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## RESEARCH ARTICLE

# A COMPARATIVE STUDY OF ONE FACTOR ANOVA MODEL UNDER FUZZY ENVIRONMENTS USING TRAPEZOIDAL FUZZY NUMBERS

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**ABSTRACT**

This paper deals with the problem of one factor ANOVA test using Trapezoidal Fuzzy Numbers (TFNs.). The proposed ANOVA test is analysed under various types of trapezoidal fuzzy models such as Alpha Cut Interval, Ranking Function, Total Integral Value and Graded Mean Integration Representation (GMIR). Finally a comparative view of all conclusions obtained from various tests is given. Moreover, two numerical examples having different conclusions have been illustrated for a concrete comparative study.



## INTRODUCTION

Statistical analysis, in traditional form, is based on crispness of data, random variable, point estimation, hypotheses and parameter and so on. And there are many different situations in which the above mentioned concepts are imprecise. On the other hand, the theory of fuzzy sets is a well-known tool for formulation and analysis of imprecise and subjective concepts. Therefore the hypotheses testing for mean and variance with fuzzy data can be important. The problem of statistical inference in fuzzy environment is developed in different approaches. The statistical hypotheses testing under fuzzy environments has been studied by many authors using the fuzzy set theory concepts introduced by Zadeh [38].

The application by using fuzzy set theory to statistics has been widely studied in Manton *et al*. [22], Buckley [8] and Viertl [31]. Arnold [6] proposed the fuzzification of usual statistical hypotheses and considered the testing hypotheses under fuzzy constraints on the type I and type II errors. Saade [26], Saade and Schwarzlander [25] considered the binary hypotheses testing and discussed the fuzzy likelihood functions in the decision making process by applying a fuzzified version of the Baye's criterion. Grzegorzewski [14], Watanabe and Imaizumi [34] proposed the fuzzy test for testing hypotheses with vague data and the fuzzy test produced the acceptability of the null and alternative hypotheses. The statistical hypotheses testing for fuzzy data by proposing the notions of degrees of optimism and pessimism was proposed by Wu [37]. Viertl [31] investigated some methods to construct confidence intervals and statistical tests for fuzzy data. Wu [36] proposed some approaches to construct fuzzy confidence intervals for the unknown fuzzy parameter. Arefi and Taheri [5] developed an approach to test fuzzy hypotheses upon fuzzy test statistic for vague data. A new approach to the problem of testing statistical hypotheses is introduced by Chachi *et al*. [9]. Dubois and Prade [12] defined any of the fuzzy numbers as a fuzzy subset of the real line. Chen and Chen [11] presented a method for ranking generalized trapezoidal fuzzy numbers. The symmetric triangular approximation was presented by Ma *et al*. [21]. Chanas [10] derived a formula for determining the interval approximations under the Hamming distance. The trapezoidal approximation was proposed by Abbasbandy *et al*. [1-4]. Grzegorzewski *et al*. [15] proposed the trapezoidal approximation of a fuzzy number, which

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is considered as a reasonable compromise between two opposite tendencies: to lose too much information and to introduce too sophisticated form of approximation from the point of view of computation. Mikihiko Konishi *et al*. [24] proposed a method of ANOVA for the fuzzy interval data by using the concept of fuzzy sets. Hypothesis testing of one factor ANOVA model for fuzzy data was proposed by Wu [35] using the h-level set and the notions of pessimistic degree and optimistic degree by solving optimization problems. Parthiban and Gajivaradhan [23] analysed one-factor ANOVA model under fuzzy environments using trapezoidal fuzzy numbers.

Wang *et al*. [33] presented the method for centroid formulae for a generalized fuzzy number. Iuliana Carmen BĂRBĂCIORU [18] dealt with the statistical hypotheses testing using membership function of fuzzy numbers. Salim Rezvani [28] analysed the ranking functions with trapezoidal fuzzy numbers. Wang [33] arrived some different approach for ranking trapezoidal fuzzy numbers. Thorani *et al*. [29] approached the ranking function of a trapezoidal fuzzy number with some modifications. Salim Rezvani and Mohammad Molani [27] presented the shape function and Graded Mean Integration Representation for trapezoidal fuzzy numbers. Liou and Wang [20] proposed the Total Integral Value of the trapezoidal fuzzy number with the index of optimism and pessimism.

In the proposed $\alpha$-cut interval method, we convert the given fuzzy hypothesis test of one factor ANOVA model with fuzzy data into two hypotheses tests of one factor ANOVA models with crisp data namely, lower level model and upper level model. And the decision rules which are used to accept or reject the fuzzy null and alternative hypotheses are also provided here. We test the hypothesis of each of the one factor ANOVA models with crisp data and obtain the results based on the decision rules. In the decision rules of the proposed testing technique, we do not use the degrees of optimism, pessimism and h-level set but they are used in Wu [35]. In fact we would like to counter an argument that $\alpha$-cut interval method can be general enough to deal with one-factor ANOVA model under fuzzy environments. For better understanding, the proposed fuzzy hypothesis test is illustrated with two numerical examples having different decisions. Moreover, as the observed samples are in terms of tfns., we can evenhandedly use the centroid point/ranking grades of tfns. for statistical hypothesis testing. In arriving the centroid point/ranking grades of tfns., various methods are used to test which could be the best fit. Therefore, in the proposed approach, the centroid point/ranking grades of tfns. are used in one-factor ANOVA model. That is, the proposed method is analysed using ranking grades of TFNs. [17, 18], Graded Mean Integration Representation (GMIR) of tfns. and Total Integral Value (TIV) of tfns. [23]. Finally, a comparative study all these methods based on the decisions obtained from various approaches is given. **In order to present this paper in nutshell, we only present the necessary data and explanations by avoiding elementary, surplus mathematical calculations and repetitive tables.**

### *Preliminaries*

#### *Definition*

#### *Generalized fuzzy number*

A generalized fuzzy number $\widetilde{A}$ is described as any fuzzy subset of the real line $\mathbb{R}$, whose membership function $\mu_{\widetilde{A}}(x)$ satisfies the following conditions:

1. $\mu_{\widetilde{A}}(x)$ is a continuous mapping from $\mathbb{R}$ to the closed interval $[0, \omega],\ 0 \le \omega \le 1$,
2. $\mu_{\widetilde{A}}(x) = 0, \text{ for all } x \in (-\infty, a]$,
3. $\mu_L(x) = L_{\widetilde{A}}(x)$ is strictly increasing on $[a, b]$,
4. $\mu_{\widetilde{A}}(x) = \omega, \text{ for all } [b, c],\ \omega$, is a constant and $0 < \omega \le 1$,
5. $\mu_R(x) = R_{\widetilde{A}}(x)$ is strictly decreasing on $[c, d]$,
6. $\mu_{\widetilde{A}}(x) = 0, \text{ for all } x \in [d, \infty)$ where a, b, c, d are real numbers such that $a < b \le c < d$.

#### *Definition*

A fuzzy set $\widetilde{A}$ is called ***normal*** fuzzy set if there exists an element (member) 'x' such that $\mu_{\widetilde{A}}(x) = 1$. A fuzzy set $\widetilde{A}$ is called ***convex*** fuzzy set if $\mu_{\widetilde{A}}(\alpha x_1 + (1 - \alpha)x_2) \ge \min\{\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)\}$ where $x_1, x_2 \in X \text{ and } \alpha \in [0, 1]$. The set $\widetilde{A}_\alpha = \{x \in X / \mu_{\widetilde{A}}(x) \ge \alpha\}$ is said to be the $\alpha$ - cut of a fuzzy set $\widetilde{A}$.

***Definition***

A fuzzy subset $\widetilde{A}$ of the real line $\mathbb{R}$ with ***membership function*** $\mu_{\widetilde{A}}(x)$ such that $\mu_{\widetilde{A}}(x):\mathbb{R}\rightarrow[0, 1]$, is called a ***fuzzy number*** if $\widetilde{A}$ is normal, $\widetilde{A}$ is fuzzy convex, $\mu_{\widetilde{A}}(x)$ is upper semi-continuous and $\mathrm{Supp}(\widetilde{A})$ is bounded, where $\mathrm{Supp}(\widetilde{A}) = \mathrm{cl}\{x \in \mathbb{R}: \mu_{\widetilde{A}}(x) > 0\}$ and 'cl' is the closure operator.

***Definition***

**$\alpha$-cut of a fuzzy number:** A useful notion for dealing with a fuzzy number is a set of its $\alpha$-cuts. The $\alpha$-cut of a fuzzy number $\widetilde{A}$ is a non-fuzzy set defined as $A_\alpha=\{x\in\mathbb{R}:\mu_{\widetilde{A}}(x)\geq\alpha\}$. A family of $\{A_\alpha:\alpha\in(0,1]\}$ is a set representation of the fuzzy number $\widetilde{A}$. According to the definition of a fuzzy number, it is easily seen that every $\alpha$-cut of a fuzzy number is a closed interval. Hence we have, $A_\alpha=[A_\alpha^L, A_\alpha^U]$ where $A_\alpha^L=\inf\{x\in\mathbb{R}:\mu_{\widetilde{A}}(x)\geq\alpha\}$ and $A_\alpha^U=\sup\{x\in\mathbb{R}:\mu_{\widetilde{A}}(x)\geq\alpha\}$. A space of all fuzzy numbers will be denoted by $F(\mathbb{R})$.

