RESEARCH ARTICLE

NEW FUNCTIONS IN CECH ƒGꜱ1- CLOSURE SPACES

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ABSTRACT

In this paper we initiate πgβ - continuous maps, πgβ - irresolute maps furthermore extend and study their characterizations.

INTRODUCTION

N.Levine [5] introduced g-closed sets. The concept of generalized closed sets and generalized continuous maps of topological spaces were extended to closure spaces in [1]. ech closure spaces were introduced by E. ech [2] and then studied by many authors [3][4][6][7].

A map k: P(X) → P(X) defined on the power set P(X) of a set X is called a closure operator on X and the pair (X, k) is called a Cech closure space if the following axioms are satisfied.

1. k (φ) = φ,
2. A⊆ k(A) for every A⊆ X
3. k(A∪B) = k(A) ∪ k(B) for all A,B ⊆ X

A closure operator k on a set X is called idempotent if k (A) =k [k (A)] for all A⊆ X.

Definitions: A subset A of a ech closure space (X, k) is said to be

1. ech closed if k(A) =A
2. ech open if k(X-A) =X-A

3. ech semi-open if A ⊆ k int (A)
4. ech pre-open if A ⊆ int [k(A)]
5. ech pre-closed if k[int (A)] ⊆ A

Definition: A ech closure space (Y,l) is said to be a subspace of (X, k) if Y⊆ X and k(A) = k(A)∩Y for each subset A ⊆Y. If Y is closed in (X, k) then the subspace (Y,l) of (X,k) is said to be closed too.

Definition: Let (X,k) and (Y,l) be ech closure spaces. A map f: (X,k) →(Y,k) is said to be continuous, if f (kA) ⊆ k f(A) for every subset A ⊆ F.

Definition: Let (X,k) and (Y,l) be ech closure spaces. A map f: (X,k) →(Y,l) is said to be closed (resp.open) if f(F) is a closed(resp.open) subset of (Y,l) whenever F is a closed (resp.open) subset of (X,k).

ƒGꜱ - Continuous And ƒGꜱ - Irresolute Functions

Definition: Let (X, u) and (Y, v) be closure spaces. A map f: (X, u) → (Y, v) is called πgβ - continuous if the inverse image of every open set in (Y, v) is πgβ - open in (X, u).

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Proposition

1. Every continuous function is $\pi g\beta$ - continuous.
2. Every g-continuous function is $\pi g\beta$ - continuous.
3. Every π-continuous function is $\pi g\beta$ - continuous.

Remark

The converses need not be true may be seen by the following example.

Example

Let X= {1,2,3}, Y={a,b,c}. Define a closure operator u on X by u( ) = , u({1}) = u({3}) = u({1,2}) = u({1,3}) = u({2,3}) = uX = X and u({2})={2}.Define a closure operator v on Y by v( ) = ,v({a}) = {a,c}, v({b}) = {b},v({c}) = {a,c} & v({a,b}) = v({a,c}) = v({b,c}) = vY = Y.Let f: (X,u) → (Y,v) be defined by f(1) = b, f(2) = a & f(3) = c.

1. Then f is $\pi g\beta$ – continuous but not continuous. Since for the open set {a,c} in Y, the inverse image $f^{-1}\{a,c\}$ = {2,3} is not open in X.
2. f is $\pi g\beta$ – continuous but not g-continuous. Since for the open set {a,c} in Y, the inverse image $f^{-1}\{a,c\}$ = {2,3} is not g- open in X
3. f is $\pi g\beta$ – continuous but not π- continuous. Since the inverse image $f^{-1}\{b\}$ = {1} is not π-closed in X

Proposition

Let (X, u) and (Y, v) be closure spaces and let f :(X, u) →(Y, v) be a map. Then f is

$\pi g\beta$ - continuous if and only if the inverse image of every closed subset of (Y, v) is $\pi g\beta$ - closed in (X, u).

