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CONTRA-(1, 2)*-$M_{\delta\pi}$-CONTINUOUS FUNCTIONS IN BITOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to study some new generalization of continuous functions in bitopological space namely, (1, 2)*-$M_{\delta\pi}$-continuous functions, (1, 2)*-$M_{\delta\pi}$-irresolute functions, contra (1, 2)*-$M_{\delta\pi}$-continuous and contra (1, 2)*-$M_{\delta\pi}$-irresolute functions. Also we investigate the relationships between these functions and other existing functions in bitopological spaces.

INTRODUCTION

Dontchev and Ganster [4] introduced δ-generalized closed set in topological spaces. Balachandran *et al* [3] and Dontchev *et al* [4] investigated generalized continuity, δ-generalized continuity, δ-generalized irresolute functions respectively. Dontchev [5] obtained a new notion of continuous functions called contra-continuous functions in the recent past. Lellis Thivagar [12] have developed the concepts of (1, 2)*-semi-generalized continuous functions in bitopological spaces. Recently Arockiarani and Mohana [2, 7] discussed (1, 2)*-πgα-continuous and contra (1, 2)*-πgα-continuous functions in bitopological spaces. In this paper, we study the notion of new class of functions called (1, 2)*- $M_{\delta\pi}$-continuous functions and (1, 2)*- $M_{\delta\pi}$-irresolute functions in Bitopological space. Also we introduce few types of generalizations of contra-functions called contra (1, 2)*-$M_{\delta\pi}$-continuous, contra (1, 2)*-$M_{\delta\pi}$-irresolute functions. Further, We discuss some properties of these functions in bitopological spaces.

Preliminaries

Throughout this paper the spaces X and Y represent non-empty bitopological spaces on which no separation axioms are assumed, unless otherwise mentioned. We recall the following definitions and results which are useful in the sequel.

Definition: [6] A subset S of a bitopological space X is said to be $\tau_{1,2}$-open if

S=A∪B where $A \in {}^{2}{}_{1}$ and $B \in {}^{2}{}_{2}$ .A subset S of X is said to be (i) $\tau_{1,2}$-closed if the complement of S is $\tau_{1,2}$-open. (ii) $\tau_{1,2}$-clopen if S is both $\tau_{1,2}$-open and $\tau_{1,2}$-closed.

Definition: [6] Let S be a subset of the bitopological space X. Then the $\tau_{1,2}$-interior of S denoted by $\tau_{1,2}$-int(S) is defined by ∪{G: G⊆S and G is $\tau_{1,2}$-open} and the $\tau_{1,2}$-closure of S denoted by $\tau_{1,2}$-cl(S) is defined by ∩{F: S⊆F and F is $\tau_{1,2}$-closed}.

The family of all $\tau_{1,2}$-closed sets of X will be denoted by $\tau_{1,2}$-C(X).

The set $\tau_{1,2}$-C(X, x) = {V ∈ $\tau_{1,2}$-C(X) / x ∈ V} for x ∈ X.

Definition: A subset A of a bitopological space X is called

1. (1, 2)*-regular open [6] if A= $\tau_{1,2}$-int ($\tau_{1,2}$-cl (A)).

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2. (1, 2)*-∼ -open [6] if A ⊆ $\tau_{1,2}$-int ($\tau_{1,2}$-cl ($\tau_{1,2}$-int (A))).
3. (1, 2)*-semi-open [6] if A ⊆ $\tau_{1,2}$-cl ($\tau_{1,2}$-int(A))

The complement of the sets mentioned from (1) to (3) are called their respective closed sets.

Definition: **[2]** Let S be a subset of the bitopological space X. Then

- The (1, 2)*-α-interior of S denoted by (1, 2)*-α-int(S) is defined by
  ∪{G: G ⊆ S and G is (1, 2)*-α-open}.
- The (1, 2)*-α-closure of S denoted by (1,2)*-α-cl (S) is defined by
  ∩{F: S ⊆ F and F is (1, 2)*-α-closed}.

Definition: **[10]** The (1, 2)*-δ-interior of a subset A of X is the union of all (1, 2)*-regular open set of X contained in A and is denoted by (1, 2)*-δ-int (A). The subset A is called (1, 2)*-δ-open if A = (1, 2)*-δ-int (A), (i. e), a set is (1, 2)*-δ-open if it is the union of (1, 2)*-regular open sets. The complement of a (1, 2)*-δ-open set is called (1, 2)*-δ-closed. Alternatively, a subset A in X is called (1, 2)*-δ-closed if A = (1, 2)*-δ-cl (A), where (1, 2)*-δ-cl (A) = {x ∈ X: $\tau_{1,2}$-int ($\tau_{1,2}$-cl (U)) ∩ A ≠ ¤ , U ∈ $\tau_{1,2}$ and x ∈ U}.

Definition: A subset A of a bitopological space X is called

1. (1, 2)*- πg∼ - closed [1] if (1, 2)*-αcl (A) ⊆ U whenever A ⊆ U and U is $\tau_{1,2}$-π-open.
2. (1, 2)*-πg- closed [8] if $\tau_{1,2}$-cl (A) ⊆ U whenever A ⊆ U and U is $\tau_{1,2}$- π-open.
3. (1, 2)*- strongly- πg∼ - closed [9] if (1, 2)*-αcl (A) ⊆ U whenever A ⊆ U and U is (1, 2)*-πg-open.
4. (1, 2)*- gα- closed [6] if (1, 2)*-αcl (A) ⊆ A whenever A ⊆ U and U is (1, 2)*-α-open in X.
5. (1, 2)*-$M_{\delta\pi}$-closed set [10] if (1, 2)*-δcl (A) ⊆ U whenever A ⊆ U and U is (1, 2)*-πg-open in X.

and the complement of the sets mentioned from (1) to (5) are called their respective open sets.

