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RESEARCH ARTICLE

TOTAL EDGE LUCAS IRREGULAR LABELING FOR SOME CYCLE RELATED GRAPHS

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ABSTRACT

Let G= (V, E) be a (p, q) – graph. A total edge Lucas irregular labeling $f: V(G) \cup E(G) \rightarrow \{1,2,3, \ldots, K\}$ of a graph G=(V,E) is a labeling of vertices and edges of G in such a way that for any different edges xy and x'y' their weights f(x)+f(xy)+f(y) and f(x')+f(x'y')+f(y') are distinct Lucas numbers. The total edge Lucas irregularity strength, tels (G), is defined as the minimum K for which G has a total edge Lucas irregular labeling. In this paper, we prove that the graphs such as $C_m@P_n$, $C_m@K_{1,n}$, $C_m@2P_n$ and $C_n \odot K_1$ admit the total edge Lucas irregular labeling.

INTRODUCTION

By a graph, we mean a finite, undirected graph without loops and multiple edges, for terms not defined here, we refer to Harary [2]. By labeling we mean any mapping that carries a set of graph elements to a set of numbers (usually positive integers), called labels. The notion of a total vertex irregular labeling and total edge irregular labeling are introduced by Baca *et al*. [1]A total vertex irregular labeling on a graph G with v vertices and e edges is an assignment of integer labels to both vertices and edges so that the weights calculated at vertices are distinct. The weight of a vertex v in G is defined as the sum of the label of v and the labels of all the edges incident with v, that is wt(v)=$\lambda(v) + \sum_{uv \epsilon E} \lambda(uv)$.The total vertex irregularity strength of G, denoted by tvs(G), is the minimum value of the largest label over all such irregular assignments.

For a graph G = (V,E), define a labeling f :V(G) ∪E (G) → {1,2,…,K} to be an edge irregular total K-labeling of the graph G if for every two different edges xy and $x'y'$ of G the edge weights $wt(xy) \neq wt(x'y')$. The total edge irregularity strength, tels(G), is defined as the minimum K for which has an edge irregular total K- labeling.We defined the total edge Lucas irregular labeling [3].

MAIN RESULTS

Definition: 2.1

A total edge Lucas irregular labeling $f: V(G) \cup E(G) \rightarrow \{1,2,3, \ldots, K\}$ of a graph G=(V,E) is a labeling of vertices and edges of G in such a way that for any different edges xy and x'y' their weights f(x)+f(xy)+f(y) and f(x')+f(x'y')+f(y') are distinct Lucas numbers where Lucas series is $L_1 = 1$, $L_2 = 3$, $L_3 = 4$, L_4=7, L_5= 11, L_6=18, L_7=29 etc.,The total edge Lucas irregularity strength, tels is defined as the minimum K for which G has total edge Lucas irregular labeling[3].Note that if f is a total edge Lucas irregular labeling of $G = (V, E)$ with $|V(G)| = p$ and $|E(G)| = q$ then $L_2(3) \leq wt(xy) \leq L_{q+1}$ which implies that $tels \geq \left\lceil \frac{L_{q+1}}{3} \right\rceil$

Theorem 2.2

The graph $C_m@P_n$ admits a total edge Lucas irregular labeling and $tels(C_m@P_n) = L_{m+n-1} \quad for\ all\ m\ and\ n.$

Proof

Let $G = C_m@P_n$

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Let $u_1, u_2,\ldots,u_m$ be the vertices of a cycle C_m and $v_1, v_2,\ldots,v_n, v_{n+1}$ be the vertices of a path P_n
Which is attached with a vertex $(u_m = v_1)$ of C_m
Here,$E(G) = \{x_i = u_i u_{i+1}: 1 \le i \le m-1\} \cup \{u_m u_1\} \cup \{y_i = v_j v_{j+1}: 1 \le j \le n\}$
Then,$|V(G)| = m+n \ and \ |E(G)| = m+n$
Define $f: V(G) \cup E(G) \to \{1,2,3,\ldots,L_{m+n-1}\}$ by