Proof

Let F be closed subset in (Y,v).Then Y-F is open in (Y,v).Since f is $\pi g\beta$ - continuous, f^{-1}(Y-F) is $\pi g\beta$ - open. But f^{-1}(Y-F) = X- f^{-1}(F) thus f^{-1}(F) is $\pi g\beta$ - closed in space (X, u). Conversely let G be an open subset in (Y, v).Then Y - G is closed in (Y, v). Since the inverse image of each closed subset in (Y, v) is $\pi g\beta$ - closed in (X, u). Hence f^{-1}(Y-G) is $\pi g\beta$-closed in (X,u). But f^{-1}(Y-G) =X – f^{-1}(G). Thus f^{-1}(G) is $\pi g\beta$ - open. Therefore f is $\pi g\beta$ - continuous.

Definition

Let (X, u) and (Y, v) be closure spaces and a map f : (X, u) →(Y, v) is called $\pi g\beta$ - irresolute, if f^{-1}(G) is $\pi g\beta$ -open (closed) in (X, u) for every $\pi g\beta$ -open set (closed set) G in (Y, v).

Definition

Let (X, u) and (Y, v) be closure spaces and a map f: (X, u) →(Y, v) is called $\pi g\beta$ - open map(closed map) if f (B) is $\pi g\beta$ - open(closed) in (Y, v) for every open set(closed set) B in (X, u).

Proposition

Consider (X,u), (Y,v) and (Z,w) to be closure spaces, let f:(X,u)→(Y,v) g:(Y,v)→(Z,w) be two maps. If g ∘ f is open and g is a $\pi g\beta$ - continuous injection, then f is $\pi g\beta$ - open.

Proof

Let G be an open subset of (X, u). Since gof is open, g (f (G)) is open in (Z, w).

as g is $\pi g\beta$ - continuous, g^{-1}(g (f (G))) is $\pi g\beta$ - open in (Y, v). But g is injective, so

g^{-1}(g (f (G))) = f (G) is $\pi g\beta$ - open in (Y, v). Hence f is $\pi g\beta$ - open.

Remark

The composition of two $\pi g\beta$ - continuous map need not be $\pi g\beta$ - continuous.

Definition

A closure space (X, u) is said to be a T_f – space if every $\pi g\beta$ - open set in (X, u) is open.

Proposition

Let (X, u) and (Z, w) be closure spaces and (Y, v) be a T_f – space. If f : (X,u) → (Y,v) and g : (Y,v) → (Z,w) are $\pi g\beta$ - continuous, then g ∘ f is $\pi g\beta$ - continuous.

Proof

Let H be open in (Z, w). Since g is $\pi g\beta$ - continuous, g^{-1}(H) is $\pi g\beta$ -open in (Y, v). But (Y, v) is a T_f – space, hence g^{-1}(H) is open in (Y, v).Thus $f^{-1}(g^{-1}$(H)) = (g ∘ f)$^{-1}$(H) is $\pi g\beta$ -open in (X, u).Therefore, g ∘ f is $\pi g\beta$ - continuous.

Proposition

Let (X, u), (Y, v) and (Z, w) be closure spaces. If f : (X,u) → (Y,v) is $\pi g\beta$ - continuous and g : (Y,v) → (Z,w) is continuous then g ∘ f is $\pi g\beta$ - continuous.

Proof

Let H be an open subset of (Z, w). Since g is continuous, g^{-1}(H) is open in (Y, v).

Since f is $\pi g\beta$ - continuous, $f^{-1}(g^{-1}$(H)) is $\pi g\beta$ - open in (X, u).But $f^{-1}(g^{-1}$(H)) = (g ∘ f)$^{-1}$(H).
Therefore, g ∘ f is $\pi g\beta$ - continuous.

