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RESEARCH ARTICLE

STOCHASTIC MODEL FOR THE PREDICTION OF EXPECTED TIME OF PERSONNEL TO LEAVE THE ORGANISATION

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ABSTRACT

Attrition is a common event in any organization or industry. The leaving of personnel may be due to unsatisfactory decisions regarding pay revision, promotion and work targets as observed by employees. So the successive decisions contribute to the propensity of individual to leave the organization and also, the emotion developed by any employee at the decision epoch. There is a maximum level of these factors. When any one of these two feelings crosses the threshold level then a person decides to leave the organization. So recruitment should be made at random time intervals to compensate the loss of manpower. In this paper the expected time to leave the organization is derived using the shock model and cumulative damage process.

INTRODUCTION

In any organisation or industry manpower otherwise called the human resource is an important factor. A number of individuals are employed in different sections of the organization or industry. Decisions regarding the production schedule, marketing and related aspects are taken by the management and accordingly activities are formulated. Periodical review regarding the work schedule, promotions and pay revisions are also made. These decisions may be of satisfactory nature for some individuals and may not be satisfactory and up to the expectations of some employees. This results in leaving of personnel which otherwise is called attrition. This phenomenon is very much pronounced in software organisations. The propensity to leave the organisation also changes due to the decisions of the management at decision making epochs. As and when the cumulative propensity to leave the organisation crosses a particular level called the threshold the individual leaves the organisation.

Another factor which contributes to the exit of a person from the organisation is the emotion which occurs at decision epochs which are at random time intervals. The level of the emotion suffered by an individual is of random character. But when it crosses a particular level called the threshold then the individuals quit the organisation. The maximum level of emotion may occur at a particular epoch of decision making and cross the threshold level. It is also likely that it is not of cumulative type. Under these assumptions a stochastic model is developed to estimate the expected time of an individual to leave the organisation.

The use of statistical methods for manpower planning is discussed in detail by Bartholomew (1971). The concept of shock model and wear process discussed by Esary et.al (1973) is very much useful in the application of shock model in manpower planning. The concept of Setting the Clock Back to Zero Property has been discussed by Raja rao and Talwalker (1990), Sathiyamoorthi and Elangovan (1998) have used the Shock model approach for the determination of expected time to recruitment in on organization.

Assumptions

- At every decision epoch a person is subjected to disappointment and it influences the propensity to leave the organisation. It is of random character and cumulative.
- The feeling of emotion due to disappointment is possible. If the level of emotional feeling at a decision

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epoch is higher than the threshold the attrition occurs.

- Decisions by the management are taken at certain epochs which have random inter arrival times.
- The two characteristics namely disappointment and emotion are independent of each other.

Notations

X_i	:	The random amount of propensity to leave on the i^{th} decision epoch, i = 1,2,3,.….,k with probability density function q(.) with cumulative distribution function Q(.)
Y_i	:	The random amount of emotional feeling on the i^{th} decision epoch, i = 1,2,3,……,k with probability density function p(.) and cumulative distribution function P(.)
Z_1	:	The thershold level of propensity to leave, a random variable with probability density function m(.) and cumulative distribution function M(.)
Z_2	:	The thershold level of emotion feeling, a random variable with probability density function $n(.)$ and cumulative distribution function N(.)
U_i	:	The inter arrival time between successive decision epochs, which are i.i.d with probability density function $f(.)$ and cumulative distribution function F (.).
T	:	Time to leave the organisation

RESULTS

The survivor function is defined as S(t) = P[T > t] = P [The cumulative level of propensity to leave the organisation due 'k' decisions at 'k' epochs not greater than the threshold level and maximum level of emotion does not cross the corresponding threshold].

Therefore

S(t) = Pr [that there exactly 'k' decision & epochs in (0, t)]

$$S(t) = P\left[\sum_{i=1}^{k} x_i < z_1 \cap \max(y_1, y_2, \dots y_k) < z_2\right]$$

$$= \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)] \times P\left[\sum_{i=1}^{k} x_i < z_1\right] P\left[\sum_{i=1}^{k} \max(y_1, y_2, \dots y_k) < z_2\right]$$

$$= \sum_{k=0}^{\infty}[\mathrm{F_k(t) - F_{k+1}(t)}] \times \mathrm{P}\left[\sum_{i=1}^{k} \mathrm{x_i} < \mathrm{z_1}\right] \bigcap_{i=1}^{k} \mathrm{P}[\mathrm{y_i} < \mathrm{z_2}]$$

Hence the expression for S(t) is given by

$$S(t) = \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)] \left[\int_0^{\infty} q_k(x)\overline{M(x)}\, dx\right]\left[\int_0^{\infty} p_k(y)\overline{N(y)}\, dy\right]$$

let us assume that $Z_1 \sim \exp(\theta)$ *and hence* $M(x) = 1 - e^{-\theta x}, \overline{M(x)} = e^{-\theta x}$