Definition: A function f: X → Y is called

1. (1, 2)*-πg-continuous [8] if the inverse image of every $\sigma_{1,2}$-closed set of Y is (1, 2)*-πg-closed set in X.
2. (1, 2)*-πgα-continuous [2] if the inverse image of every $\sigma_{1,2}$-closed set of Y is (1, 2)*-πgα-closed in X.
3. (1, 2)*-πgα^s-continuous [11] if the inverse image of every $\sigma_{1,2}$-closed set of Y is (1, 2)*-strongly-πgα-closed in X.
4. (1, 2)*-gα-continuous if the inverse image of every $\sigma_{1,2}$-closed set of Y is (1, 2)*-gα-closed in X.
5. Contra (1, 2)*-πgα-continuous [7] if the inverse image of every $\sigma_{1,2}$-open set of Y is (1, 2)*-πgα-closed in X.
6. Contra (1, 2)*- continuous [7] if the inverse image of every $\sigma_{1,2}$-open set of Y is $\tau_{1,2}$-closed in X.
7. Contra (1, 2)*-πgα^S-continuous [11] if the inverse image of every $\sigma_{1,2}$-open set of Y is (1, 2)*-strongly-πgα-closed in X.

Definition: [10] A space X is called (1, 2)*-$T_{\delta\pi g}$-space if every (1, 2)*-$M_{\delta\pi}$-closed set in it is an (1, 2)*-δ-closed.

(1, 2)*-$M_{\delta\pi}$-Continuous Functions

Definition: A function f: X→Y is called (1, 2)*-$M_{\delta\pi}$-continuous if the inverse image of
every $\sigma_{1,2}$-closed set in Y is (1, 2)*-$M_{\delta\pi}$-closed set in X.

Example: Let X = {a, b, c} =Y with topologies τ_1 = {Φ, X, {a}, {a, b}}, τ_2 = {Φ, X, {b}, {a, c}}, σ_1= {Φ, Y, {b}}, σ_2 = {Φ, Y, {a, c}} and let f be the identity map. Clearly, f is (1, 2)*-$M_{\delta\pi}$-continuous.

Definition: A function f: X→Y is called (1, 2)*-$M_{\delta\pi}$-irresolute if the inverse image of (1, 2)*-$M_{\delta\pi}$-closed set in Y is (1, 2)*-$M_{\delta\pi}$-closed set in X.

Example: Let X = {a, b, c, d} =Y with τ_1 = {Φ, X, {a}, {a, b, d}}, τ_2 = {Φ, X, {a, b}, {b}}, σ_1= {Φ, Y, {a}}, σ_2 = {Φ, Y, {b}} and let f be the identity map. Clearly, f is (1, 2)*-$M_{\delta\pi}$-irresolute functions.

Definition: A function f: X → Y is called (1, 2)*-δ-continuous if the inverse image of every $\sigma_{1,2}$-closed set of Y is (1, 2)*-δ-closed in X.

Theorem: Every (1, 2)*-δ-continuous function is (1, 2)*-$M_{\delta\pi}$-continuous function.

Proof. The proof is obvious, since every (1, 2)*-δ-closed set is (1, 2)*-$M_{\delta\pi}$-closed set.

Remark: The converse of the above theorem is not true in general as shown in the following example.

Example: Let X = {a, b, c, d}=Y with τ_1 = {Φ, X, {a}, {d}, {a, d}}, τ_2 = {Φ, X, {d}, {a, d}, {c, d}, {a, c, d}}, σ_1 = {Φ, Y, {a}, {a, b, d}}, σ_2 = {Φ, Y, {b}, {a, b}}. Define a function f: X→Y by f (a) = a, f (b) = c, f(c) = b, f(d) = d. Then f is (1, 2)*-$M_{\delta\pi}$-continuous function, but not (1, 2)*-δ-continuous function, since f^{-1}({a, c, d}) = {a, b, d} is not (1, 2)*-δ-closed set in X, for
$\sigma_{1,2}$-closed set {a, c, d} in Y.

Theorem: Every (1, 2)*-$M_{\delta\pi}$-continuous function is (1, 2)*-πg-continuous function.

Proof. It is true that, every (1, 2)*- $M_{\delta\pi}$-closed set is (1, 2)*-πg-closed set.

Remark: The converse of the above theorem need not be true as shown in the following example.

Example: Let X = {a, b, c} =Y with τ_1 = {Φ, X, {b}, {a, c}}, τ_2 = {Φ, X, {a}, {a, b}},

σ_1 = {Φ, Y, {a}}, σ_2 = {Φ, Y, {a, c}} and let f: X→Y be an identity function. Then f is not (1, 2)*-$M_{\delta\pi}$-continuous function, because f^{-1}({b, c}) = {b, c} is not (1, 2)*- $M_{\delta\pi}$-closed set in X, for $\sigma_{1,2}$-closed set {b, c} in Y. However, f is (1, 2)*- πg-continuous.

Theorem: Every (1, 2)*-$M_{\delta\pi}$-continuous function is (1, 2)*-πgα-continuous function.

Proof. The proof is immediate, since every (1, 2)*- $M_{\delta\pi}$-closed set is (1, 2)*- πgα-closed set.

Remark: The converse of the Theorem 3. 12 need not be true as shown in the following example.