It is known that for a ***normalized tfn.*** $\widetilde{A}=(a, b, c, d; 1)$, there exists four numbers $a, b, c, d \in \mathbb{R}$ and two functions $L_{\widetilde{A}}(x), R_{\widetilde{A}}(x):\mathbb{R}\rightarrow[0, 1]$, where $L_{\widetilde{A}}(x)$ and $R_{\widetilde{A}}(x)$ are non-decreasing and non-increasing functions respectively. And its membership function is defined as follows:
$\mu_{\widetilde{A}}(x)=\{L_{\widetilde{A}}(x)=(x-a)/(b-a)$ for $a\leq x\leq b$; $1$ for $b\leq x\leq c$; $R_{\widetilde{A}}(x)=(x-d)/(c-d)$ for $c\leq x\leq d$ and $0$ otherwise$\}$. The functions $L_{\widetilde{A}}(x)$ and $R_{\widetilde{A}}(x)$ are also called the ***left*** and ***right side*** of the fuzzy number $\widetilde{A}$ respectively [12]. In this paper, we assume that $\int_{-\infty}^{\infty}\widetilde{A}(x)dx<+\infty$. The left and right sides of the fuzzy number $\widetilde{A}$ are strictly monotone, obviously, $\widetilde{A}_L$ and $\widetilde{A}_U$ are inverse functions of $L_{\widetilde{A}}(x)$ and $R_{\widetilde{A}}(x)$ respectively. Another important type of fuzzy number was introduced in [7] as follows:
Let $a, b, c, d \in \mathbb{R}$ such that $a<b\leq c<d$. A fuzzy number $\widetilde{A}$ defined as $\mu_{\widetilde{A}}(x):\mathbb{R}\rightarrow[0, 1]$,
$\mu_{\widetilde{A}}(x)=\left(\frac{x-a}{b-a}\right)^n$ for $a\leq x\leq b$; $1$ for $b\leq x\leq c$; $\left(\frac{d-x}{d-c}\right)^n$ for $c\leq x\leq d$; $0$ otherwise where $n>0$ is denoted by $\widetilde{A}=(a, b, c, d)_n$. And $L(x)=\left(\frac{x-a}{b-a}\right)^n$; $R(x)=\left(\frac{d-x}{d-c}\right)^n$ can also be termed as left and right spread of the tfn. [Dubois and Prade in 1981].
If $\widetilde{A}=(a, b, c, d)_n$, then[1-4],
$\widetilde{A}_\alpha=\left[\widetilde{A}_L(\alpha), \widetilde{A}_U(\alpha)\right]=\left[a+(b-a)\sqrt[n]{\alpha},\ d-(d-c)\sqrt[n]{\alpha}\right];\ \alpha\in[0, 1]$.

When $n=1$ and $b=c$, we get a triangular fuzzy number. The conditions $r=1, a=b$ and $c=d$ imply the closed interval and in the case $r=1, a=b=c=d=t$ (some constant), we can get a crisp number 't'. Since a trapezoidal fuzzy number is completely characterized by $n=1$ and four real numbers $a\leq b\leq c\leq d$, it is often denoted as $\widetilde{A}=(a, b, c, d)$. And the family of trapezoidal fuzzy numbers will be denoted by $F^T(\mathbb{R})$. Now, for $n=1$ we have a normal trapezoidal fuzzy number $\widetilde{A}=(a, b, c, d)$ and the corresponding $\alpha$-cut is defined by
$\widetilde{A}_\alpha=\left[a+\alpha(b-a),\ d-\alpha(d-c)\right];\ \alpha\in[0, 1]---(2.5)$. And we need the following results which can be found in [17, 19].

## RESULT

Let D = {[a, b], a$\leq$b and a, b$\in\mathbb{R}$ }, the set of all closed, bounded intervals on the real line $\mathbb{R}$.

## RESULT

Let A = [a, b] and B = [c, d] in D. Then A = B if a = c and b = d.

***One-Factor ANOVA Model***

The Analysis of Variance (ANOVA) is a powerful statistical tool for tests of significance. The term "Analysis of Variance" was introduced by Prof. R. A. Fisher in 1920's to deal with problems in the analysis of agronomical data. Variation is inherent in nature. The total variation in any set of numerical data is due to a number of causes which may be classified as (i) Assignable causes and (ii) Chance causes.

The variation due to assignable causes can be detected and measured whereas the variation due to chance is beyond the control of human hand and cannot be traced separately. In general, ANOVA studies mainly the homogeneity of populations by separating the total variance into its various components. That is, this technique is to test the difference among the means of populations by studying the amount of variation within each of the samples relative to the amount of variation between the samples. Samples under employing in ANOVA model are assumed to be drawn from 'normal populations of equal variances'. The variation of each value around its own grand mean should be independent for each value. A one-factor ANOVA is used when the analysis involves only one factor with more than two levels and different subjects in each of the experimental conditions.

Let a sample of N values of a given random variable X drawn from a normal population with variance $\sigma^2$ which is subdivided into 'h' classes according to some factor of classification with which the classes are homogeneous, that is, there is no difference between various classes.

Now, let $\mu_i$ be the mean of $i^{th}$ population class. The test of hypotheses are:

Null hypothesis: $H_0 : \mu_1 = \mu_2 = \cdots = \mu_h$ against Alternative hypothesis: $H_A : \mu_1 \neq \mu_2 \neq \cdots \neq \mu_h$.

Let $x_{ij}$ be the value of the $j^{th}$ member of the $i^{th}$ class, which contains $n_i$ members. Let the general mean of all the N values be $\overline{x}$ and the mean of $n_i$ values in the $i^{th}$ class be $\overline{x}_i$. Now,

$$\sum_i \sum_j \left(x_{ij} - \overline{x}\right)^2 = \sum_i \sum_j \left\{\left(x_{ij} - \overline{x}\right) + \left(x_i - \overline{x}\right)\right\}^2 = \sum_i \sum_j \left(x_{ij} - \overline{x}_i\right)^2 + \sum_i n_i \left(\overline{x}_i - \overline{x}\right)^2 = Q_2 + Q_1 \text{ where}$$

$Q_1 = \sum_i n_i \left(\overline{x}_i - \overline{x}\right)^2$ is the sum of the squared deviations of class means from the general mean (variation between classes) and $Q_2 = \sum_i \sum_j \left(x_{ij} - \overline{x}_i\right)^2$ is the sum of the squared deviations of variates from the corresponding class means (variation within classes). Q is total variation.

Now, it is known from the theory of estimation that $\left(\frac{ns^2}{n\text{-}1}\right)$ is an unbiased estimate of $\sigma^2$, where $s^2$ is the variance of a sample of size 'n' drawn from a population with variance $\sigma^2$. That is, $E\left(ns^2 / n\text{-}1\right) = \sigma^2$. Since the items in the $i^{th}$ class with variance $\frac{1}{n_i}\sum_{j=1}^{n_i}\left(x_{ij} - \overline{x}_i\right)^2$ may be considered as a sample of size $n_i$ drawn from a population with variance $\sigma^2$. That is, $E\left\{\frac{n_i}{n_i - 1} \cdot \frac{1}{n_i}\sum_{j=1}^{n_i}\left(x_{ij} - \overline{x}_i\right)^2\right\} = \sigma^2$.

i.e. $E\left[\sum_i \sum_j \left(x_{ij} - \bar{x}_i\right)^2\right] = \sum_{i=1}^{h} (n_i - 1)\sigma^2$ i.e. $E[Q_2] = (N - h)\sigma^2$ i.e. $E\left\{\frac{Q_2}{N - h}\right\} = \sigma^2$.

Hence, $\frac{Q_2}{N - h}$ is an **unbiased estimate** of $\sigma^2$ with $(N - h)$ degrees of freedom.

Let us consider the entire group of N items with variance $\frac{1}{N}\sum_i \sum_j \left(x_{ij} - \bar{x}\right)^2$ as the sample of size N drawn from the same population. Now, $E\left\{\frac{N}{N-1} \cdot \frac{1}{N}\sum_i \sum_j \left(x_{ij} - \bar{x}\right)^2\right\} = \sigma^2$. That is, $E\left[\frac{Q}{N-1}\right] = \sigma^2$, this states that $\frac{Q}{N-1}$ is an **unbiased estimate** of $\sigma^2$ with $(N-1)$ degrees of freedom. Now, $E(Q_1) = E(Q) - E(Q_2) = (N-1)\sigma^2 - (N-h)\sigma^2$ $\Rightarrow E\left(\frac{Q_1}{h - 1}\right) = \sigma^2$.

Thus, $\frac{Q_1}{h - 1}$ is also an **unbiased estimate** of $\sigma^2$ with $(h - 1)$ degrees of freedom. If we assume that the sample drawn from a normal population, then the estimates $\frac{Q_1}{h - 1}$ and $\frac{Q_2}{N - h}$ are independent and hence the ratio $\frac{Q_1 / (h - 1)}{Q_2 / (N - h)}$ follows F-distribution with $(h - 1, N - h)$ degrees of freedom. Choosing the ratio which is greater than one, we employ the F-test. For simplicity, let us choose, $M_1 = \frac{Q_1}{h - 1}$ and $M_2 = \frac{Q_2}{N - h}$.