$Z_2 \sim \exp(\alpha)$ *and so* $\overline{N(y)} = e^{-\lambda y}$

$$\therefore\ \mathrm{S(t)} = \sum_{k=0}^{\infty}[\mathrm{F_k(t) - F_{k+1}(t)}] \left[\int_0^{\infty} \mathrm{q_k(x)e^{-\theta x}\, dx}\right]\left[\int_0^{\infty} \mathrm{p_k(y)e^{-\lambda y}\, dy}\right]$$

$$= \sum_{k=0}^{\infty}[\mathrm{F_k(t) - F_{k+1}(t)}]\ [\mathrm{q_k}^*(\theta)\mathrm{p_k}^*(\lambda)]$$

$$= \sum_{k=0}^{\infty}[\mathrm{F_k(t) - F_{k+1}(t)}]\ [\mathrm{q^*(\theta)p^*(\lambda)}]^{k}$$

$$= [\mathrm{F_0(t) - F_1(t)}](1) + [\mathrm{F_1(t) - F_2(t)}][\mathrm{q^*(\theta)p^*(\lambda)}] + [\mathrm{F_1(t) - F_2(t)}][\mathrm{q^*(\theta)p^*(\lambda)}]^2 + \dots\dots\dots\dots\dots\dots$$

$$= 1 - [1 - \mathrm{q^*(\theta)p^*(\lambda)}]\mathrm{F_1(t)} + [1 - \mathrm{q^*(\theta)p^*(\lambda)}]\mathrm{F_2(t)q^*(\theta)p^*(\lambda)}$$

$$= 1 - [1 - \mathrm{q^*(\theta)p^*(\lambda)}]\sum_{k=0}^{\infty}\mathrm{F_k(t)}[\mathrm{q^*(\theta)p^*(\lambda)}]^{k-1} \quad \dots\dots\dots\dots(1.1)$$

Now $L(t)$ *is defined as* $P[T > t]$

Therefore

$$L(t) = 1 - S(t)$$

$$= [1 - \mathrm{q^*(\theta)p^*(\lambda)}]\sum_{k=0}^{\infty}\mathrm{F_k(t)}[\mathrm{q^*(\theta)p^*(\lambda)}]^{k-1}$$

Taking Laplace transform of both sides $L(t)$ *we have*

$$l^*(s) = [1 - \mathrm{q^*(\theta)p^*(\lambda)}]\sum_{k=0}^{\infty}[\mathrm{f}^*(s)]^{k}[\mathrm{q^*(\theta)p^*(\lambda)}]^{k-1}$$

By Simplification of the above we get

$$l^*(s) = \frac{[1 - \mathrm{q^*(\theta)p^*(\lambda)}]\mathrm{f}^*(s)}{[1 - \mathrm{f}^*(s)\mathrm{q^*(\theta)p^*(\lambda)}]} \quad \dots\dots\dots\dots(1.2)$$

To find the $E(T)$ *and* $V(T)$, *we have*

$$E(T) = \left.\frac{-dl^*(s)}{ds}\right|_{s=0}$$

$$V(T) = E(T^2) - [E(T)]^2$$

$$E(T^2) = \left.\frac{-d^2 l^*(s)}{ds^2}\right|_{s=0}$$

we assume that $f(.) \sim \exp(\mu)$ *and* $f^*(s) = \frac{\mu}{\mu + s}$

$q(.) \sim \exp(\varphi)$ *and* $q^*(\theta) = \frac{\varphi}{\theta + \varphi}$

$p(.) \sim \exp(\gamma)$ *and* $p^*(\lambda) = \frac{\gamma}{\lambda + \gamma}$

$$[1 - q^*(\theta)p^*(\lambda)]f^*(s) = \left[1 - \frac{\varphi}{\theta + \varphi}.\frac{\gamma}{\lambda + \gamma}\right]\frac{\mu}{\mu + s}$$

$$[1 - f^*(s)q^*(\theta)p^*(\lambda)] = \left[1 - \frac{\varphi}{\theta + \varphi}.\frac{\gamma}{\lambda + \gamma}.\frac{\mu}{\mu + s}\right]$$

$$l^*(s) = \frac{\left[1 - \frac{\varphi}{\theta + \varphi}.\frac{\gamma}{\lambda + \gamma}\right]\frac{\mu}{\mu + s}}{\left[1 - \frac{\varphi}{\theta + \varphi}.\frac{\gamma}{\lambda + \gamma}.\frac{\mu}{\mu + s}\right]}$$