$f(u_1) = 1$
$f(u_2) = 1$
$f(u_3) = 2$
$f(u_i) = L_{i-2}, \quad 4 \le i \le m$
$f(x_1) = 1$
$f(x_2) = 1$
$f(x_3) = 2$
$f(x_i) = L_{i-1}, \quad 4 \le i \le m-1$
$f(x_m) = L_{m+1} - L_{m-2} - 1$
$f(u_m) = L_{m-2} = f(v_1)$
$f(v_i) = L_{m+i-2}, \quad 2 \le i \le n+1$
$f(y_1) = L_{m+1} - L_{m-2}$
$f(y_i) = L_{m+i-1}, \quad 2 \le i \le n$
By this labeling,
$wt(x_1) = f(u_1) + f(x_1) + f(u_2)$
$= 1+1+1$
$= 3$
$= L_2$
$wt(x_2) = f(u_2) + f(x_2) + f(u_3)$
$= 1+1+2$
$= 4$
$=L_3$
$wt(x_3) = f(u_3) + f(x_3) + f(u_4)$
$= 2+2+3$
$= 7$
$= L_4$

In general,
$wt(x_i) = f(u_i) + f(x_i) + f(u_{i+1})$
$= L_{i-2} + L_{i-1} + L_{i-1}$
$=L_i + L_{i-1}$
$= L_{(i+1)-1} + L_{(i+1)-2}$,
$= L_{i+1}, \quad 4 \le i \le m-1$
$wt(x_m) = f(u_m) + f(x_m) + f(u_1)$
$= L_{m-2} + L_{m+1} - L_{m-2} - 1 + 1$
$= L_{m+1}$

Thus, the weights o $x_1, x_2, x_3,\ldots, x_m$ are $L_2, L_3, L_4,\ldots, L_m, L_{m+1}$.
$wt(y_1) = f(v_1) + f(y_1) + f(v_2)$
$= L_{m-2} + L_{m+1} - L_{m-2} + L_m$
$= L_{m+1} + L_m$
$=L_{(m+2)-2} + L_{(m+2)-1}$
$= L_{m+2}$
$wt(y_2) = f(v_2) + f(y_2) + f(v_3)$
$= L_m + L_{m+1} + L_{m+1}$
$= L_{(m+2)-2} + L_{(m+2)-1} + L_{m+1}$
$=L_{m+2} + L_{m+1}$
$= L_{(m+3)-1} + L_{(m+3)-2}$
$=L_{m+3}$

In general,
$wt(y_i) = f(v_i) + f(y_i) + f(v_{i+1})$
$= L_{m+i-2} + L_{m+i-1} + L_{m+i-1}$
$= L_{m+i} + L_{m+i-1}$
$= L_{(m+i+1)-1} + L_{(m+i+1)-2}$
$= L_{m+i+1}$, $\quad 2 \le i \le n$

Thus, the weights of $y_1, y_2, y_3,\ldots, y_n$ are $L_{m+2}, L_{m+3}, L_{m+4},\ldots, L_{m+n+1}$.
Therefore,the weights of $x_1, x_2, x_3,\ldots, x_m, y_1, y_2, y_3, \ldots, y_n$ are $L_2, L_3, L_4,\ldots, L_m, L_{m+1}, L_{m+2}, L_{m+3}, L_{m+4},\ldots, L_{m+n+1}$ respectively.

Hence, The graph $C_m@P_n$ admits a total edge Lucas irregular labelingand $tels(C_m@P_n) = L_{m+n-1} \quad for\ all\ m\ and\ n.$

Theorem 2.3

The graph $C_m@K_{1,n}$admits a total edge Lucas irregular labeling and $tels(C_m@K_{1,n}) = L_{m+n+1} - L_{m-2} - \left\lfloor \frac{L_{m+n+1} - L_{m-2}}{2} \right\rfloor \quad for\ all\ m\ and\ n.$

Proof

Let $G = C_m@K_{1,n}$
Let $V(G) = \{u_i : 1 \le i \le m\} \cup \{v_i : 1 \le i \le n\}$ and $E(G) = \{x_i = u_i u_{i+1}: 1 \le i \le m-1\} \cup \{u_m u_1\} \cup \{y_i = u_m v_i: 1 \le i \le n\}$
Then,$|V(G)| = m+n \ and \ |E(G)| = m+n$
Define $f: V(G) \cup E(G) \to \left\{1,2,3,\ldots,L_{m+n+1} - L_{m-2} - \left\lfloor \frac{L_{m+n+1} - L_{m-2}}{2} \right\rfloor\right\}$ by

