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FatemehMohammadiAghjeh Mashhad

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SOME CHARACTERIZATIONS OF COHEN-MACAULAY MODULES AND GORENSTEIN MODULES

FatemehMohammadiAghjeh Mashhad

Islamic Azad University, Parand Branch, Tehran, Iran

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ABSTRACT

Let $(R,\mathfrak{m},k)$ be a commutative Noetherian local ring. In this paper, we will give some characterizations for specifying Cohen-Macaulay R-modules and Gorenstein R-modules.

INTRODUCTION

Throughout this paper, R is a commutative Noetherian ring with nonzero identity.The $\mathfrak{m}$-adic completion of a local ring $(R,\mathfrak{m},k)$ will be denoted by $\hat{R}$.

In the classical homological algebra, it is important to characterize Cohen-Macaulay and Gorenstein local rings. So, we are interest to find some ways to characterize these rings. As an extension of our goal, we will give some characterizations for specifying Cohen-Macaulay R-modules and Gorenstein R-modules. For an R-module N, we fix the notation $N^{\perp}$ for the full subcategory of $\mathbf{D}_0^f(R)$ whose objects are exactly those complexes X for which $H_{\mathfrak{m}}^i(X,N)=0$ for all $i \neq \operatorname{depth}_R N$. We will show that a nonzero finitely generated R-module N of finite either projective or injective dimension is Cohen-Macaulay if and only if there is a nonzero R-module $M \in N^{\perp}$ of finite Gorenstein projective dimension such that $\operatorname{Supp}_R(M) \cap \operatorname{Assh}_R(N) \neq \phi$ this, partially improves (Herzog *et al*, 2003). Also, we establish the Gorenstein analogue of this. We prove that a non-zero finitely generated R-module N is Gorenstein if and only if $N^{\perp} = \mathbf{D}_0^f(R)$. It improves (Herzog *et al*, 2003).

PREREQUISITES

Hyperhomology

Throughout, we will work within $\mathbf{D}(R)$, the derived category of R-modules. The objects in $\mathbf{D}(R)$ are complexes of R-modules and symbol denotes isomorphisms in this category. For a complex

$$X = \cdots \to X_{n+1} \xrightarrow{\partial_{n+1}^X} X_n \xrightarrow{\partial_n^X} X_{n-1} \to \cdots$$

in $\mathbf{D}(R)$, its supremum and infimum are defined respectively by $\sup X := \sup\{i \in \mathbb{Z} \mid H_i(X) \neq 0\}$ and $\inf X := \inf\{i \in \mathbb{Z} \mid H_i(X) \neq 0\}$, with the usual

Corresponding author:* **FatemehMohammadiAghjeh Mashhad
Islamic Azad University, Parand Branch, Tehran, Iran

convention that $\sup \phi = -\infty$ and $\inf \phi = \infty$. Modules will be considered as complexes concentrated in degree zero and we denote the full subcategory of complexes with homology concentrated in degree zero by $\mathsf{D}_0(R)$. The full subcategory of complexes homologically bounded to the right (resp. left) is denoted by $\mathsf{D}_{\,]}(R)$ (resp. $\mathsf{D}_{[}\,(R)$). Also, the full subcategories of homologically bounded complexes and of complexes with finitely generated homology modules will be denoted by $\mathsf{D}_{[\,]}(R)$ and $\mathsf{D}^f(R)$, respectively. Throughout for any two properties $*$ and $\bullet$ of complexes, we set $\mathsf{D}^*_\bullet(R) := \mathsf{D}_\bullet(R) \cap \mathsf{D}^*(R)$. So for instance, $\mathsf{D}^f_{[\,]}(R)$ stands for the full subcategory of homologically bounded complexes with finitely generated homology modules.