Aggregating the above results, the ANOVA table for one factor classification is given below ([16, 30]):

| **Source of Variation (S.V.)** | **Sum of Squares (S.S.)** | **Degrees of freedom (d.f.)** | **Mean Square (M.S.)** | **Variance Ratio (F-value)** |
|---|---|---|---|---|
| **Between Classes** | $Q_1$ | $h - 1$ | $M_1 = \frac{Q_1}{(h - 1)}$ | $F = \left(\frac{M_1}{M_2}\right)^{\pm 1}$ |
| **Within Classes** | $Q_2$ | $N - h$ | $M_2 = \frac{Q_2}{(N - h)}$ | |
| **Total** | $Q$ | $N - 1$ | -- | |

***The decision rules of F-test are given below***

1. If $M_2 < M_1$ and $F = \frac{M_1}{M_2} < F_t$ where $F_t$ is the tabulated value of F with $(h - 1, N - h)$ degrees of freedom at 'k' level of significance, then we accept the null hypothesis $H_0$, otherwise the alternative hypothesis $H_A$ is accepted.
2. If $M_1 < M_2$ and $F = \frac{M_2}{M_1} < F_t$ where $F_t$ is the tabulated value of F with $(N - h, h - 1)$ degrees of freedom at 'k' level of significance, then we accept the null hypothesis $H_0$, otherwise the alternative hypothesis $H_A$ is accepted.

Note that here we use the notation for level of significance is to be "k" instead of "$\alpha$" so as to avoid confusion with '$\alpha$ - cut' value that can be seen in trapezoidal fuzzy numbers (tfns.). For simplicity of calculations, the following formulae for $Q$ , $Q_1$ and $Q_2$ are used:

$$Q = \sum_i \sum_j x_{ij}^2 - \frac{T^2}{N} \text{ where } T = \sum_i \sum_j x_{ij} \,;\; Q_1 = \sum_i \left( \frac{T_i^2}{n_i} \right) - \frac{T^2}{N} \text{ where } T_i = \sum_j x_{ij} \text{ and } Q_2 = Q - Q_1 .$$

***One-Factor ANOVA model with TFNs. using $\alpha$ - cut method***

The fuzzy test of hypotheses of one-factor ANOVA model where the sample data are trapezoidal fuzzy numbers is proposed here. Using the relation (2.5), we transform the fuzzy ANOVA model to interval ANOVA model. Fetching the upper limit of the fuzzy interval, we construct upper level crisp ANOVA model and considering the lower limit of the fuzzy interval, we construct the lower level crisp ANOVA model. Thus, in this proposed approach, two crisp ANOVA models are designated in terms of upper and lower levels. Finally, we analyse lower level and upper level model using crisp one-factor ANOVA technique. Let there be N values of samples for a given random variables 'X' which are subdivided into 'h' classes according to some kind of classification. Then the lower level data and upper level data for given trapezoidal fuzzy numbers using $\alpha$ - cut method can be assigned as follows:

***Lower level data***

$$\begin{array}{cccc}
a_{11} + \alpha\left(b_{11} - a_{11}\right) & a_{12} + \alpha\left(b_{12} - a_{12}\right) & \ldots & a_{1j} + \alpha\left(b_{1j} - a_{1j}\right) \\
a_{21} + \alpha\left(b_{21} - a_{21}\right) & a_{22} + \alpha\left(b_{22} - a_{22}\right) & \ldots & a_{2j} + \alpha\left(b_{2j} - a_{2j}\right) \\
 & \ldots & & \\
a_{i1} + \alpha\left(b_{i1} - a_{i1}\right) & a_{i2} + \alpha\left(b_{i2} - a_{i2}\right) & \ldots & a_{ij} + \alpha\left(b_{ij} - a_{ij}\right) \\
\multicolumn{4}{c}{\text{where } 0 \le i \le h,\ 0 \le j \le n}
\end{array}$$

***Upper level data***

$$\begin{array}{cccc}
d_{11} - \alpha\left(d_{11} - c_{11}\right) & d_{12} - \alpha\left(d_{12} - c_{12}\right) & \ldots & d_{1j} - \alpha\left(d_{1j} - c_{1j}\right) \\
d_{21} - \alpha\left(d_{21} - c_{21}\right) & d_{22} - \alpha\left(d_{22} - c_{22}\right) & \ldots & d_{2j} - \alpha\left(d_{2j} - c_{2j}\right) \\
 & \ldots & & \\
d_{i1} - \alpha\left(d_{i1} - c_{i1}\right) & d_{i2} - \alpha\left(d_{i2} - c_{i2}\right) & \ldots & d_{ij} - \alpha\left(d_{ij} - c_{ij}\right) \\
\multicolumn{4}{c}{\text{where } 0 \le i \le h,\ 0 \le j \le n}
\end{array}$$

***The one-factor ANOVA formulae using $\alpha$ - cut can be tabulated as follows***

| Lower level model | Upper level model |
|---|---|
| $Q^L = \sum_i \sum_j \left[a_{ij} + \alpha\left(b_{ij} - a_{ij}\right)\right]^2 - \frac{T^2}{N}$ where $0 \le i \le h,\ 0 \le j \le n$ . | $Q^U = \sum_i \sum_j \left[d_{ij} - \alpha\left(d_{ij} - c_{ij}\right)\right]^2 - \frac{T^2}{N}$ where $0 \le i \le h,\ 0 \le j \le n$ . |
| $T_i = \sum_j \left[a_{ij} + \alpha\left(b_{ij} - a_{ij}\right)\right]$ ; i =1, 2, …, h. | $T_i = \sum_j \left[d_{ij} - \alpha\left(d_{ij} - c_{ij}\right)\right]$ ; i = 1, 2, ..., h |
| And $T = \sum_{r=1}^{h} T_r$ , $Q_1^L = \sum_i \frac{T_i^2}{n_i} - \frac{T^2}{N}$ | And $T = \sum_{r=1}^{h} T_r$ , $Q_1^U = \sum_i \frac{T_i^2}{n_i} - \frac{T^2}{N}$ |
| $Q_2^L = Q^L - Q_1^L$ | $Q_2^U = Q^U - Q_1^U$ |

Let 'k' be the level of significance.

Now, the null hypothesis: $\widetilde{H}_0 : \tilde{\mu}_1 = \tilde{\mu}_2 = \cdots = \tilde{\mu}_h$ against the alternative hypothesis: $\widetilde{H}_A : \tilde{\mu}_1 \ne \tilde{\mu}_2 \ne \cdots \ne \tilde{\mu}_h$

$$\Rightarrow \quad \left[\widetilde{H}_0\right] : \left[\tilde{\mu}_1\right] = \left[\tilde{\mu}_2\right] = \cdots = \left[\tilde{\mu}_h\right] \text{ against } \left[\widetilde{H}_A\right] : \left[\tilde{\mu}_1\right] \ne \left[\tilde{\mu}_2\right] \ne \cdots \ne \left[\tilde{\mu}_h\right] .$$

$\Rightarrow$ $\left[H_0^L,\ H_0^U\right]:\ \left[\mu_1^L,\ \mu_1^U\right]=\left[\mu_2^L,\ \mu_2^U\right]=\cdots=\left[\mu_h^L,\ \mu_h^U\right]$ against

$\left[H_A^L,\ H_A^U\right]:\ \left[\mu_1^L,\ \mu_1^U\right]\neq\left[\mu_2^L,\ \mu_2^U\right]\neq\cdots\neq\left[\mu_h^L,\ \mu_h^U\right]$

$\Rightarrow$ The following two sets of hypotheses can be obtained.

1. The null hypothesis $H_0^L:\ \mu_1^L=\mu_2^L=\cdots=\mu_h^L$ against the alternative hypothesis $H_A^L:\mu_1^L\neq\mu_2^L\neq\cdots\neq\mu_h^L$.
2. The null hypothesis $H_0^U:\ \mu_1^U=\mu_2^U=\cdots=\mu_h^U$ against the alternative hypothesis $H_A^U:\mu_1^U\neq\mu_2^U\neq\cdots\neq\mu_h^U$.

***Decision rules for proposed test***

1. If $F^L<F_t$ at 'k' level of significance with $(N-h,\ h-1)$ degrees of freedom then the null hypothesis $H_0^L$ is accepted for certain value of $\alpha\in[0,\ 1]$, otherwise the alternative hypothesis $H_A^L$ is accepted.
2. If $F^U<F_t$ at 'k' level of significance with $(N-h,\ h-1)$ degrees of freedom then the null hypothesis $H_0^U$ is accepted for certain value of $\alpha\in[0,\ 1]$, otherwise the alternative hypothesis $H_A^U$ is accepted.

***Decision table***

| Acceptance of null hypotheses $\widetilde{H}_0$ | | |
|---|---|---|
| **Lower Level Model** | **Upper Level Model** | **Conclusion** |
| If $H_0^L$ is accepted for all $\alpha$, $\alpha\in[0,1]$ | and $H_0^U$ is accepted for all $\alpha$, $\alpha\in[0,1]$ | then $\widetilde{H}_0$ is accepted for all $\alpha$, $\alpha\in[0,1]$ |
| If $H_0^L$ is accepted for all $\alpha$, $\alpha\in[0,1]$ | and $H_0^U$ is rejected for all $\alpha$, $\alpha\in[0,1]$ | then $\widetilde{H}_0$ is rejected for all $\alpha$, $\alpha\in[0,1]$ |
| If $H_0^L$ is rejected for all $\alpha$, $\alpha\in[0,1]$ | and $H_0^U$ is accepted for all $\alpha$, $\alpha\in[0,1]$ | then $\widetilde{H}_0$ is rejected for all $\alpha$, $\alpha\in[0,1]$ |
| If $H_0^L$ is rejected for all $\alpha$, $\alpha\in[0,1]$ | or $H_0^U$ is rejected for all $\alpha$, $\alpha\in[0,1]$ | then $\widetilde{H}_0$ is rejected for all $\alpha$, $\alpha\in[0,1]$ |

Partial acceptance of null hypothesis $H_0$ at the intersection of certain level of $\alpha$ at both upper level and lower level models can be taken into account for the acceptance of the null hypothesis $\widetilde{H}_0$.