$f(u_1) = 1$
$f(u_2) = 1$
$f(u_3) = 2$
$f(u_i) = L_{i-2}, \quad 4 \le i \le m$
$f(x_1) = 1$
$f(x_2) = 1$
$f(x_3) = 2$
$f(x_i) = L_{i-1}, \quad 4 \le i \le m-1$
$f(x_m) = L_{m+1} - L_{m-2} - 1$
$f(v_i) = \left\lfloor \frac{L_{m+1+i} - L_{m-2}}{2} \right\rfloor, \quad 1 \le i \le n$
$f(y_i) = L_{m+i+1} - L_{m-2} - \left\lfloor \frac{L_{m+i+1} - L_{m-2}}{2} \right\rfloor, \quad 1 \le i \le n$
By this labelling,
$wt(x_1) = f(u_1) + f(x_1) + f(u_2)$
$= 1+1+1$
$= 3$
$= L_2$
$wt(x_2) = f(u_2) + f(x_2) + f(u_3)$
$= 1+1+2$
$= 4$
$=L_3$
$wt(x_3) = f(u_3) + f(x_3) + f(u_4)$
$= 2+2+3$
$= 7$
$= L_4$
In general,
$wt(x_i) = f(u_i) + f(x_i) + f(u_{i+1})$

$= L_{i-2} + L_{i-1} + L_{i-1}$
$= L_i + L_{i-1}$
$= L_{(i+1)-1} + L_{(i+1)-2}$,
$= L_{i+1}, \quad 4 \le i \le m-1$
$wt(x_m) = f(u_m) + f(x_m) + f(u_1)$
$= L_{m-2} + L_{m+1} - L_{m-2} - 1 + 1$
$= L_{m+1}$
Thus, the weights of $x_1, x_2, x_3, \dots, x_m$ are $L_2, L_3, L_4, \dots, L_m, L_{m+1}$.
In general,
$wt(y_i) = f(u_m) + f(y_i) + f(v_i)$
$= L_{m-2} + L_{m+i+1} - L_{m-2} - \left\lfloor \frac{L_{m+i+1} - L_{m-2}}{2} \right\rfloor + \left\lfloor \frac{L_{m+i+1} - L_{m-2}}{2} \right\rfloor$
$= L_{m+i+1}, \quad 1 \le i \le n$

Thus, the weights of $y_1, y_2, y_3, \dots, y_n$ are $L_{m+2}, L_{m+3}, L_{m+4}, \dots, L_{m+n+1}$.
Therefore, the weights of $x_1, x_2, x_3, \dots, x_m, y_1, y_2, y_3, \dots, y_n$ are $L_2, L_3, L_4, \dots, L_m, L_{m+1}, L_{m+2}, L_{m+3}, L_{m+4}, \dots, L_{m+n+1}$ respectively. Hence, the graph $C_m@K_{1,n}$ admits a total edge Lucas irregular labelling and $tels(C_m@K_{1,n}) = L_{m+n+1} - L_{m-2} - \left\lfloor \frac{L_{m+n+1} - L_{m-2}}{2} \right\rfloor$ *for all m and n.*

Theorem 2.4

The graph $C_m@2P_n$ admits a total edge Lucas irregular labeling and
$tels(C_m@2P_n) =$
$L_{m+2n+1} - L_{m+2n-1} - L_{m+2n-3}$ *for all m and n.*

Proof

Let $G = C_m@2P_n$
Let $V(G) = \{w_i: 1 \le i \le m\} \cup \{u_i: 1 \le i \le n\} \cup \{v_i: 1 \le i \le n\}$ be the vertex set of G and the vertices of w_1 and w_m of C_m are identified with v_1 and u_1 of two paths of length n respectively.

Here, $E(G) = \{x_i = w_i w_{i+1}: 1 \le i \le m-1\} \cup \{w_m w_1\} \cup \{z_i = v_i v_{i+1}, y_i = u_i u_{i+1}: 1 \le i \le n\}$
Then, $|V(G)| = m + 2n$ *and* $|E(G)| = m + 2n$
Define $f: V(G) \cup E(G) \to \{1,2,3,\dots, L_{m+2n+1} - L_{m+2n-1} - L_{m+2n-3}\}$ by