For any complex X in $\mathsf{D}_{\,]}(R)$ (resp. $\mathsf{D}_{[}\,(R)$), there is a bounded to the right (resp. left) complex P (resp. I) consisting of projective (resp. injective) R-modules which is isomorphic to X in $\mathsf{D}(R)$. A such complex P (resp. I) is called a projective (resp. injective) resolution of X . A complex X is said to have finite projective (resp. injective) dimension, if X possesses a bounded projective (resp. injective) resolution. The left derived tensor product functor $- \otimes^{\mathbf{L}}_R \sim$ is computed by taking a projective resolution of the first argument or of the second one. The right derived homomorphism functor $\mathbf{R}\mathrm{Hom}_R(-,\sim)$ is computed by taking a projective resolution of the first argument or by taking an injective resolution of the second one. For any two complexes X and Y and any integer i, set $\mathrm{Ext}^i_R(X,Y) := H_{-i}(\mathbf{R}\mathrm{Hom}_R(X,Y))$. Also, we recall that for any homologically finite complex like X and any ideal of R like $\mathfrak{a}$, $\mathrm{depth}(\mathfrak{a}, X)$ is defined by $\mathrm{depth}(\mathfrak{a}, X) := -\sup \mathbf{R}\mathrm{Hom}_R(R/\mathfrak{a}, X)$.

Gorenstein Homological Dimension

We recall some definitions from the theory of Gorenstein homological dimensions from the text book (Enochset *al*, 2000). An R-module M is said to be Gorenstein projective if there exists an exact complex P of projective R-modules such that $M \cong \mathrm{im}(P_0 \to P_{-1})$ and $\mathrm{Hom}_R(P,Q)$ is exact for all projective R-modules Q. Also, an R-module N is said to be Gorenstein injective if there exists an exact complex I of injective R-modules such that $N \cong \mathrm{im}(I_1 \to I_0)$ and $\mathrm{Hom}_R(E,I)$ is exact for all injective R-modules E. A complex X is said to have finite Gorenstein projective (resp. injective) dimension if it is isomorphic (in $\mathsf{D}(R)$) to a bounded complex of Gorenstein projective (resp. injective) R-modules. Obviously, if projective (resp. injective) dimension of a complex X is finite, then its Gorenstein projective (resp. injective) dimension is also finite.

Auslander Category

Let $(R, \mathfrak{m})$ be a local ring. A dualizing complex for R is a complex $D \in \mathsf{D}^f_{[\,]}(R)$ such that the homothetymorphism, $R \to \mathbf{R}\mathrm{Hom}_R(D,D)$ is an isomorphism in $\mathsf{D}(R)$ and D has finite injective dimension. So, R is Gorenstein if and only if R is a dualizing complex of R . A dualizing complex D is said to be normalized if $\sup D = \dim R$. Assume that R possesses a normalized dualizing complex D . The Auslander category $\mathsf{A}(R)$ (with respect to D) is the full subcategory of $\mathsf{D}_{[\,]}(R)$ whose objects are exactly those complexes $X \in \mathsf{D}_{[\,]}(R)$ for which $D \otimes^{\mathbf{L}}_R X \in \mathsf{D}_{[\,]}(R)$ and the natural morphism $\eta_X : X \to \mathbf{R}\mathrm{Hom}_R(D, D \otimes^{\mathbf{L}}_R X)$ is an isomorphism in $\mathsf{D}(R)$.

