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Reenu Sharma

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A CLASS OF BÉZIER-TYPE CUBIC TRIGONOMETRIC CURVES AND SURFACES WITH SHAPE PARAMETER

Reenu Sharma

Department of Mathematics, Mata Gujri Mahila Mahavidyalaya, Jabalpur, M.P., India

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ABSTRACT

Shape modelling by Bézier-type cubic trigonometric curves with a shape parameter over the space $\Omega = \text{span}\{1, \sin t, \cos t, \sin^2 t, \cos^2 t, \sin^3 t, \cos^3 t\}$ is studied in this paper and the corresponding cubic trigonometric Bézier surfaces are defined. These curves not only inherit most properties of the usual cubic Bézier curves with the Bernstein basis in the polynomial space, but also enjoy some other advantageous properties for shape modelling. The shape parameter provides freedom in terms of design and shape control of the curve. Thus we can construct smooth curves of almost any shape. The shape of the curve can be adjusted by altering the values of shape parameter while the control polygon is kept unchanged. These curves can be used as an efficient model for geometric design in the fields of CAGD.

INTRODUCTION

In the field of Computer Aided Geometric Design (CAGD) and computer graphics (CG), the cubic Bézier curves and surfaces have been widely used because they are easy to compute and very stable. However, once the control points of usual cubic Bézier curves with the Bernstein basis are specified, their shapes cannot be changed. Also, usual cubic Bézier curves cannot represent exactly the transcendental curves. With the development of geometric design industry, designers need a curve representation that is directly related to the control points and is flexible enough to bend, twist or change the shapes of curves and surfaces by changing one or more control points. These shortcomings of Bézier curves restrict their practical applications in geometric modelling. To overcome these shortcomings, many bases (Schoenberg *et al*, 1964, Zhang *et al*, 1996, Zhang *et al*, 2005, Mainar *et al*, 2001, Lü *et al*, 2002, Wang *et al*, 2004) are presented using trigonometric functions or the blending of polynomial and trigonometric functions.

Some of these existing methods have no shape parameters; hence the shape of the curves or surfaces cannot be modified when once their control points are determined. Many authors have studied different kinds of spline for curve and surface with shape parameters through incorporating some parameters into the classical basis functions, where the parameters can adjust the shape of the curves and surfaces without changing the control points (Xuli Han *et al*, 2006, Xumin Liu *et al*, 2008, Imre Juhász *et al*, 2009). Recently, many papers investigate the trigonometric Bézier-like polynomial, trigonometric spline and their applications.

A quadratic trigonometric Bézier curve with shape parameter is constructed which is G^1 continuous (Wei Xiang Xu *et al*, 2011). The generalized basis functions of degree $n+1$ with two shape parameters are presented by L.L. Yan *et al*, 2008. The cubic trigonometric polynomial spline curve of G^1 continuity is constructed by Xiaoqin Wu *et al* 2007, which can be G^3 continuity under special condition. Xiaoqin Wu *et al*, 2008 constructed the cubic trigonometric polynomial curve similar to the cubic Bézier curves. The shape features of the cubic trigonometric polynomial curves with a shape parameter are analyzed by *Xi-An Han et al*, 2010. An extension of the Bézier model is studied by L.Yan *et al*, 2011. Dube *et al*, 2013 presented the quartic trigonometric Bézier curve with shape parameter and analysed the effect of shape parameter. A new rational cubic trigonometric Bézier curve with four shape parameters are defined by Beibei Wu *et al*, 2015. Juncheng Li

Corresponding author:* ***Reenu Sharma*
Department of Mathematics, Mata Gujri Mahila Mahavidyalaya, Jabalpur, M.P., India

et al, 2014 presented another novel extension of the cubic Bézier curve and surface with shape parameters. Simple quartic trigonometric polynomial blending functions, with a tension parameter, are presented by Lamnii *et al*, 2015. The main purpose of this work is to present a Bézier-type cubic trigonometric curve and surface with a shape parameter. The shape of the proposed curve and surface can be adjusted by altering values of the shape parameter when the control points are fixed. These proposed curves have the same geometric properties as the usual cubic Bézier models.