Example: Let X = {a, b, c} =Y with τ_1 = {Φ, X, {b}, {a, b}}, τ_2 = {Φ, X, {b, c}},

σ_1 = {Φ, Y, {b}}, σ_2 = {Φ, Y, {c}} and let f: X→Y be an identity function. Then f is (1, 2)*- πgα-continuous, but not (1, 2)*-$M_{\delta\pi}$-continuous function, since for the $\sigma_{1,2}$-closed sets {a}, {a, c}, {a, b} of Y, f^{-1}({a}, {a, c}, {a, b}) = {a}, {a, c}, {a, b} are not (1, 2)*- $M_{\delta\pi}$-closed set in X.

Theorem: Every (1, 2)*-$M_{\delta\pi}$-continuous function is (1, 2)*-$\pi g\alpha^S$-continuous function.

Proof. The proof is clear, since every (1, 2)*- $M_{\delta\pi}$-closed set is (1, 2)*- strongly-πgα-closed set.

Remark: The converse of the Theorem 3. 15 need not be true as shown in the following example.

Example: Let X = {a, b, c} =Y with τ_1 = {Φ, X, {b}, {b, c}}, τ_2 = {Φ, X, {a, b}},
σ_1 = {Φ, Y, {a}}, σ_2 = {Φ, Y, {a, c}}. Define a function f: X→Y by f(a)=b, f(b)=a, f(c)=c. Then f is (1, 2)*- $\pi g\alpha^s$-continuous, but not (1, 2)*-$M_{\delta\pi}$-continuous function, since for the $\sigma_{1,2}$-closed sets {b}, {b, c} of Y, f^{-1}({b}, {b, c}) = {a}, {a, c} are not (1, 2)*- $M_{\delta\pi}$-closed set in X.

Definition: A subset A of a bitopological space X is called (1, 2)*-M_π-closed set if $\tau_{1,2}$-cl (A) ⊆ U whenever A ⊆ U and U is (1, 2)*-πg-open in X.

Remark: Every (1, 2)*-$M_{\delta\pi}$ –closed set is (1, 2)*-M_π-closed set, but not conversely.

Theorem: Every (1, 2)*-$M_{\delta\pi}$-continuous function is (1, 2)*- M_π -continuous function.

Proof. By Remark 3. 19, the proof is clear.

Remark: The converse of the Theorem 3. 20 need not be true as shown in the following example.

Example: In example 3. 17, f is (1, 2)*- M_π-continuous, but not (1, 2)*-$M_{\delta\pi}$-continuous function, since for the $\sigma_{1,2}$-closed sets {b}, {b, c} of Y, f^{-1}({b}, {b, c}) = {a}, {a, c} are not (1, 2)*-$M_{\delta\pi}$-closed set in X.

Remark: From the above discussions, we have the following table. The symbol “1” in a cell means that a function on the corresponding row implies a function on the corresponding column. Finally, the symbol “0” means that a function on the corresponding row does not implies a function on the corresponding column.

a. (1, 2)*-δ-continuous b. (1, 2)*- πg-continuous c. (1, 2)*-πgα-continuous
d. (1, 2)*- $\pi g\alpha^s$-continuous e. (1, 2)*-$M_{\delta\pi}$-continuous. f. (1, 2)*-M_π –continuous

(1, 2)*-continuous functions	a	b	c	d	e	f
a	1	1	1	1	0	0
b	0	1	1	1	0	0
c	0	0	1	1	0	0
d	0	0	0	1	0	0
e	0	1	1	1	1	1
f	0	1	1	1	0	1

Remark: The following examples show that (1, 2)*-$M_{\delta\pi}$ –continuity is independent of (1, 2)*-continuity & (1, 2)*-gα-continuity.

Example: Let X = {a, b, c, d} =Y, τ_1 = {Φ, X, {a}, {d}, {a, d}, {a, c, d}}, τ_2 = {Φ, X, {c, d}}, σ_1 = {Φ, Y, {c}}, σ_2 = {Φ, Y, {b, c, d}}. Define a function f: X→Y by f (a) =b, f (b) =a, f(c) =c, f (d) = d. Then f is (1, 2)*-$M_{\delta\pi}$-continuous function, but not (1, 2)*-continuous and (1, 2)*-gα-continuous, because f^{-1}{a, b, d} = {a, b, d} is $M_{\delta\pi}$-closed set in X, but not $\tau_{1,2}$ –closed and (1, 2)*-gα-closed set in X.

Example: Let X = {a, b, c} =Y, τ_1 = {Φ, X, {a}, {a, b}}, τ_2 = {Φ, X, {b}, {a, c}}, σ_1 = {Φ, Y, {b}}, σ_2 = {Φ, Y, {b, c}}. Define a function f: X→Y by f (a) = b, f (b) =a, f(c) =c. Then f is (1, 2)*-continuous and (1, 2)*-gα-continuous, but not (1, 2)*-$M_{\delta\pi}$-continuous function.

Theorem: A function f: X→Y is (1, 2)*-$M_{\delta\pi}$ –continuous iff f^{-1}(U) is (1, 2)*-$M_{\delta\pi}$ –open in X, for every $\sigma_{1,2}$ –open set in Y.
Proof. Let f be an (1, 2)*-$M_{\delta\pi}$ –continuous function and U be an $\sigma_{1,2}$ –open set in Y. Then

$f^{-1}(U^C)$ is (1, 2)*-$M_{\delta\pi}$ –closed set in X. But $f^{-1}(U^C) = [f^{-1}(U)]^C$ and hence f^{-1}(U) is (1, 2)*-$M_{\delta\pi}$ –open in X. Conversely, f^{-1}(U) is (1, 2)*-$M_{\delta\pi}$ –open in X, for every $\sigma_{1,2}$ –open set U in Y. U^C is $\sigma_{1,2}$ –closed set in Y. Then $[f^{-1}(U)]^C$ is (1, 2)*-$M_{\delta\pi}$ –closed in X. But $[f^{-1}(U)]^C = f^{-1}(U^C)$ and hence $f^{-1}(U^C)$ is (1, 2)*-$M_{\delta\pi}$ –closed set in X. Therefore, f is (1, 2)*-$M_{\delta\pi}$ –continuous.