Local Cohomology

Let $\mathfrak{a}$ be an ideal of R. One can easily check that the section functor $\Gamma_{\mathfrak{a}}(-) = \varinjlim_n \mathrm{Hom}_R(R/\mathfrak{a}^n, -)$ defines an additive, left exact functor on the category of complexes of R-modules. So, we may consider its right derived functor in the category $\mathsf{D}(R)$. For any complex $X \in \mathsf{D}_{[}\,(R)$, the complex $\mathbf{R}\Gamma_{\mathfrak{a}}(X) \in \mathsf{D}_{[}\,(R)$ is defined by $\mathbf{R}\Gamma_{\mathfrak{a}}(X) := \Gamma_{\mathfrak{a}}(I)$, where I is an (every) injective resolution of X . Also, for any integer i , the i-th local cohomology module of X with respect to $\mathfrak{a}$ is defined by $H^i_{\mathfrak{a}}(X) := H_{-i}(\mathbf{R}\Gamma_{\mathfrak{a}}(X))$. The notion of generalized section functors was introduced by Yassemi (Yassemi, 1994). For any two complexes $X \in \mathsf{D}_{\,]}(R)$ and $Y \in \mathsf{D}_{[}\,(R)$, he defined $\mathbf{R}\Gamma_{\mathfrak{a}}(X,Y)$ by $\mathbf{R}\Gamma_{\mathfrak{a}}(X,Y) := \mathbf{R}\Gamma_{\mathfrak{a}}(\mathbf{R}\mathrm{Hom}_R(X,Y))$. Since $\mathbf{R}\Gamma_{\mathfrak{a}}(-) = \mathbf{R}\Gamma_{\mathfrak{a}}(R,-)$, this notion extends the usual notion of section functor. For any integer i, set $H^i_{\mathfrak{a}}(X,Y) := H_{-i}(\mathbf{R}\Gamma_{\mathfrak{a}}(X,Y))$.

RESULTS

In this section, we establish some characterizations for Cohen-Macaulay R-modules and Gorenstein R-modules. We start with the following lemma.

Lemma 1 Let $X \in \mathsf{D}_{[}\,(R)$ and $N \in \mathsf{D}^f_0(R)$. Then

$$-\sup \mathbf{R}\operatorname{Hom}_R(N,X)=\inf\{\operatorname{depth}_{R_\mathfrak{p}} X_\mathfrak{p} \mid \mathfrak{p}\in \operatorname{Supp}_R(N)\}=\operatorname{depth}_R(\operatorname{Ann}_R(H_0(N)),X).$$

Proof. As $N \simeq H_0(N)$, we have $-\sup \mathbf{R}\operatorname{Hom}_R(N,X)=-\sup \mathbf{R}\operatorname{Hom}_R(H_0(N),X)$ and $\operatorname{Supp}_R(N)=\operatorname{Supp}_R(H_0(N))$. By (Foxby, 1981) and (Foxby*et al*, 2003), we have

$$-\sup \mathbf{R}\operatorname{Hom}_R(H_0(N),X)=\inf\{\operatorname{depth}_{R_\mathfrak{p}} X_\mathfrak{p} \mid \mathfrak{p}\in \operatorname{Supp}_R(H_0(N))\}=\operatorname{depth}_R(\operatorname{Ann}_R(H_0(N)),X).$$

Lemma 2 Let $(R,\mathfrak{m})$ be a local ring possessing a normalized dualizing complex D and M,N two nonzero finitely generated R-modules. Assume that Gorenstein projective dimension of M is finite. If $\operatorname{Supp}_R(M)\cap \operatorname{Assh}_R(N)\neq \phi$, then

$$\sup \mathbf{R}\operatorname{Hom}_R(N, D\otimes^{\mathbf{L}}_R M)\geq \dim_R N.$$

Proof. Let $\mathfrak{p}$ be a prime ideal of R. From (Foxby,1981), one has $\inf D_\mathfrak{p}=\dim R/\mathfrak{p}+\operatorname{depth} R_\mathfrak{p}$. By (Christensen, 2000), $\operatorname{Gpd}_{R_\mathfrak{p}} M_\mathfrak{p}$ is finite and so $M_\mathfrak{p}\in \mathsf{A}(R_\mathfrak{p})$ by (Christensen *etal*, 2006).So, $M_\mathfrak{p}\simeq \mathbf{R}\operatorname{Hom}_{R_\mathfrak{p}}(D_\mathfrak{p}, D_\mathfrak{p}\otimes^{\mathbf{L}}_{R_\mathfrak{p}} M_\mathfrak{p})$. Also, by (Christensen *et al*, 2006) and (Christensen, 2000), $\operatorname{Gpd}_{R_\mathfrak{p}} M_\mathfrak{p}=\operatorname{depth} R_\mathfrak{p}-\operatorname{depth}_{R_\mathfrak{p}} M_\mathfrak{p}$. So, by applying (Foxby*et al*, 2003) and (Christensen *et al*, 2002) we have