$f(w_1) = 1$
$f(w_2) = 1$
$f(w_3) = 2$
$f(w_i) = L_{i-2}, \quad 4 \le i \le m$
$f(x_1) = 1$
$f(x_2) = 1$
$f(x_3) = 2$
$f(x_i) = L_{i-1}, \quad 4 \le i \le m-1$
$f(x_m) = L_{m+1} - L_{m-2} - 1$
$f(u_1) = L_{m+1}$
$f(u_i) = L_{m+2i-2}, \quad 2 \le i \le n$
$f(y_1) = L_{m+2} - L_{m+1} - 1$
$f(y_2) = L_{m+4} - L_{m+1} - L_{m+2}$
$f(y_i) = L_{m+2i} - L_{m+2i-2} - L_{m+2i-4}, \quad 3 \le i \le n$
$f(v_1) = L_{m+2}$
$f(v_i) = L_{m+2i-1}, \quad 2 \le i \le n$
$f(z_1) = L_{m+3} - L_{m-2} - L_{m+2}$
$f(z_2) = L_{m+5} - L_{m+3} - L_{m+2}$
$f(z_i) = L_{m+2i+1} - L_{m+2i-1} - L_{m+2i-3}, \quad 3 \le i \le n$

By this labeling,
$wt(x_1) = f(w_1) + f(x_1) + f(w_2)$
$= 1+1+1$
$= 3$
$= L_2$
$wt(x_2) = f(w_2) + f(x_2) + f(w_3)$
$= 1+1+2$
$= 4$
$= L_3$
$wt(x_3) = f(w_3) + f(x_3) + f(w_4)$
$= 2+2+3$
$= 7$
$= L_4$
In general,
$wt(x_i) = f(w_i) + f(x_i) + f(w_{i+1})$
$= L_{i-2} + L_{i-1} + L_{i-1}$
$= L_i + L_{i-1}$
$= L_{(i+1)-1} + L_{(i+1)-2}$,
$= L_{i+1}, \quad 4 \le i \le m-1$
$wt(x_m) = f(w_m) + f(x_m) + f(w_1)$
$= L_{m-2} + L_{m+1} - L_{m-2} - 1 + 1$
$= L_{m+1}$
Thus, the weights of $x_1, x_2, x_3, \dots, x_m$ are $L_2, L_3, L_4, \dots, L_m, L_{m+1}$.
$wt(y_1) = f(w_1) + f(y_1) + f(u_1)$
$= 1 + L_{m+2} - L_{m+1} - 1 + L_{m+1}$
$= L_{m+2}$
$wt(y_2) = f(u_1) + f(y_2) + f(u_2)$
$= L_{m+1} + L_{m+4} - L_{m+1} - L_{m+2} + L_{m+2}$
$= L_{m+4}$
In general,
$wt(y_i) = f(u_{i-1}) + f(y_i) + f(u_i)$
$= L_{m+2i-4} + L_{m+2i} - L_{m+2i-2} - L_{m+2i-4} + L_{m+2i-2}$
$= L_{m+2i}, \quad 3 \le i \le n$
Thus, the weights of $y_1, y_2, y_3, \dots, y_n$ are $L_{m+2}, L_{m+4}, \dots, L_{m+2n}$.
$wt(z_1) = f(w_m) + f(z_1) + f(v_1)$
$= L_{m-2} + L_{m+3} - L_{m-2} - L_{m+2} + L_{m+2}$
$= L_{m+3}$
$wt(z_2) = f(v_1) + f(z_2) + f(v_2)$
$= L_{m+2} + L_{m+5} - L_{m+3} - L_{m+2} + L_{m+3}$
$= L_{m+5}$
In general,
$wt(z_i) = f(v_{i-1}) + f(z_i) + f(v_i)$
$= L_{m+2i-3} + L_{m+2i+1} - L_{m+2i-1} - L_{m+2i-3} + L_{m+2i-1}$
$= L_{m+2i+1}, \quad 3 \le i \le n$
Thus, the weights of $z_1, z_2, z_3, \dots, z_n$ are $L_{m+3}, L_{m+5}, \dots, L_{m+2n+1}$.
Therefore, the weights of $x_1, x_2, x_3, \dots, x_m, y_1, z_1, y_2, z_2, \dots, y_n, z_n$ are $L_2, L_3, L_4, \dots, L_m, L_{m+1}, L_{m+2}, L_{m+3}, L_{m+4}, \dots, L_{m+2n}, L_{m+2n+1}$ respectively.