$$l^*(s) = \frac{\mu(\varphi + \theta)(\gamma + \lambda) - \mu\varphi\gamma}{(\varphi + \theta)(\gamma + \lambda)(\mu + s) - \mu\varphi\gamma} \quad On\ Simplification \quad \dots\dots(1.3)$$

$$l^*(s) = [\mu(\varphi + \theta)(\gamma + \lambda) - \mu\varphi\gamma]\,[(\varphi + \theta)(\gamma + \lambda)(\mu + s) - \mu\varphi\gamma]^{-1} \quad \dots\dots(1.4)$$

$$\frac{\mathrm{d}l^*(\mathrm{s})}{\mathrm{ds}} = [\mu(\varphi + \theta)(\gamma + \lambda) - \mu\varphi\gamma](-1)[(\varphi + \theta)(\gamma + \lambda)(\mu + s) - \mu\varphi\gamma]^{-2}(\varphi + \theta)(\gamma + \lambda)$$

$$-\frac{dl^*(s)}{ds} = (-1)[\mu(\varphi + \theta)(\gamma + \lambda) - \mu\varphi\gamma](-1)[(\varphi + \theta)(\gamma + \lambda)(\mu + s) - \mu\varphi\gamma]^{-2}(\varphi + \theta)(\gamma + \lambda)$$

$$-\frac{dl^*(s)}{ds} = \frac{(\varphi + \theta)(\gamma + \lambda)[\mu(\varphi + \theta)(\gamma + \lambda) - \mu\varphi\gamma]}{[\mu(\varphi + \theta)(\gamma + \lambda) - \mu\varphi\gamma]^2}$$

Therefore $\left.\frac{-dl^*(s)}{ds}\right|_{s=0}$

$$= \frac{(\varphi + \theta)(\gamma + \lambda)[\mu(\varphi + \theta)(\gamma + \lambda) - \mu\varphi\gamma]}{[\mu(\varphi + \theta)(\gamma + \lambda) - \mu\varphi\gamma]^2} \quad on\ simplification$$

$$E(T) = \frac{(\varphi + \theta)(\gamma + \lambda)}{(\mu)(\varphi + \theta)(\gamma + \lambda) - \mu\varphi\gamma} \quad \dots\dots(1.5)$$

From equation (3.4)

$$\left.\frac{-d^2 l^*(s)}{ds^2}\right|_{s=0} = [\mu(\varphi + \theta)(\gamma + \lambda) - \mu\varphi\gamma](-1)(-2)[(\varphi + \theta)(\gamma + \lambda)(\mu + s) - \mu\varphi\gamma]^{-3}[(\varphi + \theta)(\gamma + \lambda)][(\varphi + \theta)(\gamma + \lambda)]$$

$$= \frac{2[(\varphi + \theta)(\gamma + \lambda)]^2}{[(\mu)(\varphi + \theta)(\gamma + \lambda) - \mu\varphi\gamma]^2} \quad on\ simplification$$

$$E(T^2) = 2\left[\frac{(\varphi + \theta)(\gamma + \lambda)}{(\mu)(\varphi + \theta)(\gamma + \lambda) - \mu\varphi\gamma}\right]^2 \quad \dots\dots(1.6)$$

$$V(T) = E(T^2) - [E(T)]^2$$

$$V(T) = 2\left[\frac{(\varphi + \theta)(\gamma + \lambda)}{(\mu)(\varphi + \theta)(\gamma + \lambda) - \mu\varphi\gamma}\right]^2 - \left[\frac{(\varphi + \theta)(\gamma + \lambda)}{(\mu)(\varphi + \theta)(\gamma + \lambda) - \mu\varphi\gamma}\right]^2$$

$$V(T) = \left[\frac{(\varphi + \theta)(\gamma + \lambda)}{(\mu)(\varphi + \theta)(\gamma + \lambda) - \mu\varphi\gamma}\right]^2 \quad \dots..(1.7)$$

Numerical illustrations

The behavior of E(T) and V(T) due to the changes in the different parameters associated with the distribution of the random variables in the model is explained by taking a numerical example.