Proposition

Let (X, u) and (Y, v) be closure spaces .If f : (X, u) →(Y, v) be a bijection, then the following statements are equivalent

1. The inverse map f^{-1}: (Y, v) →(X, u) is πgβ - continuous.
2. f is a πgβ - open map.
3. f is a πgβ - closed map.

Proof

(i) ⇒(ii)
Let f^{-1}: (Y, v) →(X, u) be πgβ - continuous and A be an open set in X. Then $(f^{-1})^{-1}$(A) is πgβ - open, which implies f (A) is πgβ - open. Thus (i) ⇒(ii)

(ii) ⇒(iii)
Let B be closed in X. Then X-B is open in X. Since f is πgβ - open, f(X-B) is πgβ - open in Y. Then Y-f (B) is πgβ - open in X. Hence f (B) is πgβ - closed in X.

Thus (ii) ⇒(iii)
(iii) ⇒(i)
Let A be closed in X. As f is πgβ - closed, f(A) is πgβ - closed in Y. But f (A) = $(f^{-1})^{-1}$(A). Thus f^{-1} is πgβ - continuous. Therefore (iii) ⇒(i).

Proposition

Let (X,u) and (Y,v) be closure spaces and f : (X,u) → (Y,v) be a map. Then f is

πg -irresolute if and only if f^{-1}(B) is πg - closed in (X,u) whenever B is πg - closed in (Y,v).

Proof

Suppose B be a πg - closed subset of (Y,v). Then Y-B is πg - open in (Y,v). Since f: (X,u) → (Y,v) is πg -irresolute, f^{-1}(Y-B) is πg open in (X,u). But f^{-1}(Y-B) = X - f^{-1}(B), so that f^{-1}(B) is πg - closed in (X,u). Conversely, Let A be a πg -open subset in (Y,v). Then Y - A is πg - closed in (Y, v).By the assumption, f^{-1}(Y-A) is πg - closed in (X,u). But f^{-1}(Y-A) = X- f^{-1}(A).Thus f^{-1}(A) is πg - open in (X,u).Therefore, f is πg -irresolute.

Note: Every πg -irresolute map is πg -continuous.

Proposition

Let (X,u), (Y,v) and (Z,w) be closure spaces. If f : (X,u) → (Y,v) is a πg -irresolute map and g:(Y,v)→(Z,w) is a πg -continuous map, then the composition
g ∘ f : (X,u) → (Z,w) is πg -continuous.

Proof

Let G be an open subset of (Z,w). Then g^{-1}(G) is a πg -open in (Y,v) as g is πg -continuous. Hence, f^{-1} (g^{-1}(G)) is πg -open in (X,u) because f is πg -irresolute.
Thus g ∘ f is πg -continuous.

Proposition

Let (X,u), (Y,v) and (Z,w) be closure spaces. If f:(X,u)→(Y,v) and g:(Y,v) → (Z,w) are πg -irresolute, Then g ∘ f: (X,u) → (Z,w) is πg - irresolute.

Proof

Let F be πgβ open set in (Z,w). As g is πgβ- irresolute,g^{-1}(F) is πgβ open in (Y,v). Since, f is πg -irresolute, f^{-1}(g^{-1}(F)) is πgβ- open in (Y,v) implies $(g∘f)^{-1}$F= f^{-1} (g^{-1}(F)) is πgβ- open in (X,u). Hence g ∘ f is πgβ- irresolute.

Proposition

Let (X,u) and (Z,w) be closure spaces and (Y,v) be a T_f -space. If f: (X,u) → (Y,v) be a πg -continuous map and g:(Y,v) → (Z,w) is a πgβ -irresolute, Then the composition
g ∘ f: (X,u) → (Z,w) is πgβ - irresolute.

Proof

Let V be πgβ- open in Z. Since g is πgβ - irresolute, g^{-1}(V) is πgβ- open in Y. As Y is a T_f -space, g^{-1}(V) is open in Y. Since f is πgβ-continuous, f^{-1}(g^{-1}(V)) is πgβ - open in X. Thus (g ∘ f)$^{-1}$(V) is πgβ- open in X. Hence g ∘ f is πgβ- irresolute

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