Hence, The graph $C_m@2P_n$ admits a total edge Lucas irregular labelingand $tels(C_m@2P_n) = L_{m+2n+1} - L_{m+2n-1} - L_{m+2n-3}$ $for\ all\ m\ and\ n$

Theorem 2.5

The graph $C_n \odot K_1$ admits a total edge Lucas irregular labeling and $tels(C_n \odot K_1) = L_{2n} - L_{2n-3} - 1$ $for\ all\ n \geq 3$.

Proof

Let $G = C_n \odot K_1$
Let the vertex set be $V(C_n) = \{u_1, u_2, \ldots, u_n\}$and $v_1, v_2, \ldots, v_n$ be the vertices adjacent to each vertex of C_n, the edge set $E(G) = \{x_i = u_i u_{i+1}: 1 \leq i \leq n-1\} \cup \{y_i = u_i v_i: 1 \leq i \leq n\} \cup \{x_n = u_1 u_n\}$

Then $|V(G)| = 2n$ and $|E(G)| = 2n$

Define $f: V(G) \cup E(G) \to \{1,2,3,\ldots,L_{2n} - L_{2n-3} - 1\}$ by
$f(u_1) = 1$
$f(u_2) = 1$
$f(u_i) = L_{2i-3}, \quad 3 \leq i \leq n$
$f(v_1) = 2$
$f(v_2) = 5$
$f(v_i) = \left\lceil \frac{L_{2i+1} - L_{2i-3}}{2} \right\rceil, \quad 3 \leq i \leq n$
$f(x_1) = 1$
$f(x_2) = 2$
$f(x_i) = L_{2i-4}, \quad 3 \leq i \leq n-1$
$f(x_n) = L_{2n} - L_{2n-3} - 1$
$f(y_1) = 1$
$f(y_2) = 5$
$f(y_i) = L_{2i+1} - L_{2i-3-} \left\lceil \frac{L_{2i+1} - L_{2i-3}}{2} \right\rceil, \quad 3 \leq i \leq n$

By this labeling,
$wt(x_1) = f(u_1) + f(x_1) + f(u_2)$
$= 1 + 1 + 1$
$= 3$
$= L_2$
$wt(x_2) = f(u_2) + f(x_2) + f(u_3)$
$= 1 + 2 + 4$
$= 7$
$= L_4$
$wt(x_3) = f(u_3) + f(x_3) + f(u_4)$
$= 4 + 3 + 11$
$= 18$
$= L_6$

In general,
$wt(x_i) = f(u_i) + f(x_i) + f(u_{i+1})$
$= L_{2i-3} + L_{2i-4} + L_{2i-1}$
$= L_{(2i-2)-1} + L_{(2i-2)-2} + L_{2i-1}$
$= L_{2i-2} + L_{2i+1}$
$= L_{2i}, \quad 4 \leq i \leq n-1$

$wt(x_n) = f(u_n) + f(x_n) + f(u_1)$
$= L_{2n-3} + L_{2n} - L_{2n-3} - 1 + 1$
$= L_{2n}$

Thus, the weights of$x_1, x_2, \ldots, x_n$ are $L_2, L_4, \ldots, L_{2n}$ respectively.
$wt(y_1) = f(u_1) + f(y_1) + f(v_1)$
$= 1 + 1 + 2$
$= 4$
$= L_3$
$wt(y_2) = f(u_2) + f(y_2) + f(v_2)$
$= 1 + 5 + 5$
$= 11$
$= L_5$
$wt(y_3) = f(u_3) + f(y_3) + f(v_3)$
$= 4 + 12 + 13$
$= 29$
$= L_7$

In general,
$wt(y_i) = f(u_i) + f(y_i) + f(v_i)$
$= L_{2i-3} + L_{2i+1} - L_{2i-3} - \left\lceil \frac{L_{2i+1} - L_{2i-3}}{2} \right\rceil + \left\lceil \frac{L_{2i+1} - L_{2i-3}}{2} \right\rceil$
$= L_{2i+1}, \quad 4 \leq i \leq n$

Thus, the weights of$y_1, y_2, \ldots, y_n$ are $L_3, L_5, \ldots, L_{2n+1}$ respectively.
Therefore, the weights of $x_1, y_1, x_2, y_2, x_3, y_3, \ldots, x_n, y_n$ are $L_2, L_3, L_4, \ldots, L_{2n}, L_{2n+1}$
respectively.

Hence, The graph $G = C_n \odot K_1$ admits a total edge Lucas irregular labelling and $tels(C_n \odot K_1) = L_{2n} - L_{2n-3} - 1$ $for\ all\ n \geq 3$.

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