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RESEARCH ARTICLE

ON NANO GENERALIZED STAR PRE-CLOSED SETS IN NANO TOPOLOGICAL SPACES

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ABSTRACT

The purpose of this paper is to define and study the notion of nano g^*p-closed sets and nano g^*p-open sets in nano topological spaces. Also to investigate nano g*p-interior and nano g*p-closure.

INTRODUCTION

Levine [6] introduced the class of g-closed sets in 1970. Lellis Thivagar [5] introduced Nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximations of X. He has also defined Nano closed sets Nano-interior and Nano-closure of a set. Bhuvaneswari [1, 2, 3& 4, 9] introduced Nano g-closed, Nano gs-closed, Nano αg-closed, Nano gα-closed, Nano gr-closed and Nano rg-closed sets, Nano gp-closed sets and Nano pg-closed sets and studied some their properties.

Definition 1.1 [10]

A subset A of a topological space (X,τ) is called a generalized star closed set if cl(A) ⊆U whenever A ⊆ U and U is g-open in (X, τ).

Definition 1.2 [5]

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided intodisjoint equivalence classes. Elements belongings to the same equivalence class are said to be indiscernible with one another.

The pair (U, R) is said to be the approximation space. Let X ⊆U

1. The lower approximation of X with respect to R is the set of all objects, which can be for certainly classified as X with respect to R and its denoted by $L_R(X)$.
   That is $L_R(X) = U_{X\in U}\{R(x): R(x) \subseteq X\}$,where R(x) denotes the equivalence class determined by X∈ U.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$.
3. That is $U_R(X) = U_{X\in U}\{R(x): R(x) \cap X \neq \Phi\}$
4. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$.
   That is $B_R(X) = U_R(X) - L_R(X)$.

Property 1.3 [5]

If (U, R) is an approximation space and X, Y ⊆ U, then

- $L_R(X) \subseteq X \subseteq U_R(X)$
- $L_R(\Phi) = U_R(\Phi) = \Phi$ and $L_R(U) = U_R(U) = U$
- $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
- $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
- $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$

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- $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
- $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
- $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$
- $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
- $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

Definition 1.4 [5]

Let U be the universe, R be an equivalence relation on U and$\tau_R(X) = \{U, \Phi, U_R(X), L_R(X), B_R(X)\}$ where $X \subseteq U$. Then by Property 1.3, $\tau_R(X)$ satisfies the following axioms.

- U and$\Phi \in \tau_R(X)$.
- The union of the elements of any sub collection of $\tau_R(X)$ is in$\tau_R(X)$.
- The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in$\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. we call $(U, \tau_R(X))$ is called the Nano topological space. Elements of $\tau_R(X)$ are called as Nano open sets in U. Elements of $[\tau_R(X)]^C$ are called Nano closed sets with $[\tau_R(X)]^C$ being called dual Nano topology of$\tau_R(X)$.

Remark 1.5 [5]

If $\tau_R(X)$ is the Nano topology on U with respect to X. Then the set
$B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 1.6 [5]

If $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

- The Nano interior of the set A is defined as the union of all nano open subsets contained in A and is denoted by Nint (A). Nint (A) is the largest nano open subset of A.
- The Nano closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by Ncl (A). Ncl (A) is the smallest nano closed set containing A.

Definition 1.7

Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then A is said to be

1. Ng-closed **[1]** ifNcl (A) ⊆ V whenever A ⊆ V and V is Nano-open in U.
2. Ngr-closed **[9]**if Nrcl (A) ⊆ V whenever A ⊆ V and V is Nano-open in U.
3. Ngs-closed **[2]** if Nscl (A) ⊆ V whenever A ⊆ V and V is Nano-open in U.
4. Ngα-closed **[3]** if Nαcl (A) ⊆ V whenever A ⊆ V and V is Nano-open in U.
5. Ng*-closed **[7]** if Ncl (A) ⊆ V whenever A ⊆ V and V is Nano g-open in U.
6. Ng*s-closed**[8]**if Nscl(A) ⊆ V whenever A ⊆ V and V is Nano g-open in U

Nano Generalized Star Pre-Closed Sets

Throughout this paper (U,τ_R(X) is a nano topological space with respect to X where X⊆U,R isan equivalence relation on U,U/R denotes the family of equivalence classes of U by R.