***Example-1*** A food company wished to test four different package designs for a new product. Ten stores with approximately equal sales volumes are selected as the experimental units. Package designs 1 and 4 are assigned to three stores each and package designs 2 and 3 are assigned to two stores each. We cannot record the exact sales volume in a store due to some unexpected situations, but we have the fuzzy data for sales volumes. The fuzzy data are given below [35]:

| **Package design (i)** | **Store (Observation j)** | | |
|---|---|---|---|
| | **1** | **2** | **3** |
| **1** | $(9,\ 10,\ 12,\ 13)$ | $(14,\ 15,\ 17,\ 18)$ | -- |
| **2** | $(11,\ 13,\ 16,\ 19)$ | $(10,\ 14,\ 16,\ 20)$ | $(11,\ 12,\ 14,\ 15)$ |
| **3** | $(15,\ 17,\ 19,\ 21)$ | $(14,\ 16,\ 19,\ 20)$ | $(17,\ 20,\ 21,\ 23)$ |
| **4** | $(15,\ 18,\ 21,\ 23)$ | $(21,\ 23,\ 25,\ 27)$ | -- |

We test the hypothesis whether the fuzzy mean sales are same for four designs of package or not. Let $\widetilde{\mu}_i$ be the mean sales for the $i^{th}$ design. Then the null hypothesis $\widetilde{H}_0:\ \widetilde{\mu}_1=\widetilde{\mu}_2=\widetilde{\mu}_3=\widetilde{\mu}_4$ against the alternative hypothesis $\widetilde{H}_A:\ \widetilde{\mu}_1\neq\widetilde{\mu}_2\neq\widetilde{\mu}_3\neq\widetilde{\mu}_4$.

***Example-2*** In order to determine whether there is significant difference in the durability of 3 makes of computers, samples of size 5 are selected from each make and the frequency of repair during the first year of purchase is observed. The results are obtained in terms of fuzzy data due to different kinds of maintenance and usage. The results are as follows:

| Makes | | |
|---|---|---|
| **A** | **B** | **C** |
| $(3, 5, 7, 8)$ | $(6, 8, 10, 13)$ | $(4, 6, 8, 9)$ |
| $(4, 6, 9, 10)$ | $(8, 9, 11, 12)$ | $(2, 4, 5, 7)$ |
| $(6, 8, 10, 11)$ | $(9, 11, 13, 15)$ | $(2, 5, 7, 9)$ |
| $(8, 10, 12, 14)$ | $(9, 12, 14, 15)$ | $(2, 5, 8, 10)$ |
| $(5, 7, 9, 12)$ | $(2, 4, 6, 9)$ | $(1, 2, 4, 7)$ |

In view of the above data, the testing procedure is proposed to check "is there any significant difference in the durability of the 3 makes of computers?"

We test the hypothesis whether the fuzzy means of the 3 makes of computers differ or not. Here, the null hypothesis $\widetilde{H}_0 : \tilde{\mu}_1 = \tilde{\mu}_2 = \tilde{\mu}_3$ against the alternative hypothesis $\widetilde{H}_A : \tilde{\mu}_1 \neq \tilde{\mu}_2 \neq \tilde{\mu}_3$.

***One-factor ANOVA model using alpha cut interval method***

***Example*** let us consider example-1, the interval model for the given trapezoidal fuzzy number using $\alpha$ - cut method is:

| Package design (i) | Store (Observation j) | | |
|---|---|---|---|
| | **1** | **2** | **3** |
| **1** | $[9 + \alpha, 13 - \alpha]$ | $[14 + \alpha, 18 - \alpha]$ | -- |
| **2** | $[11+ 2\alpha, 19 - 3\alpha]$ | $[10 + 4\alpha, 20 - 4\alpha]$ | $[11 + \alpha, 15 - \alpha]$ |
| **3** | $[15 + 2\alpha, 21 - 2\alpha]$ | $[14 + 2\alpha, 20 - \alpha]$ | $[17 + 3\alpha, 23 - 2\alpha]$ |
| **4** | $[15 + 3\alpha, 23 - 2\alpha]$ | $[21 + 2\alpha, 27 - 2\alpha]$ | -- |

Now, the ANOVA tables for "lower level $\alpha$ - cut interval" and "upper level $\alpha$ - cut interval" are given below:

***Lower level model***

| Package design (i) | Store (Observation j) | | |
|---|---|---|---|
| | **1** | **2** | **3** |
| **1** | $[9 + \alpha]$ | $[14 + \alpha]$ | -- |
| **2** | $[11+ 2\alpha]$ | $[10 + 4\alpha]$ | $[11 + \alpha]$ |
| **3** | $[15 + 2\alpha]$ | $[14 + 2\alpha]$ | $[17 + 3\alpha]$ |
| **4** | $[15 + 3\alpha]$ | $[21 + 2\alpha]$ | -- |

The null hypothesis $H_0^L : \mu_1^L = \mu_2^L = \mu_3^L = \mu_4^L$ against the alternative hypothesis $H_A^L : \mu_1^L \neq \mu_2^L \neq \mu_3^L \neq \mu_4^L$.

***Upper level model***

| Package design (i) | Store (Observation j) | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| 1 | $[13 - \alpha]$ | $[18 - \alpha]$ | -- |
| 2 | $[19 - 3\alpha]$ | $[20 - 4\alpha]$ | $[15 - \alpha]$ |
| 3 | $[21 - 2\alpha]$ | $[20 - \alpha]$ | $[23 - 2\alpha]$ |
| 4 | $[23 - 2\alpha]$ | $[27 - 2\alpha]$ | -- |

The null hypothesis $H_0^U: \mu_1^U = \mu_2^U = \mu_3^U = \mu_4^U$ against the alternative hypothesis $H_A^U: \mu_1^U \neq \mu_2^U \neq \mu_3^U \neq \mu_4^U$.

***The ANOVA table for lower level model***

| Source of Variance (S.V.) | Sum of Squares (S.S.) | Degrees of freedom (d.f.) | Mean Square (M.S.) | F-ratio $\left(F_C^L\right)$ |
|---|---|---|---|---|
| Between Classes | $Q_1^L$ | $h - 1 = (4-1) = 3$ | $M_1^L = \frac{Q_1^L}{3}$ | $F_C^L = \frac{M_1^L}{M_2^L}$ |
| Within Classes | $Q_2^L$ | $N - h = (10-4) = 6$ | $M_2^L = \frac{Q_2^L}{6}$ | |

Here, $N = 10$ and $n_i = 2, 3, 3, 2$ for the package designs 1, 2, 3, 4 respectively.

$T = 137 + 21\alpha$; $\sum_i \frac{T_i^2}{n_i} = \frac{1}{6}\left[283\alpha^2 + 3540\alpha + 11755\right]$ and

$\sum_i \sum_j \left[a_{ij} + \alpha\left(b_{ij} - a_{ij}\right)\right]^2 = 53\alpha^2 + 584\alpha + 1995$ $Q^L = \frac{1}{10}\left[89\alpha^2 + 86\alpha + 1181\right]$; $Q_1^L = \frac{1}{60}\left[184\alpha^2 + 876\alpha + 4936\right]$ and

$Q_2^L = \frac{1}{60}\left[350\alpha^2 - 360\alpha + 2150\right]$. And $M_1^L = \frac{1}{180}\left[184\alpha^2 + 876\alpha + 4936\right]$;

$M_2^L = \frac{1}{360}\left[350\alpha^2 - 360\alpha + 2150\right]$ and $F_C^L = \left(\frac{4}{5}\right)\left[\frac{46\alpha^2 + 219\alpha + 1234}{35\alpha^2 - 36\alpha + 215}\right]$ where $0 \le \alpha \le 1$ and $F_C^L$ is the calculated value of 'F' at lower level model. Now, the tabulated value of 'F' at k = 5% level of significance with $(h - 1, N - h) = (3, 6)$ degrees of freedom is $F_{t(\text{at } 5\%)} = 4.76$. Here, $F_C^L < F_t$ at $\alpha = 0.1$ and $F_C^L > F_t$ for $0.2 \le \alpha \le 1$.

***Hence, the null hypothesis $H_0^L$ is rejected at 5% level of significance for $0.2 \le \alpha \le 1$.***

***The ANOVA table for upper level model***

| Source of Variance (S.V.) | Sum of Squares (S.S.) | Degrees of freedom (d.f.) | Mean Square (M.S.) | F-ratio $\left(F_C^U\right)$ |
|---|---|---|---|---|
| Between Classes | $Q_1^U$ | $h - 1 = (4-1) = 3$ | $M_1^U = \frac{Q_1^U}{3}$ | $F_C^U = \frac{M_1^U}{M_2^U}$ |
| Within Classes | $Q_2^U$ | $N - h = (10-4) = 6$ | $M_2^U = \frac{Q_2^U}{6}$ | |

Here, $N = 10$ and $n_i = 2, 3, 3, 2$ for the package designs 1, 2, 3, 4 respectively.

$$T = 199 - 19\alpha; \ \sum_i \frac{T_i^2}{n_i} = \frac{1}{6}\left[238\alpha^2 - 4580\alpha + 24407\right] \text{ and}$$

$$\sum_i \sum_j \left[d_{ij} - \alpha\left(d_{ij} - c_{ij}\right)\right]^2 = 45\alpha^2 - 782\alpha + 4107$$

$$Q^U = \frac{1}{10}\left[89\alpha^2 - 258\alpha + 1469\right]; \ Q_1^U = \frac{1}{30}\left[107\alpha^2 - 214\alpha + 3232\right] \text{ and}$$

$$Q_2^U = \frac{1}{6}\left[32\alpha^2 - 112\alpha + 235\right]. \text{ And } \quad M_1^U = \frac{1}{90}\left[107\alpha^2 - 214\alpha + 3232\right]; \quad M_2^U = \frac{1}{36}\left[32\alpha^2 - 112\alpha + 235\right] \text{ and}$$

$F_C^U = \left(\frac{2}{5}\right)\left[\frac{107\alpha^2 - 214\alpha + 3232}{32\alpha^2 - 112\alpha + 235}\right]$ where $0 \le \alpha \le 1$ and $F_C^U$ is the calculated value of 'F' at upper level model.