$$\begin{aligned}\operatorname{depth}_{R_\mathfrak{p}} M_\mathfrak{p} &=\operatorname{depth}_{R_\mathfrak{p}}(\mathbf{R}\operatorname{Hom}_{R_\mathfrak{p}}(D_\mathfrak{p}, D_\mathfrak{p}\otimes^{\mathbf{L}}_{R_\mathfrak{p}} M_\mathfrak{p}))\\ &=\operatorname{depth}_{R_\mathfrak{p}}(D_\mathfrak{p}\otimes^{\mathbf{L}}_{R_\mathfrak{p}} M_\mathfrak{p})+\dim R/\mathfrak{p}+\operatorname{depth} R_\mathfrak{p}.\end{aligned}$$

By (Christensen *et al*, 2006) $M\in \mathsf{A}(R)$ and so, $M\otimes^{\mathbf{L}}_R D\in \mathsf{D}_{[\,]}(R)$. Thus by the assumption, Lemma 1 and (Christensen, 2010), it turns out that:

$$\begin{aligned}\sup \mathbf{R}\operatorname{Hom}_R(N,D\otimes^{\mathbf{L}}_R M) &=-\inf\{\operatorname{depth}_{R_\mathfrak{p}}(D\otimes^{\mathbf{L}}_R M)_\mathfrak{p} \mid \mathfrak{p}\in\operatorname{Supp}_R(N)\}\\ &=-\inf\{\operatorname{depth}_{R_\mathfrak{p}}(D_\mathfrak{p}\otimes^{\mathbf{L}}_{R_\mathfrak{p}} M_\mathfrak{p}) \mid \mathfrak{p}\in\operatorname{Supp}_R(N)\}\\ &=-\inf\{-\dim R/\mathfrak{p}-\operatorname{Gpd}_{R_\mathfrak{p}} M_\mathfrak{p} \mid \mathfrak{p}\in\operatorname{Supp}_R(N)\}\\ &=\sup\{\dim R/\mathfrak{p}+\operatorname{Gpd}_{R_\mathfrak{p}} M_\mathfrak{p} \mid \mathfrak{p}\in\operatorname{Supp}_R(N)\}\\ &=\sup\{\dim R/\mathfrak{p}+\operatorname{Gpd}_{R_\mathfrak{p}} M_\mathfrak{p} \mid \mathfrak{p}\in\operatorname{Supp}_R(M)\cap\operatorname{Supp}_R(N)\}\\ &\geq \dim_R N.\end{aligned}$$

Now, we are ready to establish a characterization of Cohen-Macaulay R -modules. It partially improves (Herzog *et al*, 2003). To this end, for a complex $Y\in \mathsf{D}_{[}\,(R)$, we fix the notation $Y^{\perp}$ for the full subcategory of $\mathsf{D}^f_0(R)$ whose objects are exactly those complexes $X\in \mathsf{D}^f_0(R)$ for which $H^i_\mathfrak{m}(X,Y)=0$ for all $i\neq \operatorname{depth}_R Y$.

Theorem 3 (Herzog *et al*, 2003) Let $(R,\mathfrak{m})$ be a Cohen-Macaulay local ring and N a nonzero finitely generated R-module. Then N is Cohen-Macaulay if and only if there exists a nonzero R-module $M\in N^{\perp}$ of finite projective dimension.

Theorem 4 Let $(R,\mathfrak{m})$ be a local ring and N a nonzero finitely generated R-module. Consider the following conditions.

1. N is Cohen-Macaulay.
2. There is a nonzero R-module $M\in N^{\perp}$ of finite projectivedimension such that $\operatorname{Supp}_R(M)\cap\operatorname{Assh}_R(N)\neq\phi$.
3. There is a nonzero R -module $M\in N^{\perp}$ of finite Gorenstein projective dimension such that $\operatorname{Supp}_R(M)\cap\operatorname{Assh}_R(N)\neq\phi$.