Definition 2.1 **Let** (U,τ_R(X) be a nano topological space.A subset Aof (U,τ_R(X) is called nano generalized star pre –closed set(briefly Ng*p-closed) if Npcl(A)⊆V where A⊆V and V is nano g-open

Example 2.2 Let U={a,b,c,d}withU/R={{a},{c},{b,d}} and X={a,b}.Thenτ_R(X)={ U,∅.{a},{b,d},{a,b,d}} whichnano open sets. Then

1. The nano closed set={ U,∅.{c},{a,c},{b,c,d}}
2. The nano semi closed sets={ U,∅.{a},{c},{a,c}{b,d}{b,c,d}}
3. The nano pre-closed set={U,∅,{a},{b},{c},{d},{a,c},{b,c},{c,d},{a,b,c},{a, c,d},{b,c,d}}
4. The nano regular closed set = { U,∅,{b},{c},{d},{a,c},{b,c},{c,d},{a,b,c},{a, c,d},{b,c,d}}
5. The nano g-closed set={ U,∅,{c},{a,c},{b,c},{c,d},{a,b,c},{a,c,d},{b,c,d}}
6. The nano gp-closed set ={ U,∅,{b},{c},{d},{a,c},{b,c},{c,d},{a,b,c},{a,c,d},{b,c,d }}
7. The nano gs-closed set ={ U,{a},{b},{c},{d},{a,c},{b,c},{b,d},{c,d},{a,b,c},{a, c,d},{b,c,d}}
8. The nano gr-closed set ={ U,∅,{c},{b,c},{c,d},{a,c},{a,b,c},{b,c,d},{a, c,d}}
9. The nano g^*-closed set ={ U,∅,{c},{a,c},{b,c},{c,d},{a,b,c},{a, c,d},{b,c,d}}
10. The nano g^*s-closed set ={ U,∅,{a},{c},{a,c},{b,d},{a,b,d},{b,c,d}}
11. The nano g^*p-closed set ={ U,∅,{a},{b},{c},{d},{a,b},{a,c},{a,d}, {b,c},{b,d},{c,d}, {a,b,c},{a, b,d},{a, c,d},{b,c,d}}

Theorem2.3. If A is nano closed set in(U,τ_R(X),then it is a nano g^*p-closed set but not the converse.

Proof. Let A be a nano closed set of U andA⊆V, where V is nano g-open in U.Since A is nano closed, we haveNcl (A) = A⊆V,That is Ncl (A) ⊆V.Also Npcl (A) ⊆Ncl (A) implies Npcl (A) ⊆V,where V is nano g-open in U.Therefore A is a nano g*p-closed set.

Example 2.4.In Example 2.2, the subsets {a}, {b}, {d} are nano g*p-closed butnot nano g-closed set.

Theorem 2.5.If A is nano g-closed in(U, τ_R(X), then it is nano g*p-closed set but not the converse.

Proof. Let A be a nano g-closed set. ThenNcl (A) ⊆V whenever A⊆V and Vis nano open in U,since every nano open set is nano g-open set. So V is nano g-open set in U.We have Npcl(A) ⊆Ncl(A) which implies

Npcl(A) ⊆V,A⊆V,V is nano g-open in U.Hence A is nano g*p-closed set.

Example 2.6.In Example 2.2, the subsets {b}, {d} are nano g*p-closed but not a nano g-closed set.

Theorem 2.7.If A is nano g*-closed in (U, τ_R(X), then it is nano g*p-closedset but the converse is not true.