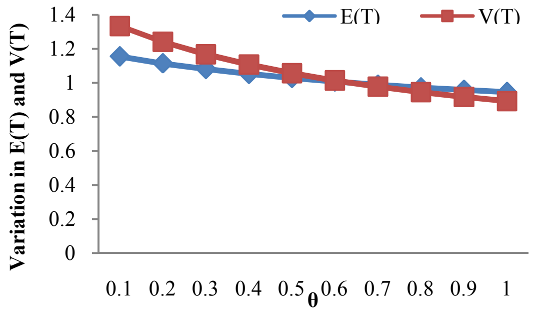

Figure 1 Variation in E (T) and V (T) for Changes in θ

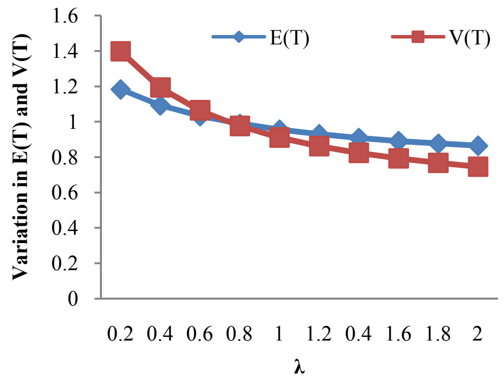

Figure 2 Variation in E (T) and V (T) for Changes in λ

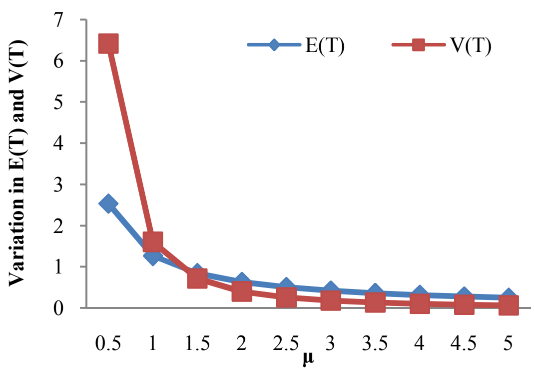

Figure 3 Variation in E (T) and V(T) for Changes in μ

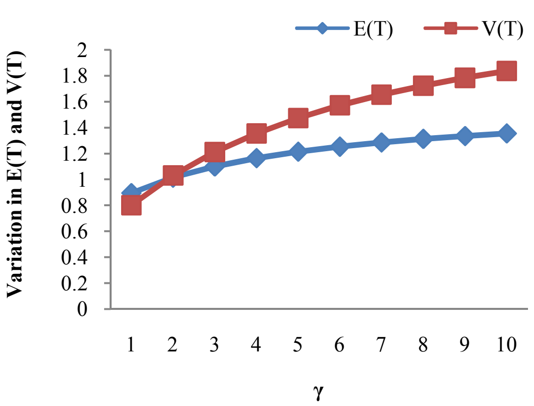

Figure 4 Variation in E (T) and V (T) for Changes in γ

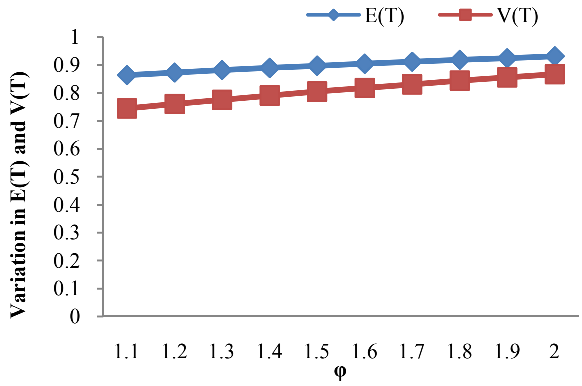

Figure 5 Variation in E(T) and V(T) for Changes in φ

CONCLUSION

1. If the value of the θ which is the parameter of the distribution of the random variable Z_1 denoting the threshold level of propensity to leave the organisation increases, E(T) decreases. This is due to the fact that $Z_1 \sim \exp(\theta)$ and $E(Z_1) = \frac{1}{\theta}$ decreases. Hence it takes less of time to cross the threshold level of propensity, and indicated in Table 1 and Fig (1).
2. If λ which is the parameter of the random variable that represents the contribution to propensity to leave increases than $E(X) = \frac{1}{\lambda}$ decreases. Hence there is smaller contribution to the propensity to leave and so E (T) is on the increase. This is found in Table 2 and Fig (2).
3. If μ which is the parameter of the distribution of the random variableU_i, denoting the inter arrival times between decision epochs increases then$E(U) = \frac{1}{\mu}$ decreases. This implies that the mean time interval between decisions is smaller. Hence more frequent contribution to propensity makes E(T) to decrease. This is indicated in Table 3 and Fig (3).
4. If the value of γ which is the parameter of the distribution of the random variable Z_2 representing the emotion threshold increases then E(T) increases. This is due to the fact that $E(Y) = \frac{1}{\gamma}$ and as γ increases the average level of emotion decreases Hence E(T) is increasing. This is indicated in Table 4 and Fig (4).
5. If φ which is the parameter of the distribution of the random variable X denoting the propensity to leave in each decision epoch increases than $E(X) = \frac{1}{\varphi}$ decreases. Hence the average contribution on each decision epoch becomes smaller. Hence E(T) increases as φ increases. This is indicated in Table 5 and Fig (5).

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