Theorem: Let f: X→Y and g: Y→Z be two functions. Then

1. gof: X→Z is (1, 2)*-$M_{\delta\pi}$ –continuous, if g is (1, 2)*-continuous and f is (1, 2)*-$M_{\delta\pi}$ –continuous function.
2. gof: X→Z is (1, 2)*-$M_{\delta\pi}$ –irresolute, if g is (1, 2)*- $M_{\delta\pi}$ –irresolute and f is (1, 2)*-$M_{\delta\pi}$ –irresolute.
3. gof: X→Z is (1, 2)*-$M_{\delta\pi}$ –continuous, if g is (1, 2)*-$M_{\delta\pi}$ –continuous and f is (1, 2)*-$M_{\delta\pi}$ –irresolute.

Proof. The proof follows from the definitions.

Lemma: The product of two (1, 2)*- $M_{\delta\pi}$ –open sets is (1, 2)*-$M_{\delta\pi}$ –open sets in the product space.

Proof. Let A and B be (1, 2)*- $M_{\delta\pi}$ –open sets of two space X and Y respectively and V = A x B $\subseteq$ X x Y. Let F $\subseteq$ V be a (1, 2)*-πg-closed in X x Y, then there exists two (1, 2)*-πg-closed sets $F_1 \subseteq$ A, $F_2 \subseteq$ B. So, $F_1 \subseteq$ (1, 2)*-δint (A) and $F_2 \subseteq$ (1, 2)*-δint (B). Hence, F_1 x $F_2 \subseteq$ A x B and F_1 x $F_2 \subseteq$ (1, 2)*-δint (A) x (1, 2)*-δint (B) = (1, 2)*-δint (A x B). Therefore, A x B is (1, 2)*- $M_{\delta\pi}$ –open subset of a space X x Y.

Definition: A function f: X→Y is called (1, 2)*-δ^S-closed if the image of (1, 2)*-δ-closed set
in X is (1, 2)*-δ-closed set in Y.

Theorem: Let f: X→Y be (1, 2)*-continuous and (1, 2)*-δ^S-closed. Then for every (1, 2)*-$M_{\delta\pi}$ –closed subset A of X, f (A) is (1, 2)*-$M_{\delta\pi}$ –closed set in Y.

Proof. Let A be (1, 2)*-$M_{\delta\pi}$ –closed in X. Let f (A) $\subseteq$ W, where W is $\sigma_{1,2}$-open set in Y. Since A $\subseteq$

f^{-1}(W) is $\tau_{1,2}$-open set in X, f^{-1}(W) is (1, 2)*-πg-open set in X. Since A is (1, 2)*-$M_{\delta\pi}$ –closed set and f^{-1}(W) is (1, 2)*-πg-open set in X, then (1, 2)*-δcl (A) $\subseteq f^{-1}$(W). Thus f ((1, 2)*-δcl (A)) $\subseteq$ W. Hence, (1, 2)*-δcl (f (A)) $\subseteq$ f ((1, 2)*-δcl (A)) $\subseteq$ W, since f is (1, 2)*- δ^S-closed. Hence, f (A) is (1, 2)*-$M_{\delta\pi}$ –closed in Y.

Theorem: Let f: X→Y be a function. Then the following statements are equivalent.

1. f is (1, 2)*-$M_{\delta\pi}$ –irresolute function.
2. For $x \in X$ and any (1, 2)*-$M_{\delta\pi}$ –closed set V of Y containing f(x), thereexists an (1, 2)*- $M_{\delta\pi}$ –closed set U such that $x \in U$ and f (U) $\subset$ V.
3. Inverse image of every (1, 2)*-$M_{\delta\pi}$ –open set of Y is (1, 2)*-$M_{\delta\pi}$ –open in X.

Proof. **[**1] → [2]: Let V be an (1, 2)*-$M_{\delta\pi}$ –closed set of Y and $f(x) \in V$. Since f is (1, 2)*-$M_{\delta\pi}$ –irresolute, f^{-1}(V) is (1, 2)*- $M_{\delta\pi}$ –closed in X and $x \in f^{-1}(V)$. Put U = f^{-1}(V). Then, $x \in U$ and
f (U) $\subset$ V.

[2] →[1]: Let V be an (1, 2)*- $M_{\delta\pi}$ –closed set of Y and $x \in f^{-1}(V)$. Then $f(x) \in V$. Therefore, by [2], thereexists an (1, 2)*- $M_{\delta\pi}$ –closed set U_x such that x $\in U_x$ and f (U_x) $\subset$ V. Hence x $\in U_x \subset f^{-1}$(V). This implies then, f^{-1}(V) is a union of (1, 2)*- $M_{\delta\pi}$ –closed sets of X. By Theorem 4. 1[10], f^{-1}(V) is

(1, 2)*- $M_{\delta\pi}$ –closed set. The show that, f is (1, 2)*- $M_{\delta\pi}$ –irresolute.

[2]↔[3]: It is Obvious.

Definition: A function f: X→Y is called (1, 2)*-δ-irresolute if the inverse image of (1, 2)*-δ-closed set in Y is (1, 2)*-δ-closed set in X.

Theorem: Let f: X→Y be (1, 2)*- $M_{\delta\pi}$ –irresolute. Then f is (1, 2)*-δ-irresolute if X is (1, 2)*-$T_{\delta\pi g}$-space.