Proof. Let A be a nano g*-closed set of U and A⊆V, where V isnano g-open in U. Since A is nano g*-closed we have Ncl (A) =A. So A⊆V implies Ncl(A) ⊆U. But Npcl (A) ⊆Ncl (A) implies Npcl (A)⊆V,A⊆V, Vis nano g-open in U. Therefore A isnano a g*p-closed set.

Example 2.8.In Example 2.2, the subsets {a, b}, {a, d} are nano g*p-closed set but not a nano g*-closed set.

Theorem 2.9.If A is nano gp-closed in (U, τ_R(X), then it is nano g*p-closed set but the converse is not true.

Proof. Let A be a nano gp-closed set of U and A⊆V, where V is nano open in U. But every nano open sets is nano g-open set. This implies V is nano g-open in U.So Npcl (A) ⊆V, A⊆V, V is nano g-open in U. Therefore A is nano g*p-closed set.

Example2.10. Let U={a, b,c,d} with U/R={{a},{d},{b,c}} and X={a,c}.Then τ_R(X)={U,∅,{a},{b,c},{a,b,c}}.Then in a space (U,τ_R(X)),a subset {a} is a nano g*p-closed set but it is not a nano gp-closed set.

Theorem 2.11.The union of two nano g*p-closed sets in (U, τ_R(X)) is also a nano g*p-closed set in (U,τ_R(X)).

Proof. Let A and B be two nano g*p-closed sets in (U, τ_R(X)).Let V be a nano g-open set in U, such that A⊆V and B⊆V. Then we have AUB⊆V. Since A and B are nano g*p-closed in (U, τ_R(X)).This implies Npcl (A)⊆V and Npcl(B) ⊆V.Now Npcl(A∪B)=Npcl(A)∪Npcl(B) ⊆V. Thus we have Npcl (AUB) ⊆V, whenever AUB⊆V, where V is nano g-open set in (U,τ_R(X)).This implies AUB is a nano g*p-closed set in (U,τ_R(X)).

Remark 2.12.The intersection of two nano g*p-closed sets in (U,τ_R(X)) is also a nano g*p-closed set in(U,τ_R(X)) asseen from the following example.

Example 2.13.Let U={a,b,c,d} with U/R={{a},{d},{b,c}} and X={a,c}.Then the nano topology isτ_R(X)={U,∅ {a},{b,c},{a,b,c}}.Then the nano g*p-closed sets are {a},{b},{c},{d},{a,b},{a,c}{a,d},{b,c},{b,d},{c,d},{a,b,c},{a,b,d},{a,c,d},{b,c,d}.Here {b, c}∩{c, d} ={c} is also a nano g*p-closed set.

Theorem 2.14.Let A be a nano g*p-closed subset of (U,τ_R(X)).If A ⊆B ⊆Npcl (A),then B is also a nano g*p-closed subset of (U,τ_R(X)).

Proof. Let V be a nano g-open set of a nano g*p-closed subset ofτ_R(X) such that B ⊆V,as A ⊆B, we have A ⊆V.As A is nano g*p-closed set,Npcl (A) ⊆V, Given B ⊆Npcl (A). we have Npcl(A) ⊆Npcl(B) and Npcl(A) ⊆V, we have Npcl(B) ⊆V, whenever B ⊆V and V is nano g-open .Hence B is also a nano g*p-closed subset of τ_R(X).

Theorem2.15. If a subset A is a nano g*p-closed set if and only if Npcl (A)-A contains no nonempty, nano closed set.

Proof. Necessity. Let F be nano generalized closed set in (U,τ_R(X)),such that F⊆Npcl(A)-A. Then A⊆X-F.Since A is nano g*p-closed set and X-F is nano generalized open then Npcl (A) ⊆X-F. That is F⊆X-Npcl (A).So F⊆[X-Npcl (A)]∩[Npcl (A)-A].Therefore F=∅.

Sufficiency. Let us assume that Npcl (A)-A contains no non empty nano generalized closed set.Let A⊆V, V is nano generalized open. Suppose that Npcl (A) is not contained in V, Npcl (A)∩ V^c is non empty, and nano generalized closed set of Npcl(A)-A which is a contradiction. Therefore Npcl (A) ⊆V, and hence A is nano g*p-closed set.