Now, the tabulated value of 'F' at k = 5% level of significance with $(h - 1, N - h) = (3, 6)$ degrees of freedom is $F_{t(\text{at } 5\%)} = 4.76$. Here, $F_C^U > F_t$ for all $\alpha$ where $0 \le \alpha \le 1$.

**Hence we reject the null hypothesis $H_0^U$ at 5% level of significance for all $\alpha\left(0 \le \alpha \le 1\right)$.** Thus, the rejection level of null hypotheses for lower and upper level data are given below:

$H_0^L$ is rejected for all $\alpha, 0.2 \le \alpha \le 1$ and $H_0^U$ is rejected for all $\alpha, 0 \le \alpha \le 1$. Therefore, we accept the alternative hypothesis $\widetilde{H}_A$ of the fuzzy ANOVA model.

***Conclusion***

The factor level fuzzy means $\tilde{\mu}_i$ are not equal. Hence, we conclude that there is a relation between package design and sales volumes.

***Remark***

In this proposed method, the notions of pessimistic degree and optimistic degree are not used. The whole calculation technique is fully based on $\alpha$ - cut interval method [4]. And the decision obtained in the proposed fuzzy hypothesis testing using $\alpha$ - cut interval ANOVA method for example-1 fits better when compared with Wu [35].

***Example***

Let us consider example-2, the interval model for the given trapezoidal fuzzy number using $\alpha$ - cut method is:

| **Make** | **Sample (Observation j)** | | | | |
|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** |
| **A** | 3+2α, 8-α | 4+2α, 10-α | 6+2α, 11-α | 8+2α, 14-2α | 5+2α, 12-3α |
| **B** | 6+2α, 13-3α | 8+α, 12-α | 9+2α, 15-2α | 9+3α, 15-α | 2+2α, 9-3α |
| **C** | 4+2α, 9-α | 2+2α, 7-2α | 2+3α, 9-2α | 2+3α, 10-2α | 1+α, 7-3α |

The ANOVA tables for "Lower level $\alpha$ - cut interval" and "Upper level $\alpha$ - cut interval" are:

**Lower level $\alpha$ - cut interval:**

| Make | Sample (Observation j) | | | | |
|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** |
| **A** | 3+2α | 4+2α | 6+2α | 8+2α | 5+2α |
| **B** | 6+2α | 8+α | 9+2α | 9+3α | 2+2α |
| **C** | 4+2α | 2+2α | 2+3α | 2+3α | 1+α |

The null hypothesis $H_0^L: \mu_1^L = \mu_2^L = \mu_3^L$ against the alternative hypothesis $H_0^L: \mu_1^L \neq \mu_2^L \neq \mu_3^L$.

**Upper level $\alpha$ - cut interval:**

| Make | Sample (Observation j) | | | | |
|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** |
| **A** | 8-α | 10-α | 11-α | 14-2α | 12-3α |
| **B** | 13-3α | 12-α | 15-2α | 15-α | 9-3α |
| **C** | 9-α | 7-2α | 9-2α | 10-2α | 7-3α |

The null hypothesis $H_0^U: \mu_1^U = \mu_2^U = \mu_3^U$ against the alternative hypothesis $H_A^U: \mu_1^U \neq \mu_2^U \neq \mu_3^U$.

**The ANOVA table for lower level model:**

| S.V. | S.S. | d.f. | M.S. | F-ratio $\left(F_C^L\right)$ |
|---|---|---|---|---|
| **Between Classes** | $Q_1^L$ | $h - 1 = (3-1) = 2$ | $M_1^L = \frac{Q_1^L}{2}$ | $F_C^L = \frac{M_1^L}{M_2^L}$ |
| **Within Classes** | $Q_2^L$ | $N - h = (15-3) = 12$ | $M_2^L = \frac{Q_2^L}{12}$ | |

Here, $N = 15$ and $n_i = 5, 5, 5$ for the makes A, B, C respectively.

$$T = 71 + 31\alpha; \ \sum_i \frac{T_i^2}{n_i} = \frac{1}{5}\left[321\alpha^2 + 1442\alpha + 1953\right] \text{ and}$$

$$\sum_i \sum_j \left[a_{ij} + \alpha\left(b_{ij} - a_{ij}\right)\right]^2 = 69\alpha^2 + 292\alpha + 445$$

$$Q^L = \frac{1}{15}\left[74\alpha^2 - 22\alpha + 1634\right]; \ Q_1^L = \frac{1}{15}\left[2\alpha^2 - 76\alpha + 818\right] \text{ and}$$

$$Q_2^L = \frac{1}{5}\left[24\alpha^2 + 18\alpha + 272\right]. \text{ And } M_1^L = \frac{1}{30}\left[2\alpha^2 - 76\alpha + 818\right]; \ M_2^L = \frac{1}{60}\left[24\alpha^2 + 18\alpha + 272\right] \text{ and}$$

$F_C^L = \left[\frac{2\alpha^2 - 76\alpha + 818}{12\alpha^2 + 9\alpha + 136}\right]$ where $0 \leq \alpha \leq 1$ and $F_C^L$ is the calculated value of 'F' at lower level model. Now, the tabulated value of 'F' at k = 5% level of significance with $(h - 1, N - h) = (2, 12)$ degrees of freedom is $F_{t(\text{at } 5\%)} = 3.88$. Since, $F_C^L > F_{t(\text{at } 5\%)} \ \forall\alpha, \ (0 \leq \alpha \leq 1)$, we reject the null hypothesis $H_0^L$.

**$\Rightarrow$ There is a significant difference in the durability of the 3 makes of computers at lower level of $\alpha$ - cut .**

**The ANOVA table for upper level model**

| S.V. | S.S. | d.f. | M.S. | F-ratio $\left(F_C^U\right)$ |
|---|---|---|---|---|
| **Between Classes** | $Q_1^U$ | $h - 1 = (3-1) = 2$ | $M_1^U = \frac{Q_1^U}{2}$ | $F_C^U = \frac{M_1^U}{M_2^U}$ |
| **Within Classes** | $Q_2^U$ | $N - h = (15-3) = 12$ | $M_2^U = \frac{Q_2^U}{12}$ | |

Here, $N = 15$ and $n_i = 5, 5, 5$ for the makes A, B, C respectively.

$T = 161 - 28\alpha$; $\sum_i \frac{T_i^2}{n_i} = \frac{1}{5}\left[264\alpha^2 - 3000\alpha + 8885\right]$ and

$$\sum_i \sum_j \left[d_{ij} - \alpha\left(d_{ij} - c_{ij}\right)\right]^2 = 62\alpha^2 - 596\alpha + 1829$$

$Q^U = \frac{1}{15}\left[146\alpha^2 + 76\alpha + 1514\right]$; $Q_1^U = \frac{1}{15}\left[8\alpha^2 + 16\alpha + 734\right]$ and

$Q_2^U = \frac{1}{5}\left[46\alpha^2 + 20\alpha + 260\right]$. And $M_1^U = \frac{1}{30}\left[8\alpha^2 + 16\alpha + 734\right]$; $M_2^U = \frac{1}{60}\left[46\alpha^2 + 20\alpha + 260\right]$ and $F_C^U = \left[\frac{8\alpha^2 + 16\alpha + 734}{23\alpha^2 + 10\alpha + 130}\right]$ where $0 \le \alpha \le 1$ and $F_C^U$ is the calculated value of 'F' at upper level model. And the tabulated value of 'F' at k = 5% level of significance with $(h - 1,\ N - h) = (2,\ 12)$ degrees of freedom is $F_{t(at\ 5\%)} = 3.88$. Here, $F_C^U > F_{t(at\ 5\%)}\ \forall\alpha,\ (0 \le \alpha \le 1)$, we reject the null hypothesis $H_0^U$.

$\Rightarrow$ **There is a significant difference in the durability of the 3 makes of computers at upper level of $\alpha$ - cut .**

**Conclusion 4.2.** Therefore, the null hypotheses $H_0^L$ and $H_0^U$ are rejected $\forall\alpha, (0 \le \alpha \le 1)$. We conclude in general that there is a significant difference between in the durability of the 3 makes of computers.