Proof. Let V be a (1, 2)*-δ-closed subset of Y. Every (1, 2)*-δ-closed set is (1, 2)*- $M_{\delta\pi}$ –closed and then V is (1, 2)*- $M_{\delta\pi}$ –closed in Y. Since f is (1, 2)*- $M_{\delta\pi}$ –irresolute, then f^{-1}(V) is (1, 2)*- $M_{\delta\pi}$ –closed in X. Since X is (1, 2)*-$T_{\delta\pi g}$-space, then f^{-1}(V) is (1, 2)*-δ-closed set in X. Thus, f is (1, 2)*-δ-irresolute.

Theorem: Let f: X→Y and g: Y→Z be two functions. Let Y be (1, 2)*-$T_{\delta\pi g}$-space. Then gof is

(1, 2)*- $M_{\delta\pi}$ –continuous if g is (1, 2)*- $M_{\delta\pi}$ –continuous and f is (1, 2)*- $M_{\delta\pi}$ –continuous.

Proof. The proof is obvious.

Theorem: Let f: X→Y be onto, (1, 2)*- $M_{\delta\pi}$ –irresolute and (1, 2)*-δ-irresolute. If X is a (1, 2)*-$T_{\delta\pi g}$-space then Y is also a (1, 2)*-$T_{\delta\pi g}$-space

Proof. The proof is obvious.

Theorem: If f: X→Y is bijection, (1, 2)*-open and (1, 2)*- $M_{\delta\pi}$ –continuous, then f is (1, 2)*- $M_{\delta\pi}$ –irresolute.

Proof. Let V be (1, 2)*- $M_{\delta\pi}$ –closed set in Y and let f^{-1}(V) $\subseteq$ U, where U is $\tau_{1,2}$-open set in X. Since f is (1, 2)*-open, f(U) is $\tau_{1,2}$-open set in Y. Every $\tau_{1,2}$-open set is (1, 2)*-πg-open set and hence f (U) is

(1, 2)*-πg-open. Clearly, V $\subseteq$ f (U). Then (1, 2)*-δcl (V) $\subseteq$ f (U) and thus f^{-1}((1, 2)*-δcl (V)) $\subseteq$ U. Since f is (1, 2)*- $M_{\delta\pi}$ –continuous and since (1, 2)*-δcl(V) is a $\sigma_{1,2}$-closed subset of Y, then (1, 2)*-δcl(f^{-1}(V)) $\subseteq$ (1, 2)*-δcl(f^{-1}((1, 2)*-δcl(V))) = f^{-1}((1, 2)*-δcl(V)) $\subseteq$ U. U is $\tau_{1,2}$-open set and hence (1, 2)*-πg-open set in X. Thus we have (1, 2)*-δcl(f^{-1}(V)) $\subseteq$ U whenever f^{-1}(V) $\subseteq$ U and U is (1, 2)*-πg-open set in X. This shows that f^{-1}(V) is (1, 2)*- $M_{\delta\pi}$ –closed set in X. Hence f is (1, 2)*- $M_{\delta\pi}$ –irresolute.

Contra-(1, 2)-$M_{\delta\pi}$-Continuous Functions*

Definition: 4. 1 A function f: X→Y is called contra-(1, 2)*-$M_{\delta\pi}$-continuous if the inverse image of every $\sigma_{1,2}$-open set of Y is (1, 2)*-$M_{\delta\pi}$-closed set in X.

Example: Let X = {a, b, c} =Y with topologies τ_1 = {Φ, X, {b}}, τ_2 = {Φ, X, {c}}, σ_1= {Φ, Y, {a}}, σ_2 = {Φ, Y, {a, c}} and let f be an identity function. Clearly, f is contra-(1, 2)*-$M_{\delta\pi}$-continuous function.

Definition: A function f: X→Y is called contra-(1, 2)*-$M_{\delta\pi}$-irresolute if the inverse image of (1, 2)*-$M_{\delta\pi}$-open set in Y is (1, 2)*-$M_{\delta\pi}$-closed set in X.
Example: Let X = {a, b, c} =Y with topologies τ_1 = {Φ, X, {a}, {b, c}}, τ_2 = {Φ, X, {a, b}, {c}}, σ_1= {Φ, Y, {c}}, σ_2 =

{Φ, Y, {b}} and define a function f: X → Y by f(a) = c, f(b) = b,
f(c) = a. Then f is contra-(1, 2)*-$M_{\delta\pi}$-irresolute function.

Remark: The family of all (1, 2)*-$M_{\delta\pi}$-open sets is denoted by (1, 2)*-$M_{\delta\pi}$-O(X).

The set (1, 2)*-$M_{\delta\pi}$-O(X, x) = {V Є (1, 2)*-$M_{\delta\pi}$-O(X) / x Є V} for x Є X.

Theorem: Let f: X → Y be a function. Then the following are equivalent.

1. f is contra-(1, 2)*-$M_{\delta\pi}$-continuous.
2. The inverse image of each $\sigma_{1,2}$-closed set in Y is (1, 2)*-$M_{\delta\pi}$-open set in X.
3. For each x Є X and each F Є $\sigma_{1,2}$-C(Y, f(x)), there exists U Є (1, 2)*-$M_{\delta\pi}$-O(X, x) such that f (U) ⊂ F.

Proof. 1 => 2, 2 => 1 and 2=> 3 are obvious.
3=> 2. Let F be any $\sigma_{1,2}$-closed set of Y and x Є $f^{-1}(F)$. Then f(x) Є F and there exists U_x Є (1, 2)*-$M_{\delta\pi}$-O(X, x) such that f (U_x) ⊂ F. Hence we obtain $f^{-1}(F) = \bigcup \{U_x / x \in f^{-1}(F)\}$ Є (1, 2)*-$M_{\delta\pi}$-O(X). Thus the inverse of each $\sigma_{1,2}$-closed set in Y is (1, 2)*-$M_{\delta\pi}$-open set in X.

Remark: The concepts of (1, 2)*-$M_{\delta\pi}$-continuity and contra-(1, 2)*-$M_{\delta\pi}$-continuity are independent as shown in the following example.

Example: Let X = {a, b, c} =Y with topologies τ_1 = {Φ, X, {a}, {b, c}}, τ_2 = {Φ, X, {a, b}, {c}}, σ_1= {Φ, Y, {a}, {a, b}}, σ_2 = {Φ, Y, {b}, {a, c}} respectively. Let f: X → Y be defined by

f (a) = b, f (b) = c, f(c) = a. Clearly, f is (1, 2)*-$M_{\delta\pi}$-continuous function, but f is not contra-(1, 2)*-$M_{\delta\pi}$-continuous. Because, f^{-1}({a}) = {c} is not (1, 2)*-$M_{\delta\pi}$-closed set in X, where {a} is $\sigma_{1,2}$-open set in Y

Example: Let X = {a, b, c} =Y with τ_1 = {Φ, X, {a}}, τ_2 = {Φ, X, {c}}, σ_1= {Φ, Y, {a, b}}, σ_2 = {Φ, Y, {b}} respectively. Let f: X → Y be an identity function. Clearly, f is contra-(1, 2)*-$M_{\delta\pi}$-continuous function, but f is not (1, 2)*-$M_{\delta\pi}$-continuous. Because, f^{-1}({c}, {a, c}) = {c}, {a, c} are not (1, 2)*-$M_{\delta\pi}$-closed set in X, where {c} and {a, c} are $\sigma_{1,2}$-open set in Y.

Theorem: A function f: X → Y is (1, 2)*-$M_{\delta\pi}$-continuous if for each x ЄX and each $\sigma_{1,2}$-open set V of Y containing f(x), there exists UЄ (1, 2)*-$M_{\delta\pi}$-O(X, x) such that f(U) ⊂ V.

Theorem: If a function f: X→Y is contra-(1, 2)*-$M_{\delta\pi}$-continuous and Y is (1, 2)*-regular then f is (1, 2)*-$M_{\delta\pi}$-continuous.

Proof. Let x be an arbitrary point of X and V be an $\sigma_{1,2}$-open set of Y containing f(x). Since Y is (1, 2)*-regular, there exists an $\sigma_{1,2}$-open set W of Y containing f(x) such that $\sigma_{1,2}$-cl (W) ⊂ V. Since f is contra-(1, 2)*-$M_{\delta\pi}$-continuous, by Theorem 4. 6, there exists UЄ (1, 2)*-$M_{\delta\pi}$-O(X, x) such that f (U) ⊂ $\sigma_{1,2}$-cl (W). Then f (U) ⊂ $\sigma_{1,2}$-cl (W) ⊂ V. Hence by Theorem 4. 10, f is
(1, 2)*-$M_{\delta\pi}$-continuous function.

Definition: A function f: X→Y is called contra-(1, 2)*-δ-continuous if the inverse image of every $\sigma_{1,2}$-open set of Y is (1, 2)*-δ-closed set in X.

Theorem: Every contra-(1, 2)*-δ-continuous function is contra-(1, 2)*-$M_{\delta\pi}$-continuous function.

Proof. The proof follows from definitions.

Remark: The converse of the above theorem 4. 13 need not be true as shown in the following example.

Example: Let X = {a, b, c, d} =Y with topologies τ_1 = {Φ, X, {a}, {d}, {a, d}, {a, c}, {a, c, d}}, τ_2 = {Φ, X, {c, d}}, σ_1= {Φ, Y, {a}}, σ_2 = {Φ, Y, {a, b, d}} respectively. Define f: X → Y be a function f (a) = b, f (b) = a, f(c) = c, f (d) = d. Then, f is contra-(1, 2)*-$M_{\delta\pi}$-continuous function, but not contra-(1, 2)*-δ-continuous. Since, f^{-1}({a, b, d}) = {a, b, d} is not (1, 2)*-δ-closed set in X, where {a, b, d} is $\sigma_{1,2}$-open set in Y.

Definition: A function f: X→Y is called

1. Contra-(1, 2)*- M_{π} –continuous if the inverse image of every $\sigma_{1,2}$-open set of Y is (1, 2)*- M_{π}-closed set in X.
2. Contra-(1, 2)*- πg–continuous if the inverse image of every $\sigma_{1,2}$-open set of Y is (1, 2)*- πg-closed set in X.

Theorem: Every contra-(1, 2)*-$M_{\delta\pi}$-continuous function is contra-(1, 2)*-M_{π}-continuous function.

Proof. The proof is immediate.

Remark: The converse of the above theorem 4. 15 need not be true as shown in the following example.

Example: Let X = {a, b, c} =Y with topologies τ_1 = {Φ, X, {a}}, τ_2 = {Φ, X, {a, b}, {c}}, σ_1= {Φ, Y, {c}}, σ_2 = {Φ, Y, {b, c}} respectively. Let f: X → Y be an identity function. Then, f is contra-(1, 2)*-M_{π}-continuous function, but not contra-(1, 2)*-$M_{\delta\pi}$-continuous function. Since,

f^{-1}({c}) = {c} is not (1, 2)*-$M_{\delta\pi}$-closed set in X, where {c} is $\sigma_{1,2}$-open set in Y

Theorem: Every contra-(1, 2)*-$M_{\delta\pi}$-continuous function is contra-(1, 2)*-πgα-continuous function.

Proof. The proof is clear.

Theorem: Every contra (1, 2)*-$M_{\delta\pi}$-continuous function is contra (1, 2)*-πg-continuous function.