Then $i)$ and $ii)$ are equivalent and clearly $ii)$ implies $iii)$. In addition, if either projective or injective dimension of N is finite, then all these conditions are equivalent.

Proof. $i)\Rightarrow ii)$ Assume that N is Cohen-Macaulay. Then $H^i_\mathfrak{m}(R,N)=H^i_\mathfrak{m}(N)=0$ for all $i\neq \operatorname{depth}_R N$, and so $R\in N^{\perp}$.

$ii)\Rightarrow iii)$ is clear.

Both of implications $iii)\Rightarrow i)$ and $ii)\Rightarrow i)$ have similar proofs. Because of this, we show $iii)$ implies $i)$. Assume that either projective or injective dimension of N is finite and there exists a nonzero R-module $M\in N^{\perp}$ of finite Gorenstein projective dimension such that $\operatorname{Supp}_R(M)\cap\operatorname{Assh}_R(N)\neq\phi$. So, either projective or injective dimension of the $\hat{R}$-module $\hat{N}$ is finite. Since $\dim R$ is finite, then any flat module has finite projective dimension and so (Christensen *et al*, 2006) and (Christensen *et al*, 2010) yield that $Gpd_{\hat{R}}\hat{M}=Gpd_R M$. So Gorenstein projective dimension of the $\hat{R}$-module $\hat{M}$ is finite. One can see that, $\hat{M}\in\hat{N}^{\perp}$. Now, we show that $\operatorname{Supp}_{\hat{R}}(\hat{M})\cap\operatorname{Assh}_{\hat{R}}(\hat{N})\neq\phi$ .Since $\operatorname{Supp}_R(M)\cap\operatorname{Assh}_R(N)\neq\phi$, there exists a prime ideal $\mathfrak{p}$ in $\operatorname{Supp}_R(M)$ such that $\dim_R N=\dim R/\mathfrak{p}$. Then, $\dim_{\hat{R}}\hat{N}=\dim\hat{R}/\mathfrak{p}\hat{R}$, and so there exists $q\in Spec(\hat{R})$ such that $p\hat{R}\subseteq q$ and $\dim_{\hat{R}}\hat{N}=\dim\hat{R}/\mathfrak{q}$. So,

$\mathfrak{q} \in Assh_{\hat{R}}(\hat{N})$. As $\mathfrak{p} \in \mathrm{Supp}_R(M)$, $\mathrm{Ann}_R(x) \subseteq \mathfrak{p}$ for some $x \in M$. Hence, $\mathrm{Ann}_{\hat{R}}(x) \subseteq \mathfrak{p}\hat{R} \subseteq \mathfrak{q}$, and so $\mathfrak{q} \in Supp_{\hat{R}}(\hat{M})$. Hence $\mathrm{Supp}_{\hat{R}}(\hat{M}) \cap \mathrm{Assh}_{\hat{R}}(\hat{N}) \neq \phi$. Now, without loss of generality, we may and do assume that R is $\mathfrak{m}$-adic complete. So, R possesses a normalized dualizingcomplex D. By the assumption, we have $M \in N^{\perp}$ and so $H^i_{\mathfrak{m}}(M,N)=0$ for all $i \neq \mathrm{depth}_R N$, and then $-\inf \mathbf{R}\Gamma_{\mathfrak{m}}(M,N) = \mathrm{depth}_R N$. Hence, from (Mohammadi *et al*, 2010) and Lemma 2, we deduce that

$$\mathrm{depth}_R N = \sup \mathbf{R}\,\mathrm{Hom}_R(N, D \otimes^{\mathbf{L}}_R M) \geq \dim_R N,$$

And so N is Cohen-Macaulay.

In the following immediate corollary, we characterize Cohen-Macaulay local rings.