***Wang's centroid point and ranking method***

Wang *et al*. [33] found that the centroid formulae proposed by Cheng are incorrect and have led to some misapplications such as by Chu and Tsao. They presented the correct method for centroid formulae for a generalized fuzzy number $\widetilde{A} = (a, b, c, d; w)$ as

$$\left(\bar{x}_0,\ \bar{y}_0\right) = \left[\frac{1}{3}\left((a + b + c + d) - \left(\frac{dc - ab}{(d + c) - (a + b)}\right)\right), \left(\frac{w}{3}\right)\left(1 + \left(\frac{c - b}{(d + c) - (a + b)}\right)\right)\right] \quad \text{--- (1)}$$

And the ranking function associated with $\widetilde{A}$ is $R(\widetilde{A}) = \sqrt{\bar{x}_0^2 + \bar{y}_0^2}$ --- (2)

For a normalized tfn., we put w = 1 in equations (5.1) so we have,

$$\left(\bar{x}_0,\ \bar{y}_0\right) = \left[\frac{1}{3}\left((a + b + c + d) - \left(\frac{dc - ab}{(d + c) - (a + b)}\right)\right), \left(\frac{1}{3}\right)\left(1 + \left(\frac{c - b}{(d + c) - (a + b)}\right)\right)\right] \quad \text{--- (3)}$$

And the ranking function associated with $\widetilde{A}$ is $R(\widetilde{A}) = \sqrt{\bar{x}_0^2 + \bar{y}_0^2}$ --- (4)

Let $\widetilde{A}_i$ and $\widetilde{A}_j$ be two fuzzy numbers (i) $R(\widetilde{A}_i) > R(\widetilde{A}_j)$ then $\widetilde{A}_i > \widetilde{A}_j$ (ii) $R(\widetilde{A}_i) < R(\widetilde{A}_j)$ then $\widetilde{A}_i < \widetilde{A}_j$ (iii) $R(\widetilde{A}_i) = R(\widetilde{A}_j)$ then $\widetilde{A}_i = \widetilde{A}_j$.

***One-factor ANOVA model using Wang's ranking function***

***Example 1.*** Let us consider example-1, the ranking grades of tfns. are calculated using relations (3) and (4) which are given below:

| Package design (i) | Store (Observation j) | | |
|---|---|---|---|
| | **1** | **2** | **3** |
| **1** | 11.0090 | 16.0062 | -- |
| **2** | 14.7940 | 15.0050 | 13.0076 |
| **3** | 18.0048 | 17.2280 | 20.1941 |
| **4** | 19.2168 | 24.0036 | -- |

Here, $Q_1 = 95.5770$, $Q_2 = 31.0826$, $M_1 = 31.859$, $M_2 = 5.1804$, the calculated value of F is $F_C = 6.15$ and the tabulated value of F is $F_{t(5\%)}\ (3, 6) = 4.76$. Here, $F_C > F_{t(5\%)} \Rightarrow$ **the null hypothesis $\widetilde{H}_0$ is rejected. Therefore, the factor level fuzzy means $\widetilde{\mu}_i$ are not equal. Hence, we conclude that there is a relation between package design and sales volumes.**

***Example 2.*** Let us consider example-2, the ranking grades of tfns. are calculated using relations (3) and (4) which are given below:

| Make | Sample (Observation j) | | | | |
|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** |
| **A** | 5.7303 | 7.2359 | 8.7248 | 11.0079 | 8.3063 |
| **B** | 9.3052 | 10.0099 | 12.0072 | 12.4237 | 5.3119 |
| **C** | 6.7280 | 4.5168 | 5.7182 | 6.2266 | 3.6075 |

Here, $Q_1 = 50.8173$, $Q_2 = 53.9197$, $M_1 = 25.4086$, $M_2 = 4.4933$, the calculated value of F is $F_C = 5.6548$ and the tabulated value of F is $F_{t(5\%)}\ (2, 12) = 3.89$. Here, $F_C > F_{t(5\%)} \Rightarrow$ **the null hypothesis $\widetilde{H}_0$ is rejected. Therefore, we conclude in general that there is a significant difference between the durability of the 3 makes of computers.**

***Rezvani's ranking function of TFNs.***

The centroid of a trapezoid is considered as the balancing point of the trapezoid. Divide the trapezoid into three plane figures. These three plane figures are a triangle (APB), a rectangle (BPQC) and a triangle (CQD) respectively. Let the centroids of the three plane figures be $G_1, G_2$ and $G_3$ respectively. The incenter of these centroids $G_1, G_2$ and $G_3$ is taken as the point of reference to define the ranking of generalized trapezoidal fuzzy numbers. The reason for selecting this point as a point of reference is that each centroid point are **balancing points** of each individual plane figure and the incenter of these centroid points is much more balancing point for a generalized trapezoidal fuzzy number. Therefore, this point would be a better reference point than the centroid point of the trapezoid.

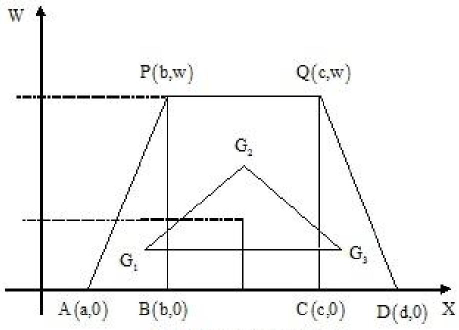

Fig.1 Centroid of centroids

Consider a generalized trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d; w)$. The centroids of the three plane figures are:

$$G_1 = \left(\frac{a+2b}{3}, \frac{w}{3}\right),\ G_2 = \left(\frac{b+c}{2}, \frac{w}{2}\right) \text{ and } G_3 = \left(\frac{2c+d}{3}, \frac{w}{3}\right) \quad \text{--- (1)}$$

Equation of the line $G_1G_3$ is $y = \dfrac{w}{3}$ and $G_2$ does not lie on the line $G_1G_3$. Therefore, $G_1, G_2$ and $G_3$ are non-collinear and they form a triangle. We define the incenter $I(\bar{x}_0, \bar{y}_0)$ of the triangle with vertices $G_1, G_2$ and $G_3$ of the generalized fuzzy number $\widetilde{A} = (a, b, c, d; w)$ as [28],

$$I_{\tilde{A}}\left(\bar{x}_0, \bar{y}_0\right) = \left[ \frac{\alpha\left(\frac{a+2b}{3}\right)+\beta\left(\frac{b+c}{2}\right)+\gamma\left(\frac{2c+d}{3}\right)}{\alpha+\beta+\gamma}, \frac{\alpha\left(\frac{w}{3}\right)+\beta\left(\frac{w}{2}\right)+\gamma\left(\frac{w}{3}\right)}{\alpha+\beta+\gamma} \right] \quad \text{--- (2)}$$

$$\text{where } \alpha = \frac{\sqrt{(c - 3b + 2d)^2 + w^2}}{6}, \beta = \frac{\sqrt{(2c + d - a - 2b)^2}}{3}, \gamma = \frac{\sqrt{(3c - 2a - b)^2 + w^2}}{6} \quad \text{--- (3)}$$

And ranking function of the trapezoidal fuzzy number $\tilde{A}=(a, b, c, d; w)$ which maps the set of all fuzzy numbers to a set of all real numbers $\left[\text{i.e. } R:[\tilde{A}] \to \mathbb{R}\right]$ is defined as $R\left(\tilde{A}\right) = \sqrt{\bar{x}_0^2 + \bar{y}_0^2}$ --- (4) which is the Euclidean distance from the incenter of the centroids. For a normalized tfn., we put w = 1 in equations (1), (2) and (3) so we have,

$$G_1 = \left(\frac{a+2b}{3}, \frac{1}{3}\right), \; G_2 = \left(\frac{b+c}{2}, \frac{1}{2}\right) \text{ and } G_3 = \left(\frac{2c+d}{3}, \frac{1}{3}\right) \quad \text{--- (5)}$$

$$I_{\tilde{A}}\left(\bar{x}_0, \bar{y}_0\right) = \left[ \frac{\alpha\left(\frac{a+2b}{3}\right)+\beta\left(\frac{b+c}{2}\right)+\gamma\left(\frac{2c+d}{3}\right)}{\alpha+\beta+\gamma}, \frac{\alpha\left(\frac{1}{3}\right)+\beta\left(\frac{1}{2}\right)+\gamma\left(\frac{1}{3}\right)}{\alpha+\beta+\gamma} \right] \quad \text{--- (6)}$$

$$\text{where } \alpha = \frac{\sqrt{(c - 3b + 2d)^2 + 1}}{6}, \beta = \frac{\sqrt{(2c + d - a - 2b)^2}}{3} \text{ and } \gamma = \frac{\sqrt{(3c - 2a - b)^2 + 1}}{6} \quad \text{--- (7)}$$

And ranking function of the trapezoidal fuzzy number $\tilde{A}=(a, b, c, d; 1)$ is defined as

$$R\left(\tilde{A}\right) = \sqrt{\bar{x}_0^2 + \bar{y}_0^2} \quad \text{--- (8)}$$

***One-factor ANOVA model using Rezvani's ranking function***

***Example 1.*** Let us consider example-1, the ranking grades of tfns. are calculated using relations (6), (7) and (8) which are given below:

| **Package design (i)** | **Store (Observation j)** | | |
|---|---|---|---|
| | **1** | **2** | **3** |
| **1** | 11.0079 | 16.0054 | -- |
| **2** | 14.5064 | 15.0058 | 13.0067 |
| **3** | 18.0048 | 17.5044 | 20.5029 |
| **4** | 19.5040 | 24.0036 | -- |

Here, $Q_1$ = 101.8644, $Q_2$ = 29.9363, $M_1$ = 33.9548, $M_2$ = 4.9894, the calculated value of F is $F_C$ = 6.8054 and the tabulated value of F is $F_{t(5\%)}$ (3, 6) = 4.76. Here, $F_C > F_{t(5\%)} \Rightarrow$ **the null hypothesis $\tilde{H}_0$ is rejected. Therefore, the factor level fuzzy means $\tilde{\mu}_i$ are not equal. Hence, we conclude that there is a relation between package design and sales volumes.**