The rest of this paper is organized as follows. In Section 2, the cubic trigonometric Bézier basis with a shape parameter is constructed, and some properties of the basis are given. In Section 3, the corresponding curve with a shape parameter is defined, the properties of the Bézier-type cubic trigonometric curve are discussed and effects of the shape parameter on cubic trigonometric Bézier curve are studied. In Section 4, the corresponding surface with a shape parameter is presented. Conclusion is given in Section 5.

The Bernstein-Type Cubic Trigonometric Basis Functions

Definition 1 For an arbitrarily selected real value of λ, where $\lambda \in [-9, 0]$, the following four functions of t ($t \in [0,\ 1]$) are defined as Bernstein-type cubic trigonometric basis functions with a shape parameter λ:

$$\begin{aligned}
\mathrm{B}_{0,3}(t) &= \left(1-\sin\tfrac{\pi}{2}\mathrm{t}\right)^3 - \lambda\left(1-\cos\tfrac{\pi}{2}\mathrm{t}\right)\left(1-\sin\tfrac{\pi}{2}\mathrm{t}\right)^2,\\
\mathrm{B}_{1,3}(t) &= \sin\tfrac{\pi}{2}\mathrm{t}\,(3-4\sin\tfrac{\pi}{2}\mathrm{t}+\sin^2\tfrac{\pi}{2}\mathrm{t}) + \lambda\left(1-\cos\tfrac{\pi}{2}\mathrm{t}\right)\left(1-\sin\tfrac{\pi}{2}\mathrm{t}\right)^2, \qquad (1)\\
\mathrm{B}_{2,3}(t) &= \cos\tfrac{\pi}{2}\mathrm{t}\,(3-4\cos\tfrac{\pi}{2}\mathrm{t}+\cos^2\tfrac{\pi}{2}\mathrm{t}) + \lambda\left(1-\sin\tfrac{\pi}{2}\mathrm{t}\right)\left(1-\cos\tfrac{\pi}{2}\mathrm{t}\right)^2,\\
\mathrm{B}_{3,3}(t) &= \left(1-\cos\tfrac{\pi}{2}\mathrm{t}\right)^3 - \lambda\left(1-\sin\tfrac{\pi}{2}\mathrm{t}\right)\left(1-\cos\tfrac{\pi}{2}\mathrm{t}\right)^2.
\end{aligned}$$

Simple calculation testifies that these Bernstein-type cubic trigonometric basis functions possess the properties similar to the Bernstein bases as follows.

Terminal Properties: At the endpoints, we know that

$$\begin{cases} B_{0,n}(0) = 1 \\ B_{n,n}(1) = 1 \end{cases} \quad \text{and} \quad \begin{cases} B_{i,n}^{(j)}(0) = 0 \\ B_{n-i,n}^{(j)}(1) = 0 \end{cases}$$

where $0 \le j \le i-1, i = 1,2,\dots,n, n = 3$ and $B_{i,n}^{(0)}(\mathrm{t}) = \mathrm{B}_{i,n}(\mathrm{t})$

Non-negativity: The Bernstein-type cubic trigonometric basis functions are non-negative on the
interval $[0,1]$

$$B_{i,n}(\mathrm{t}) \ge 0, \qquad i = 0,1,\dots,n, n = 3, t \in [0,1]$$

Symmetry: These basis functions are symmetric, namely,

$$B_{i,n}(t) = B_{n-i,\ n}(1-t), i = 0,1,\dots,n, n = 3, t \in [0,\ 1]$$

Partition of unity: These Bernstein-type cubic trigonometric basis functions possess weight property,

$$\sum_{i=0}^{\mathrm{n}} B_{i,n}(t) = 1; n = 3, t \in [0,1]$$

Maximum Value: The maximum value of $B_{i,n}(t)$ occurs at $= \frac{i}{n}, i = 0,1,\dots,n, n = 3.$

Figure 1 shows the curves of the Bernstein-type cubic trigonometric basis functions for $\lambda = -9,-5,-2,0$ where $\mathrm{B}_{0,3}(t)$, $\mathrm{B}_{1,3}(t)$, $\mathrm{B}_{2,3}(t)$ and $\mathrm{B}_{3,3}(t)$ are represented by blue, red, green and black lines respectively.

The Bézier -Type Cubic Trigonometric Curve

Definition 2. