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ABSTRACT

In this paper we initiate πgβ - continuous maps, πgβ - irresolute maps furthermore extend and study their characterizations.

INTRODUCTION

N.Levine [5] introduced g-closed sets. The concept of generalized closed sets and generalized continuous maps of topological spaces were extended to closure spaces in [1]. Čech closure spaces were introduced by E. Čech [2] and then studied by many authors [3][4][6][7].

A map k: P(X) → P(X) defined on the power set P(X) of a set X is called a closure operator on X and the pair (X, k) is called a Cech closure space if the following axioms are satisfied.

1. k (ϕ) = ϕ,
2. A⊆ k(A) for every A⊆ X
3. k(A∪B) = k(A) ∪ k(B) for all A,B ⊆ X

A closure operator k on a set X is called idempotent if k (A) =k [k (A)] for all A⊆ X.

Definitions: A subset A of a čech closure space (X, k) is said to be

1. čech closed if k(A) =A
2. čech open if k(X-A) =X-A
3. čech semi-open if A ⊆ k int (A)
4. čech pre-open if A ⊆ int [k(A)]
5. čech pre-closed if k[int (A)] ⊆ A

Definition: A čech closure space (Y,l) is said to be a subspace of (X, k) if Y⊆ X and k(A) = k(A)∩Y for each subset A ⊆Y. If Y is closed in (X, k) then the subspace (Y,l) of (X,k) is said to be closed too.

Definition: Let (X,k) and (Y,l) be čech closure spaces. A map f: (X,k) →(Y,k) is said to be continuous, if f (kA) ⊆ k f(A) for every subset A ⊆ F.

Definition: Let (X,k) and (Y,l) be čech closure spaces. A map f: (X,k) →(Y,l) is said to be closed (resp.open) if f(F) is a closed(resp.open) subset of (Y,l) whenever F is a closed (resp.open) subset of (X,k).

®G - Continuous And ®G - Irresolute Functions

Definition: Let (X, u) and (Y, v) be closure spaces. A map f: (X, u) → (Y, v) is called πgβ - continuous if the inverse image of every open set in (Y, v) is πgβ - open in (X, u).

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Proposition

1. Every continuous function is πgβ - continuous.
2. Every g-continuous function is πgβ - continuous.
3. Every π-continuous function is πgβ - continuous.

Remark

The converses need not be true may be seen by the following example.

Example

Let X= {1,2,3}, Y={a,b,c}. Define a closure operator u on X by u(φ) = φ, u({1}) = u({3}) = u({1,2}) = u({1,3}) = u({2,3}) = uX = X and u({2})={2}.Define a closure operator v on Y by v(φ) = φ,v({a}) = {a,c}, v({b}) = {b},v({c}) = {a,c} & v({a,b}) = v({a,c}) = v({b,c}) = vY = Y.Let f: (X,u) → (Y,v) be defined by f(1) = b, f(2) = a & f(3) = c.

1. Then f is πgβ – continuous but not continuous. Since for the open set {a,c} in Y, the inverse image f^{-1}{a,c} = {2,3} is not open in X.
2. f is πgβ – continuous but not g-continuous. Since for the open set {a,c} in Y, the inverse image f^{-1}{a,c} = {2,3} is not g- open in X
3. f is πgβ – continuous but not π- continuous. Since the inverse image f^{-1}{b} = {1} is not π-closed in X

Proposition

Let (X, u) and (Y, v) be closure spaces and let f :(X, u) →(Y, v) be a map. Then f is

πgβ - continuous if and only if the inverse image of every closed subset of (Y, v) is
πgβ - closed in (X, u).

Proof

Let F be closed subset in (Y,v).Then Y-F is open in (Y,v).Since f is πgβ - continuous, f^{-1}(Y-F) is πgβ - open. But f^{-1}(Y-F) = X-f^{-1}(F) thus f^{-1}(F) is πgβ - closed in space (X, u). Conversely let G be an open subset in (Y, v).Then Y - G is closed in (Y, v). Since the inverse image of each closed subset in (Y, v) is πgβ - closed in (X, u). Hence f^{-1}(Y-G) is πgβ-closed in (X,u). But f^{-1}(Y-G) =X – f^{-1}(G). Thus f^{-1}(G) is πgβ - open. Therefore f is πgβ - continuous.

Definition

Let (X, u) and (Y, v) be closure spaces and a map f : (X, u) →(Y, v) is called πgβ - irresolute, if f^{-1}(G) is πgβ -open (closed) in (X, u) for every πgβ -open set (closed set) G in (Y, v).

Definition

Let (X, u) and (Y, v) be closure spaces and a map f: (X, u) →(Y, v) is called πgβ - open map(closed map) if f (B) is πgβ - open(closed) in (Y, v) for every open set(closed set) B in (X, u).

Proposition

Consider (X,u), (Y,v) and (Z,w) to be closure spaces, let f:(X,u)→(Y,v) g:(Y,v)→(Z,w) be two maps. If g ∘ f is open and g is a πgβ - continuous injection, then f is πgβ - open.

Proof

Let G be an open subset of (X, u). Since gof is open, g (f (G)) is open in (Z, w).

as g is πgβ - continuous, g^{-1}(g (f (G))) is πgβ - open in (Y, v). But g is injective, so

g^{-1}(g (f (G))) = f (G) is πgβ - open in (Y, v). Hence f is πgβ - open.

Remark

The composition of two πgβ - continuous map need not be πgβ - continuous.

Definition

A closure space (X, u) is said to be a T_f – space if every πgβ - open set in (X, u) is open.

Proposition

Let (X, u) and (Z, w) be closure spaces and (Y, v) be a T_f – space. If f : (X,u) → (Y,v) and g : (Y,v) → (Z,w) are πgβ - continuous, then g∘f is πgβ - continuous.

Proof

Let H be open in (Z, w). Since g is πgβ - continuous, g^{-1}(H) is πgβ -open in (Y, v). But (Y, v) is a T_f – space, hence g^{-1}(H) is open in (Y, v).Thus $f^{-1}(g^{-1}(H)) = (g \circ f)^{-1}(H)$ is
πgβ -open in (X, u).Therefore, g ∘ f is πgβ - continuous.

Proposition

Let (X, u), (Y, v) and (Z, w) be closure spaces. If f : (X,u) → (Y,v) is πgβ - continuous and g : (Y,v) → (Z,w) is continuous then g∘f is πgβ - continuous.

Proof

Let H be an open subset of (Z, w). Since g is continuous, g^{-1}(H) is open in (Y, v).

Since f is πgβ - continuous, $f^{-1}(g^{-1}(H))$ is πgβ - open in (X, u).But $f^{-1}(g^{-1}(H)) = (g \circ f)^{-1}(H)$.
Therefore, g∘f is πgβ - continuous.

Proposition

Let (X, u) and (Y, v) be closure spaces .If f : (X, u) →(Y, v) be a bijection, then the following statements are equivalent

1. The inverse map $f^{-1}: (Y, v) \rightarrow (X, u)$ is πgβ - continuous.
2. f is a πgβ - open map.
3. f is a πgβ - closed map.

Proof

(i) ⇒(ii)
Let $f^{-1}: (Y, v) \rightarrow (X, u)$ be πgβ - continuous and A be an open set in X. Then $(f^{-1})^{-1}(A)$ is πgβ - open, which implies f (A) is πgβ - open. Thus (i) ⇒(ii)

(ii) ⇒(iii)
Let B be closed in X. Then X-B is open in X. Since f is πgβ - open, f(X-B) is πgβ - open in Y. Then Y-f (B) is πgβ - open in X. Hence f (B) is πgβ - closed in X.

Thus (ii) ⇒(iii)
(iii) ⇒(i)
Let A be closed in X. As f is πgβ - closed, f(A) is πgβ - closed in Y. But f (A) = $(f^{-1})^{-1}(A)$. Thus f^{-1} is πgβ - continuous. Therefore (iii) ⇒(i).

Proposition

Let (X,u) and (Y,v) be closure spaces and f : (X,u) → (Y,v) be a map. Then f is

πgβ -irresolute if and only if $f^{-1}(B)$ is πgβ- closed in (X,u) whenever B is πgβ- closed in (Y,v).

Proof

Suppose B be a πgβ- closed subset of (Y,v). Then Y-B is πgβ-open in (Y,v). Since f: (X,u) → (Y,v) is πgβ -irresolute, $f^{-1}(Y-B)$ is πgβ open in (X,u). But $f^{-1}(Y-B) = X - f^{-1}(B)$, so that $f^{-1}(B)$ is πgβ- closed in (X,u). Conversely, Let A be a πgβ-open subset in (Y,v). Then Y - A is πgβ- closed in (Y, v).By the assumption, $f^{-1}(Y-A)$ is πgβ- closed in (X,u). But $f^{-1}(Y-A) = X- f^{-1}(A)$.Thus $f^{-1}(A)$ is πgβ- open in (X,u).Therefore, f is πgβ-irresolute.

Note: Every πgβ-irresolute map is πgβ-continuous.

Proposition

Let (X,u), (Y,v) and (Z,w) be closure spaces. If f : (X,u) → (Y,v) is a πgβ-irresolute map and g:(Y,v)→(Z,w) is a πgβ-continuous map, then the composition
g∘f : (X,u) → (Z,w) is πgβ-continuous.

Proof

Let G be an open subset of (Z,w). Then $g^{-1}(G)$ is a πgβ-open in (Y,v) as g is πgβ-continuous. Hence, $f^{-1}(g^{-1}(G))$ is πgβ-open in (X,u) because f is πgβ-irresolute.
Thus g∘f is πgβ-continuous.

Proposition

Let (X,u), (Y,v) and (Z,w) be closure spaces. If f:(X,u)→(Y,v) and g:(Y,v) → (Z,w) are πgβ-irresolute, Then g∘f: (X,u) → (Z,w) is πgβ- irresolute.

Proof

Let F be πgβ open set in (Z,w). As g is πgβ- irresolute,$g^{-1}(F)$ is πgβ open in (Y,v). Since, f is πgβ-irresolute, $f^{-1}(g^{-1}(F))$ is πgβ-open in (Y,v) implies $(g\circ f)^{-1}F= f^{-1}(g^{-1}(F))$ is πgβ- open in (X,u). Hence g∘f is πgβ- irresolute.

Proposition

Let (X,u) and (Z,w) be closure spaces and (Y,v) be a T_f-space. If f: (X,u) → (Y,v) be a πgβ -continuous map and g:(Y,v) → (Z,w) is a πgβ -irresolute, Then the composition
g∘f: (X,u) → (Z,w) is πgβ - irresolute.

Proof

Let V be πgβ- open in Z. Since g is πgβ - irresolute, $g^{-1}(V)$ is πgβ- open in Y. As Y is a T_f-space, $g^{-1}(V)$ is open in Y. Since f is πgβ-continuous, $f^{-1}(g^{-1}(V))$ is πgβ - open in X. Thus $(g\circ f)^{-1}(V)$ is πgβ- open in X. Hence g∘f is πgβ- irresolute

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