***Example 2.*** Let us consider example-2, the ranking grades of tfns. are calculated using relations (6), (7) and (8) which are given below:

| **Make** | **Sample (Observation j)** | | | | |
|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** |
| **A** | 6.0134 | 7.5110 | 9.0086 | 11.0079 | 8.0115 |
| **B** | 9.0103 | 10.0087 | 12.0072 | 13.0049 | 5.0180 |
| **C** | 7.0113 | 4.5192 | 6.0137 | 6.5129 | 3.0305 |

Here, $Q_1 = 49.8492$, $Q_2 = 63.0462$, $M_1 = 24.9246$, $M_2 = 5.2538$, the calculated value of F is $F_C = 4.7441$ and the tabulated value of F is $F_{t(5\%)}$ (2, 12) = 3.89. Here, $F_C > F_{t(5\%)} \Rightarrow$ **the null hypothesis $\widetilde{H}_0$ is rejected. Therefore, we conclude that there is a significant difference between the durability of the 3 makes of computers.**

***Thorani's centroid point and ranking method***

As per the description in Salim Rezvani's ranking method, Y. L. P. Thorani *et al*. [29] presented a different kind of centroid point and ranking function of tfns. The incenter $I_{\widetilde{A}}\left(\overline{x}_0,\ \overline{y}_0\right)$ of the triangle [Fig. 1] with vertices $G_1, G_2$ and $G_3$ of the generalized tfn. $\widetilde{A} = (a, b, c, d; w)$ is given by,

$$I_{\widetilde{A}}\left(\overline{x}_0,\ \overline{y}_0\right) = \left[\frac{\alpha\left(\frac{a+2b}{3}\right)+\beta\left(\frac{b+c}{2}\right)+\gamma\left(\frac{2c+d}{3}\right)}{\alpha+\beta+\gamma},\ \frac{\alpha\left(\frac{w}{3}\right)+\beta\left(\frac{w}{2}\right)+\gamma\left(\frac{w}{3}\right)}{\alpha+\beta+\gamma}\right] \quad \text{--- (1)}$$

$$\text{where } \alpha = \frac{\sqrt{(c - 3b + 2d)^2 + w^2}}{6}, \beta = \frac{\sqrt{(2c + d - a - 2b)^2}}{3}, \gamma = \frac{\sqrt{(3c - 2a - b)^2 + w^2}}{6} \quad \text{--- (2)}$$

And the ranking function of the generalized tfn. $\widetilde{A} = (a, b, c, d; w)$ which maps the set of all fuzzy numbers to a set of real numbers is defined as $R\left(\widetilde{A}\right) = x_0 \times y_0$ --- (3). For a normalized tfn., we put w = 1 in equations (1) and (2) so we have,

$$I_{\widetilde{A}}\left(\overline{x}_0,\ \overline{y}_0\right) = \left[\frac{\alpha\left(\frac{a+2b}{3}\right)+\beta\left(\frac{b+c}{2}\right)+\gamma\left(\frac{2c+d}{3}\right)}{\alpha+\beta+\gamma},\ \frac{\alpha\left(\frac{1}{3}\right)+\beta\left(\frac{1}{2}\right)+\gamma\left(\frac{1}{3}\right)}{\alpha+\beta+\gamma}\right] \quad \text{--- (4)}$$

$$\text{where } \alpha = \frac{\sqrt{(c - 3b + 2d)^2 + 1}}{6}, \beta = \frac{\sqrt{(2c + d - a - 2b)^2}}{3} \text{ and } \gamma = \frac{\sqrt{(3c - 2a - b)^2 + 1}}{6} \quad \text{--- (5)}$$

And for $\widetilde{A} = (a, b, c, d; 1)$, $R\left(\widetilde{A}\right) = x_0 \times y_0$ --- (6)

***One-factor ANOVA model using Thorani's ranking function***

***Example 1*** Let us consider example-1, the ranking grades of tfns. are calculated using relations (4), (5) and (6) which are given below:

| **Package design (i)** | **Store (Observation j)** | | |
|---|---|---|---|
| | **1** | **2** | **3** |
| **1** | 4.5798 | 6.6615 | -- |
| **2** | 6.0403 | 6.2484 | 5.4125 |
| **3** | 7.4963 | 7.2889 | 8.5344 |
| **4** | 8.1227 | 9.9950 | -- |

Here, $Q_1 = 17.6894$, $Q_2 = 5.1889$, $M_1 = 5.8965$, $M_2 = 0.8648$, the calculated value of F is $F_C = 6.8183$ and the tabulated value of F is $F_{t(5\%)}$ (3, 6) = 4.76. Here, $F_C > F_{t(5\%)} \Rightarrow$ **the null hypothesis $\widetilde{H}_0$ is rejected. Therefore, the factor level fuzzy means $\widetilde{\mu}_i$ are not equal. Hence, we conclude that there is a relation between package design and sales volumes.**

***Example 2*** Let us consider example-2, the ranking grades of tfns. are calculated using relations (8.4), (8.5) and (8.6) which are given below:

| Make | Sample (Observation j) | | | | |
|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** |
| **A** | 2.4980 | 3.1237 | 3.7472 | 4.5811 | 3.3322 |
| **B** | 3.7487 | 4.1634 | 4.9975 | 5.4131 | 2.0828 |
| **C** | 2.9144 | 1.8731 | 2.4987 | 2.7075 | 1.2501 |

Here, $Q_1 = 8.6769$, $Q_2 = 10.9731$, $M_1 = 4.3385$, $M_2 = 0.9144$, the calculated value of F is $F_C = 4.7446$ and the tabulated value of F is $F_{t(5\%)}(2, 12) = 3.89$. Here, $F_C > F_{t(5\%)} \Rightarrow$ **the null hypothesis $\widetilde{H}_0$ is rejected. Therefore, we conclude that there is a significant difference between the durability of the 3 makes of computers.**

***Graded mean integration representation (GMIR)***

Let $\widetilde{A} = (a, b, c, d; w)$ be a generalized trapezoidal fuzzy number, then the **GMIR** [27] of $\widetilde{A}$ is defined by $P(\widetilde{A}) = \int_0^w h\left[\frac{L^{-1}(h) + R^{-1}(h)}{2}\right]dh \Big/ \int_0^w hdh$.

***Theorem 1.*** Let $\widetilde{A} = (a, b, c, d; 1)$ be a tfn. with normal shape function, where a, b, c, d are real numbers such that $a < b \le c < d$. Then the graded mean integration representation (GMIR) of $\widetilde{A}$ is $P(\widetilde{A}) = \frac{(a+d)}{2} + \frac{n}{2n+1}(b - a - d + c)$.

***Proof:*** For a trapezoidal fuzzy number $\widetilde{A} = (a, b, c, d; 1)_n$, we have $L(x) = \left(\frac{x - a}{b - a}\right)^n$ and $R(x) = \left(\frac{d - x}{d - c}\right)^n$ Then,

$$h = \left(\frac{x - a}{b - a}\right)^n \Rightarrow L^{-1}(h) = a + (b - a)h^{1/n};\ h = \left(\frac{d - x}{d - c}\right)^n \Rightarrow R^{-1}(h) = d - (d - c)h^{1/n}$$

$$\therefore P(\widetilde{A}) = \left(\frac{1}{2}\int_0^1 h\left[\left(a + (b - a)h^{1/n}\right) + \left(d - (d - c)h^{1/n}\right)\right]dh\right) \Big/ \int_0^1 hdh$$

$$= \left(\frac{1}{2}\left[\frac{(a+d)}{2} + \frac{n}{2n+1}(b - a - d + c)\right]\right) \Big/ \left(1/2\right)$$

Thus, $P(\widetilde{A}) = \frac{(a+d)}{2} + \frac{n}{2n+1}(b - a - d + c)$ Hence the proof.

***Result 1.*** If n =1 in the above theorem, we have $P(\widetilde{A}) = \frac{a + 2b + 2c + d}{6}$

**One-factor ANOVA model using GMIR of TFNs**

We now analyse the one-factor ANOVA model by using GMIR of each normalized trapezoidal fuzzy numbers and based on the GMIR of tfns., the decisions are observed.