Proof. The proof is obvious.

Remark: The converse of the theorems 4. 18 and 4. 19 need not be true as shown the following example.

Example: Let X = {a, b, c} =Y with topologies τ_1 = {Φ, X, {a}, {a, b}}, τ_2 = {Φ, X, {b, c}}, σ_1= {Φ, Y, {a}}, σ_2 = {Φ, Y, {a, c}} respectively. Let f: X → Y be an identity function. Then, f is neither contra-(1, 2)*- πg-continuous function nor contra-(1, 2)*-πgα-continuous, but not contra-(1, 2)*-$M_{\delta\pi}$-continuous function. Since, f^{-1}({a}, {a, c}) = {a}, {a, c} is not (1, 2)*-$M_{\delta\pi}$-closed set in X, where {a}, {a, c} are $\sigma_{1,2}$-open set in Y

Theorem: Every contra-(1, 2)*-$M_{\delta\pi}$-continuous function is contra-(1, 2)*- $\pi g\alpha^S$ -continuous function.

Proof. The proof follows from definitions.

Remark: The converse of the above theorem 4. 21 need not be true as shown the following example.

Example: In Example 4. 17, f is contra-(1, 2)*-$\pi g\alpha^S$-continuous, but not contra-(1, 2)*-$M_{\delta\pi}$-continuous function. Since, f^{-1}({c}) = {c} is not (1, 2)*-$M_{\delta\pi}$-closed set in X, where {c} is $\sigma_{1,2}$-open set in Y

Remark: The concepts of contra-(1, 2)*-$M_{\delta\pi}$-continuous and contra-(1, 2)*-continuous functions are independent of each other as shown in the following example.

Example: Let X = {a, b, c} =Y with topologies τ_1 = {Φ, X, {a}, {b, c}}, τ_2 = {Φ, X, {a, b}, {c}}, σ_1= {Φ, Y, {b}}, σ_2 = {Φ, Y, {b, c}} respectively. Let f: X → Y be an identity function. Clearly, f is contra-(1, 2)*-$M_{\delta\pi}$-continuous function, but not contra-(1, 2)*-continuous. Because, f^{-1}({b}) = {b} is not $\tau_{1,2}$-closed set in X, where {c} is $\sigma_{1,2}$-open set in Y

Example: Let X = {a, b, c} =Y with topologies τ_1 = {Φ, X, {b}, {a, b}}, τ_2 = {Φ, X, {b, c}}, σ_1= {Φ, Y, {a}}, σ_2 = {Φ, Y, {a, c}} respectively. Define a function f: X → Y by f(a) = c, f(b) = b, f(c) = a. Then, f is not contra-(1, 2)*-$M_{\delta\pi}$-continuous function, because, f^{-1}({a}, {a, c}) = {c}, {a, c} are not (1, 2)*-$M_{\delta\pi}$-closed set in X, where {a}, {a, c} are $\sigma_{1,2}$-open set in Y. However, f is contra-(1, 2)*-continuous.

Remark: The composition of two contra-**(**1, 2)*-$M_{\delta\pi}$-continuous functions need not be contra-(1, 2)*-$M_{\delta\pi}$-continuous as the following example shows.

Example: Le**t** X = {a, b, c} =Y=Z with τ_1 = {Φ, X, {a}, {a, b}}, τ_2 = {Φ, X, {b, c}, {c}}, σ_1= {Φ, Y, {c}}, σ_2 = {Φ, Y, {b}} and η_1= {Φ, Z, {a}}, η_2 = {Φ, Z, {a, c}}. Define a function

f: X → Y by f(a) = c, f(b) = b, f(c) = a and g: Y→Z be an identity function. Then both f and g are contra-(1, 2)*-$M_{\delta\pi}$-continuous function, but gof is not contra-(1, 2)*-$M_{\delta\pi}$-continuous. Because, $(gof)^{-1}$({a}, {a, c}) = {c}, {a, c} are not (1, 2)*-$M_{\delta\pi}$-closed set in X, where {a} and {a, c} are $\eta_{1,2}$-open set in Y.

Theorem: If f: X→ Y is contra-**(**1, 2)*-$M_{\delta\pi}$-continuous function and g: Y→ Z is a (1, 2)*-continuous function. Then gof: X→ Z is contra-**(**1, 2)*-$M_{\delta\pi}$-continuous.

Proof. The proof follows from the definitions.

Theorem: If f: X→ Y is **(**1, 2)*-$M_{\delta\pi}$-irresolute function and g: Y→ Z is a contra-**(**1, 2)*-$M_{\delta\pi}$-continuous function. Then gof: X→ Z is contra-**(**1, 2)*-$M_{\delta\pi}$-continuous.

Proof. The proof is obvious.

Theorem: Let f: X→ Y be a function then the following are equivalent.

1. f is contra-**(**1, 2)*-$M_{\delta\pi}$-irresolute.
2. For xϵX and any **(**1, 2)*-$M_{\delta\pi}$-open set V of Y containing f(x) thereexists an **(**1, 2)*-$M_{\delta\pi}$-closed set U such that x Є X and f (U) ⊂ V.
3. Inverse image of every (1, 2)*-$M_{\delta\pi}$-closed set in Y is **(**1, 2)*-$M_{\delta\pi}$-open in X.

Proof. **1 => 2.** Let V be an **(**1, 2)*-$M_{\delta\pi}$-open set in Y and f(x) Є V. Since f is contra **(**1, 2)*-$M_{\delta\pi}$-irresolute, f^{-1}(V) is **(**1, 2)*-$M_{\delta\pi}$-closed set in X and x Є f^{-1}(V). Put U= f^{-1}(V). Then x Є U and

f (U) ⊂ V.