Theorem2.16. If A is both nano generalized open and nano g*p-closed set in X, then A is nano generalized closed set.

Proof. Since A is nano generalized open and nano g*p-closed set inX, Npcl (A) ⊆V.But A⊆Npcl (A).Therefore A=Npcl(A).Since A is nano closed and Nint (A) =A, this implies Npcl (A) =A. Hence A is nano generalized closed set.

Nano Generalized Star Pre-open set

Definition 3.1.A subset A of a nano topological space (U,τ_R(X))is called nano generalized star pre-open set (briefly, nano g*p-open),if A^C is nano g*p-closed.

Theorem 3.2.

1. If A is nano open set in (U, τ_R(X)), then it is nano g*p-open
2. If A is nanog*- open set in (U, τ_R(X)), thenit is nano g*p-open
3. If A is nano gp- open set in(U, τ_R(X)),then it is nano g*p-open

Proof. It follows from the Theorem 2.3, 2.7& ` 2.9.

Remark *3.3.*For subset A of a nano topological space (U,τ_R(X)),

1. U-Ng*pint(A)=Ng*pcl(U-A)
2. U-Ng*pcl(A)=Ng* pint(U-A)

Theorem 3.4.A subset A⊆U is nano g*p-open if and only if F⊆Npint (A) whenever F is nano g-closed set and F⊆A.

Proof. Let A be nano g*p-open set and suppose F⊆A, where F is nano g-closed. Then U-A is nano g*p-closed set contained in the nano g-open set U-F. Hence Npcl (U-A) ⊆U-F and U-Npint (A) ⊆ U-F. Thus F⊆Npint (A).

Conversely, if F is nano g-closed set with F⊆Npint (A)and F⊆A. Then U-Npint (A) ⊆ U-F. Thus Npcl (U-A) ⊆U-F. Hence U-A is nano g*p-closed set and A is nano g*p-open set.

Theorem3.5. If Npint (A) ⊆B⊆A and if A is nano g*p-open, then B is nano g*p-open.

Proof. Let Npint (A) ⊆B⊆A, then A^C⊆B^C⊆Npcl (A^C), where A^C is nano g*p-closed and hence B^C is also nano g*p-closed by theorem 2.14.Therefore B is nano g*p-open.

Remark 3.6. If A and B are nano g*p-open subset of a nano topological space, then AUB is also nano g*p-open in U, as seen from the following example.

Example 3.7. In example 2.13, the sets {a} and {a, b} are nano g*p-open sets .Then {a} U {a, b} = {a, b} is also nano g*p-open sets.

Nano generalized Star pre-Interior and Nano generalized Star pre- Closure

Definition 4.1. Let U be a nano topological space and let x∈U.A subset N of U is said to be Ng*p-neighbourhood of x if there exists an Ng*p-open set G such that x∈ G⊆N.

Definition 4.2

I. Ng*p-int (A) =U {B: B is nano g*p-open set and B⊂ A}

II. Ng*p-cl (A) =∩{B: B is nano g*p closedset and A⊂ B}

Theorem 4.3 If A be a subset of U .Then Ng*p-int (A) = U {B: B is nano g*p-open set and B⊂ A}

Proof. Let A be a subset of U. x∈ Ng*p-int (A) ⇔x is a Ng*p-interior point of A.
⇔A is a Ng*p-neighbourhood of point x.
⇔There exists Ng*p-open set B such that x∈B⊂A
⇔x∈U{B: B is Ng*p-open set and B⊂A}
Hence, Ng*p-int (A) =∪ {B: B is nano g*p-open set and B⊂A}

Theorem 4.4.Let A and B be subsets of U .Then

1. Ng*p-int (U) =U and Ng*p-int (∅)= ∅
2. Ng*p-int (A) ⊂A
3. If Bis any Ng*p- open sets contained in A, then B⊂Ng*p-int (A)
4. If A⊂B, then Ng*p-int (A) ⊂Ng*p-int (B)
5. Ng*p-int (Ng*p-int (A)) =Ng*p-int (A).