***Example 1.*** Let us consider example-1, the GMIRs of tfns. are calculated using the result (1) of theorem 1 which are given below:

| Package design (i) | Store (Observation j) | | |
|---|---|---|---|
| | **1** | **2** | **3** |
| **1** | 11 | 16 | -- |
| **2** | 14.6667 | 15 | 13 |
| **3** | 18 | 17.3333 | 20.3333 |
| **4** | 19.3333 | 24 | -- |

Here, $Q_1 = 98.2845$, $Q_2 = 30.6483$, $M_1 = 32.7615$, $M_2 = 5.1080$, the calculated value of F is $F_C = 6.4138$ and the tabulated value of F is $F_{t(5\%)}(3, 6) = 4.76$. Here, $F_C > F_{t(5\%)} \Rightarrow$ **the null hypothesis $\widetilde{H}_0$ is rejected. Therefore, the factor level fuzzy means $\tilde{\mu}_i$ are not equal. Hence, we conclude that there is a relation between package design and sales volumes.**

***Example 2.*** Let us consider example-2, the GMIRs of tfns. are calculated using the result (1) of theorem 1 which are given below:

| Make | Sample (Observation j) | | | | |
|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** |
| **A** | 5.8333 | 7.3333 | 8.8333 | 11 | 8.1667 |
| **B** | 9.1667 | 10 | 12 | 12.6667 | 5.1667 |
| **C** | 6.8333 | 4.5 | 5.8333 | 6.3333 | 3.3333 |

Here, $Q_1 = 50.5454$, $Q_2 = 57.7444$, $M_1 = 25.2727$, $M_2 = 4.8120$, the calculated value of F is $F_C = 5.2520$ and the tabulated value of F is $F_{t(5\%)}(2, 12) = 3.89$. Here, $F_C > F_{t(5\%)} \Rightarrow$ **the null hypothesis $\widetilde{H}_0$ is rejected. Therefore, we conclude that there is a significant difference between the durability of the 3 makes of computers.**

***One-factor ANOVA model using total integral value (TIV) of TFNs.***

The TIV for a normalized tfn. $\widetilde{A} = (a, b, c, d; 1)$ is calculated by the relation [18]

$$\int_{Supp(\widetilde{A})} \mu_{\widetilde{A}} x dx = \int_a^b \left(\frac{x - a}{b - a}\right) dx + \int_b^c dx + \int_c^d \left(\frac{x - d}{c - d}\right) dx ---(1)$$

***Example1.*** Let us consider example-1,the TIV for the first member is calculated as follows

$$\int_{Supp(\widetilde{A}_i)} \mu_{\widetilde{A}_i}(x) dx = \int_9^{10} \left(\frac{x - 9}{1}\right) dx + \int_{10}^{12} dx + \int_{12}^{13} \left(\frac{x - 13}{-1}\right) dx = 3 = I$$

Similarly we can calculate the TIV of all other entries using $\int_{Supp(\widetilde{A}_i)} \mu_{\widetilde{A}_i}(x) dx = I$ for the given tfns. which have been tabulated below:

| Package design (i) | Store (Observation j) | | |
|---|---|---|---|
| | **1** | **2** | **3** |
| **1** | 3 | 3 | -- |
| **2** | 5.5 | 6 | 3 |
| **3** | 4 | 4.5 | 3.5 |
| **4** | 5.5 | 4 | -- |

Here, $Q_1 = 4.8083$, $Q_2 = 6.7917$, $M_1 = 1.6028$, $M_2 = 1.1319$, the calculated value of F is $F_C = 1.4160$ and the tabulated value of F is $F_{t(5\%)}(3, 6) = 4.76$. Here, $F_C < F_{t(5\%)} \Rightarrow$ **the null hypothesis $\widetilde{H}_0$ is accepted. Therefore, the factor level fuzzy means $\tilde{\mu}_i$ are equal. Hence, we conclude that the package design and sales volumes are independent.**

***Example2*** Let us consider example-2, the TIV for the the given tfns. are tabulated below:

| Make | Sample (Observation j) | | | | |
|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** |
| **A** | 3.5 | 4.5 | 3.5 | 4 | 4.5 |
| **B** | 4.5 | 3 | 4 | 4 | 4.5 |
| **C** | 3.5 | 3 | 4.5 | 5.5 | 4 |

Here, $Q_1 = 0.0333$, $Q_2 = 6.2$, $M_1 = 0.0167$, $M_2 = 0.5167$, the calculated value of F is $F_C = 30.9401$ and the tabulated value of F is $F_{t(5\%)}(12, 2) = 19.41$. Here, $F_C > F_{t(5\%)} \Rightarrow$ **the null hypothesis $\widetilde{H}_0$ is rejected. Therefore, we conclude that there is a significant difference between the durability of the 3 makes of computers.**

***Liou and Wang's centroid point method***

Liou and Wang [20] ranked fuzzy numbers with total integral value. For a fuzzy number defined by definition (2.3), the total integral value is defined as

$$I_T^{\alpha}\left(\widetilde{A}\right)=\alpha I_R\left(\widetilde{A}\right)+(1-\alpha)I_L\left(\widetilde{A}\right)---(1)$$

$I_R\left(\widetilde{A}\right)=\int_{Supp(\widetilde{A})} R_{\widetilde{A}}(x)dx=\int_c^d\left(\frac{x-d}{c-d}\right)dx---(2)$ & $I_L\left(\widetilde{A}\right)=\int_{Supp(\widetilde{A})} L_{\widetilde{A}}(x)dx=\int_a^b\left(\frac{x-a}{b-a}\right)dx---(3)$ are the ***right*** and ***left integral*** values of $\widetilde{A}$ respectively and $0\le\alpha\le1$.

(i) $\alpha\in[0,1]$ is the ***index of optimism*** which represents the ***degree of optimism*** of a decision maker. (ii) If $\alpha=0$, then the total value of integral represents a **pessimistic decision maker's view point** which is equal to left integral value. (iii) If $\alpha=1$, then the total integral value represents an **optimistic decision maker's view point** and is equal to the right integral value. (iv) If $\alpha=0.5$ then the total integral value represents a **moderate decision maker's view point** and is equal to the mean of right and left integral values. For a decision maker, the larger the value of $\alpha$ is, the higher is the degree of optimism.

***One-factor ANOVA model using Liou and Wang's centroid point method***

***Example 1*** Let us consider example-1**,** using the above equations (1), (2) and (3), we get the centroid point of first member as follows:

$$I_L\left(\widetilde{A}\right)=\int_9^{10}\left(\frac{x-9}{1}\right)dx=1/2;\quad I_R\left(\widetilde{A}\right)=\int_{12}^{13}\left(\frac{x-13}{-1}\right)dx=1/2 \text{ Therefore } I_T^{\alpha}\left(\widetilde{A}\right)=1/2.$$

Similarly we can find the centroid point for all other members and the calculated values are tabulated below:

| Package design (i) | Store (Observation j) | | |
|---|---|---|---|
| | **1** | **2** | **3** |
| **1** | 1/2 | 1/2 | -- |
| **2** | $(2+\alpha)/2$ | 2 | 1/2 |
| **3** | 1 | $(2-\alpha)/2$ | $(3-\alpha)/2$ |
| **4** | $(3-\alpha)/2$ | 1 | -- |

**The ANOVA table values using Liou and Wang's centroid point of tfns.**

Here, $Q_1=(53\alpha^2-38\alpha+92)/120$, $Q_2=(11\alpha^2-14\alpha+35)/24$, $M_1=(53\alpha^2-38\alpha+92)/360$, $M_2=(11\alpha^2-14\alpha+35)/144$, the calculated value of F is $F_C=2(53\alpha^2-38\alpha+92)/5(11\alpha^2-14\alpha+35)$ and the tabulated value of F is $F_{t(5\%)}$ (3, 6) = 4.76. Here, $F_C<F_{t(5\%)}\ \forall\alpha,\ \alpha\in[0,1]\Rightarrow$**the null hypothesis $\widetilde{H}_0$ is accepted. Therefore, the factor level fuzzy means $\widetilde{\mu}_i$ are equal. Hence, we conclude that the package design and sales volumes are independent.**

***Example 2*** Let us consider example-2**,** using the above equations (1), (2) and (3), we get the centroid points of tfns. as follows:

| Make | Sample (Observation j) | | | | |
|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** |
| **A** | $(2-\alpha)/2$ | $(2-\alpha)/2$ | $(2-\alpha)/2$ | 1 | $(2+\alpha)/2$ |
| **B** | $(2+\alpha)/2$ | 1/2 | 1 | $(3-2\alpha)/2$ | $(2+\alpha)/2$ |
| **C** | $(2-\alpha)/2$ | 1 | $(3-\alpha)/2$ | $(3-\alpha)/2$ | $(1+2\alpha)/2$ |

Here, $Q_1=(3\alpha^2+1)/30$, $Q_2=(40\alpha^2-29\alpha+12)/10$, $M_1=(3\alpha^2+1)/60$, $M_2=(40\alpha^2-29\alpha+12)/120$, the calculated value of F is $F_C=(40\alpha^2-29\alpha+12)/2(3\alpha^2+1)$ and the tabulated value of F is $F_{t(5\%)}$ (12, 2) = 19.41. Here, $F_C<F_{t(5\%)}\ \forall\alpha,\ \alpha\in[0,1]\Rightarrow$**the null hypothesis $\widetilde{H}_0$ is accepted. Therefore, we conclude that the difference between the durability of the 3 makes of computers is not significant.**

## CONCLUSION

The decisions obtained from various methods are tabulated below for the null hypothesis.

| Acceptance of null hypotheses $\widetilde{H}_0$ | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **~ cut method** | | | | **Wang** | | **Rezvani** | | **GMIR** | | **TIV** | | **L & W** | | **Thorani** | |
| Eg.1 | | Eg.2 | | Eg.1 | Eg.2 | Eg.1 | Eg.2 | Eg.1 | Eg.2 | Eg.1 | Eg.2 | Eg.1 | Eg.2 | Eg.1 | Eg.2 |
| L | U | L | U | | | | | | | | | | | | |
| × | × | × | × | × | × | × | × | × | × | ✓ | × | ✓ | ✓ | × | × |

Observing the decisions obtained from α-cut interval method, for example 1 and 2, the null hypothesis is rejected at both lower level and upper level model. Also, the one-factor ANOVA model using ranking grades of tfns. obtained from Wang's method, Rezvani's method, GMIR of tfns., Thorani's method provide a parallel discussion but Liou& Wang's method, TIV of tfns. do not conclude reliable decision as they accept the null hypothesis $\widetilde{H}_0$ while other methods are rejecting $\widetilde{H}_0$.

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