2=>1. Let V be an **(**1, 2)*-$M_{\delta\pi}$-open set in Y and x Є f^{-1}(V). Then f(x) Є V. Hence by 2, thereexists an **(**1, 2)*-$M_{\delta\pi}$-closed set U_x such that x Є U_x and f (U_x) ⊂ V. Thus x Є U_x⊂ f^{-1}(V). This implies that f^{-1} (V) is a union of **(**1, 2)*-$M_{\delta\pi}$-closed sets of X. By Theorem 4. 1[10], f^{-1} (V) is **(**1, 2)*-$M_{\delta\pi}$-closed set of X. This shows that f is contra-**(**1, 2)*-$M_{\delta\pi}$-irresolute.

1<=>3. Let V be an **(**1, 2)*-$M_{\delta\pi}$-closed in Y. Then Y-V is **(**1, 2)*-$M_{\delta\pi}$-open set in Y. Since f is contra **(**1, 2)*-$M_{\delta\pi}$-irresolute. f^{-1} (Y-V) is **(**1, 2)*-$M_{\delta\pi}$-closed set in X. Also f^{-1} (Y-V) = X – f^{-1}(V). Therefore, X- f^{-1}(V) is **(**1, 2)*-$M_{\delta\pi}$-closed set in X. Hence, f^{-1}(V) is **(**1, 2)*-$M_{\delta\pi}$-open set in X.

Remark: The concepts of contra-(1, 2)*-$M_{\delta\pi}$-irresolute function and (1, 2)*-$M_{\delta\pi}$-irresolute function are independent of each other as shown in the following example.

Example: Le**t** X = {a, b, c, d} =Y with topologies τ_1 = {Φ, X, {a}, {a, b, d}}, τ_2 = {Φ, X, {a, b}, {b}}, σ_1= {Φ, Y, {a}}, σ_2 = {Φ, Y, {b}} respectively. Let f: X → Y be an identity function. Clearly, f is (1, 2)*-$M_{\delta\pi}$-irresolute function, but not contra-(1, 2)*-$M_{\delta\pi}$-irresolute, because

f^{-1}({a}, {b}, {a, b}) = {a}, {b}, {a, b} are not (1, 2)*-$M_{\delta\pi}$-closed set in X, where {a}, {b} and {a, b} are (1, 2)*-$M_{\delta\pi}$-open set in Y

Example: From Example 4. 4, f is contra-(1, 2)*-$M_{\delta\pi}$-irresolute function, but not (1, 2)*-$M_{\delta\pi}$-irresolute, because, f^{-1}({a}, {a, c}) = {c}, {a, c} are not (1, 2)*-$M_{\delta\pi}$-closed set in X, where {a} and {a, c} are (1, 2)*-$M_{\delta\pi}$-closed set in Y.

Theorem: Let f: X→ Y and g: Y → Z be any two functions such that gof: X → Z,

1. If f is contra-(1, 2)*-$M_{\delta\pi}$-irresolute function and g is (1, 2)*-$M_{\delta\pi}$-continuous function then gof is contra-(1, 2)*-$M_{\delta\pi}$-continuous.
2. If f is (1, 2)*-$M_{\delta\pi}$-irresolute function and g is contra-(1, 2)*-$M_{\delta\pi}$-irresolute function then gof is contra-(1, 2)*-$M_{\delta\pi}$-irresolute.

Proof. The proof follows from the definitions.

References

1. Arockiarani. I and Mohana. K, (1, 2)*-πgα-closed sets and (1, 2)*-Quasi-α-normal Spaces in Bitopological settings, Antarctica J. Math., 7(3) (2010), 345-355.
2. Arockiarani. I and Mohana. K, (1, 2)*-πgα-continuous functions in Bitopological spaces, Acta ciencia Indica, Vol. XXXVII, M. No. 4,(2011), 819-829.
3. Balachandran. K, Maki. H and Devi. R, Remarks on semigeneralized closed sets and generalized semi-closed sets, Kyungpook Math. J., 36(1996), 155-163.
4. Dontchev. J and Ganster. M, On δ-generalised closed set and T3/4-spaces, Mem. Fac. Sci. Kochi Univ. Ser. A, Math., 17(1996), 15-31.
5. Dontchev. J, Contra-continuous function and strongly S-closed spaces, Internat. J. Math. Math. Sci., 19(2) (1996), 303-310.
6. Lellis Thivagar. M, Ravi. O, On stronger forms of (1, 2)*- quotient mappings in bitopological spaces. Internat. J. Math. Game theory and Algebra. Vol. 14, No. 6 (2004), 481-492.
7. Mohana. K, Arockiarani. I, Contra (1,2)*-πgα – Continuous functions in Bitopological spaces, Advances in Applied Mathematical Analysis, Vol. 6, No. 2(2011), pp. 95-105.
8. Mohana. K, Arockiarani. I, (1, 2)*-πg-homeomorphims in Bitopological spaces, CiiT Inter. J. Automation and
9. Autonomous System, April (2011). Mohana. K, Arockiarani. I, (1, 2)*-strongly -πgα-closed sets, *Journal of Advanced studies in Topology*, Vol.2, No. 2(2011), 31-36.
10. Mohana. K, Arockiarani. I, (1, 2)*- Mδπ –closed sets in bitopological spaces, *International Journal of Mathematical Archive*, 3(5), 2012, 2083-2088..
11. Mohana. K, Arockiarani. I, On some New functions of (1, 2)*- πgαS-continuity in Bitopological spaces, (Communicated).
12. Ravi. O and Lellis Thivagar. M, A bitopological (1, 2)*-semigeneralized continuous maps, Bull. Malays. Sci. Soc.,(2)29(1) (2006), 79-88.

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