Corollary 5 Let $(R,\mathfrak{m})$ be a local ring. The following are equivalent:

1. R is Cohen-Macaulay.
2. There is a nonzero R-module $M \in R^{\perp}$ of finite projective dimension such that $\dim_R M = \dim R$.
3. There is a nonzero R-module $M \in R^{\perp}$ of finite Gorenstein projective dimension such that $\dim_R M = \dim R$.

Next, we establish the Gorenstein analogue of Theorem 4. It is worth to point out that it improves (Herzog *et al*, 2003). Recall that a non-homologically trivial complex $Y \in \mathbf{D}^f_{[\,]}(R)$ over a local ring $(R,\mathfrak{m})$ is said to be Gorenstein if $\mathrm{id}_R Y = \mathrm{depth}_R Y$.

Proposition 6 **(**Herzog *et al*, 2003) Let $(R,\mathfrak{m})$ be a Cohen-Macaulay local ring and N a finitely generated R-module. Then

1. N is Gorenstein if and only if $N^{\perp}$ contains the subcategory of finitely generated R-modules.
2. Let N be a Cohen-Macaulay R-module. N is maximal Cohen-Macaulay if and only if $N^{\perp}$ contains the subcategory of finitely generated R-modules with finite projective dimension.

Theorem 7Let $(R,\mathfrak{m},k)$ be a local ring and $Y \in \mathbf{D}^f_{[\,]}(R)$ a non-homologically trivial complex. The following are equivalent:

1. Y is Gorenstein.
2. $Y^{\perp} = \mathbf{D}^f_0(R)$.
3. $k \in Y^{\perp}$.

Proof. $i) \Rightarrow ii)$ Let $X \in \mathbf{D}^f_0(R)$. By (Yassemi, 1994), $\inf\{i \in \mathbb{Z} \mid H^i_{\mathfrak{m}}(X,Y) \neq 0\} = \mathrm{depth}_R Y$. Since Y is Gorenstein, one has $\mathrm{id}_R Y = \mathrm{depth}_R Y$, and so $H^i_{\mathfrak{m}}(X,Y)=0$ for all $i \neq \mathrm{depth}_R Y$. This means that $X \in Y^{\perp}$.

$ii) \Rightarrow iii)$ is clear.

$iii) \Rightarrow i)$ Since $\mathrm{Supp}_R(Y) \cap \mathrm{Supp}_R(k) = \{\mathfrak{m}\}$, by (Yassemi, 1994) one has $H^i_{\mathfrak{m}}(k,Y) = \mathrm{Ext}^i_R(k,Y)$ for all integers i. Thus, $\mathrm{Ext}^i_R(k,Y)=0$ for all $i \neq \mathrm{depth}_R Y$. By (Christensen , 2000), this yields that $\mathrm{id}_R Y = \mathrm{depth}_R Y$.

Next, we specialize the above proposition to modules.

Corollary 8 Let $(R,\mathfrak{m},k)$ be a local ring and N a nonzero finitely generated R-module. The following are equivalent:

1. N is Gorenstein.
2. $N^{\perp} = \mathbf{D}^f_0(R)$.
3. $k \in N^{\perp}$.

Finally, we characterize Gorenstein local rings.

Proposition 9Let $(R,\mathfrak{m},k)$ be a local ring. The following are equivalent:

1. R isGorenstein.
2. There exists a nonzero cyclic R-module N of finite Gorenstein injective dimension.
3. There exists a nonzero Gorenstein R-module M of finite Gorenstein projective dimension.

Proof. Without loss of generality, we may assume that R is $\mathfrak{m}$-adic complete. So R has a dualizing complex.

$i) \Rightarrow ii)$ is clear by taking $N := R$.

$ii) \Rightarrow i)$ See (Foxbyet *al*, 2007).

$i) \Rightarrow iii)$ is clear by taking $M := R$.

$iii) \Rightarrow i)$ Since M has finite injective dimension and by (Christensen *et al*, 2006) M belongs to $\mathsf{A}(R)$, the assertion follows from (Christensen, 2000).

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