Proof.

1. Since U and∅ are Ng*p-open sets, by Theorem 4.3
Ng*p-int (U) =∪ {B: B is Ng*p-open and G⊂U}
=∪ ∪ {A: A is a Ng*p-open set}
=∪
Since,∅ is the only Ng*p-open set contained in∅,Ng*p int (∅) =∅
2. ii) Let x∈Ng*p-int (A) =>x is a Ng*p-interior point of A.
=>A is a Ng*p neighbourhood of x.
=>x∈A
Thus, x∈Ng*p-int (A)⊂A.
3. Let B be any Ng*p-open set such that B⊂ A. Let X∈B, then, B is a Ng*p-open set contained x in A is a Ng*p-interior point of A. That is B is a Ng*p-int (A).Hence B⊂Ng*p-int (A).
4. Let A and B be subsets of U such that A⊂ B.Let x∈Ng*p-int (A).Then x is a Ng*p-interior point of A and so A is Ng*p neighbourhood of x. This implies that x∈Ng*p-int (B).Thus we have shown that X∈Ng*p-int (B).Hence, Ng*p-int (A)⊂Ng*p-int(B).
5. Let A be any subset of U. Thenby definition of Ng*p-interior, Ng*p-int (A) = ∩{A⊂F∈Ng*p c(U)} if A⊂ F ∈Ng*pc (U),then Ng*p-int (A)⊂F.Since F is a Ng*p closed set containing Ng*p-int (A).By (iii), Ng*p-int (Ng*p-int (A))⊂F.Hence Ng*p-int (Ng*p-int (A))⊂∩{A⊂F∈Ng*pc(U)} =Ng*pcl(A).That is, Ng*p-int (Ng*p-int (A)) =Ng*p-int (A).

Theorem 4.5. If a subset A of a space U is Ng*p-open then Ng*p-int (A) =A

Proof. Let A be a Ng*p-open subset of U.We know that Ng*p-int (A) ⊂A.Also A is Ng*p-open set contained in A. From theorem4.4 (iii),A⊂Ng*p-int (A).Hence, Ng*p-int (A) =A.

Theorem 4.6. If A and B are subsets of U, then Ng*p-int (A)∪Ng*p-int (B) ⊂Ng*p-int (A∪B).

Proof. We know that A⊂A∪B and B⊂A∪B and we have by Theorem 4.4(iv),Ng*p-int (A) ⊂Ng*p-int (A∪B)and Ng*p-int (B) ⊂Ng*p-int (A∪B).This implies that Ng*p-int (A) ∪Ng*P-int (B) ⊂Ng*p-int (A∪B).

Theorem 4.7. If A and B are subsets of space U, Then Ng*p-int (A∩B) = (Ng*p-int (A) ∩Ng*p-int (B)

Proof. We know that A∩ B ⊂A and A∩B⊂B.We have, by Theorem4.4(iv), Ng*p-int (A∩ B) ⊂ Ng*p-int (A) and Ng*P-int (A∩B)⊂Ng*p-int(B).This implies that Ng*p-int (A∩B)⊂Ng*p-int (A)∩Ng*p-int(B) →(1).

Again, Let x∈Ng*p-int (A) ∩Ng*p-int (B).Then x∈Ng*p-int (A) and x∈Ng*p-int (B). Hence, x is a Ng*p-interior pointof each sets Aand B is Ng*p-neighbourhood of x, So that their intersection A∩B is also Ng*p-neighbourhood of xhence x∈Ng*p- int (A∩B).Therefore Ng*p-int (A) ∩Ng*p-int (B)⊂Ng*p-int (A∩ B)→(2).

From (1) & (2),wegetNg*p-int (A∩ B) = Ng*p-int (A) ∩Ng*p-int (B).

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