indices $n+1$ and $n+2$ must be replaced by 1 and 2 respectively.

The graph $ext^{(2)}(P_{n,1,2})$ has the following set of vertices and the set of edges denoted

by $V(P_{n,1,2})$ and $E(P_{n,1,2})$ for Prism graph are as under:

$V(P_{n,1,2}) = \{u_1, u_2, ..., u_n, v_1, v_2, ..., v_n,\}$

and the edge set

$E(P_{n,1,2}) = \{u_i u_{i+2}, u_i v_i\} \cup \{v_i v_{i+1}\} \cup \{v_i w_i\} \cup \{w_i w_{i+1}\}$

for $1\le i\le n$, where the indices $n+1$ and $n+2$ must be replaced by 1 and 2 respectively.

The graph $P_{n,1,2}$ for particular value of $n = 19$ for pendent and the prism graphs are shown in figure,

Representation of vertices of $ext^{(1)}(P_{n,1,2})$ and $ext^{(2)}(P_{n,1,2})$

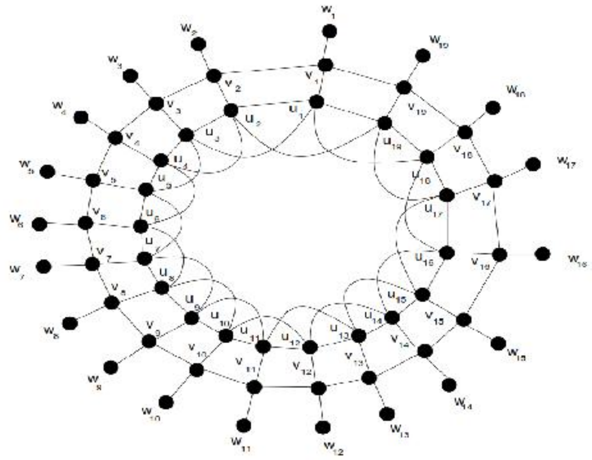

Figure 11 The pendent graph of $P_{19,1,2}$

$$r(w_{2i}/w)=\begin{cases}(3,3,k+4), & if \quad i=1;\\ (i+2,i,k-i+5), & if \quad 2\le i\le k-1;\\ (k+2,k,4), & if \quad i=k;\\ (k+3,k+1,2), & if \quad i=k+1;\\ (k+2,k+2,2), & if \quad i=k+2;\\ (k+1,k+3,4), & if \quad i=k+3;\\ (2k-i+4,2k-i+6,i-k+2), & if \quad k+4\le i\le 2k+1;\end{cases}$$

And

$$r(w_{2i-1}/w)=\begin{cases}(2,4,k+4), & if \quad i=1;\\ (3,3,k+3), & if \quad i=2;\\ (i-1,i,k-i+5), & if \quad 3\le i\le k;\\ (k+2,k+1,3), & if \quad i=k+1;\\ (k+3,k+2,1), & if \quad i=k+2;\\ (k+2,k+3,3), & if \quad i=k+3;\\ (2k-i+5,2k-i+6,i-k+1), & if \quad k+4\le i\le 2k+2;\end{cases}$$

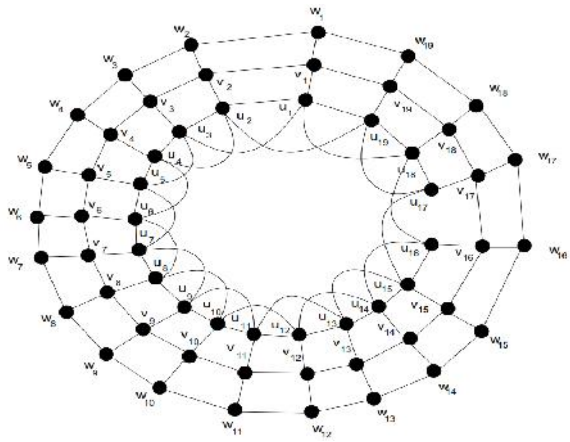

Figure 12 The prism graph of $P_{19,1,2}$

CONCLUSION

The purpose of this paper was to find the metric dimension of $P_{n,1,2}$ by using the technique of shortest distance, showing that the distinct vertices has distinct repre- sentation with respect to the resolving set W. The resolving set W is a set of vertices which is subset of the set of vertices of the graph G denoted by $V(G)$. Keeping in view the very fact that no two vertices of G has same representation with respect to the resolving set W.

We have found the metric dimension for inner and outer cycles and its first and second extensions of the graph $P_{n,1,2}$ which are named as pendent graph and prism graphs. We have also observed that the metric dimension of all these graphs is bounded and $n = 1 \pmod 4$ for $n \geq 14$ of pendent graph and prism graphs which is bounded above by 4. Note that only four vertices are appropriately chosen

suffices to resolve all the vertices of these graphs except for $n = 0 \pmod 4$ for $n \geq 16$ for pendent and the prism graph, also for $n = 2 \pmod 4$ for $n \geq 18$ and $n = 3 \pmod 4$ for $n \geq 14$ for pendent, prism graphs for which only three vertices are appropriately chosen suffices to resolve all the vertices of these graphs for which the metric dimension of these cases is bounded by 3.

We have proved that the metric dimension of $P_{n,1,2}$ is bounded for inner and outer cycles as given below:

A. $\dim(ext^{(2)}(P_{n,1,2})) \leq 3$ for $n = 0 \pmod 4$ and $n \geq 16$
B. $\dim(ext^{(2)}(P_{n,1,2})) \leq 4$ for $n = 1 \pmod 4$ and $n \geq 17$
C. $\dim(ext^{(2)}(P_{n,1,2})) \leq 3$ for $n = 2 \pmod 4$ and $n \geq 18$
D. $\dim(ext^{(2)}(P_{n,1,2})) \leq 3$ for $n = 3 \pmod 4$ and $n \geq 11$

According to results of this paper the metric dimension is bounded for all cases of $P_{n,1,2}$ and there is an open problem for constant metric dimension of these cases.

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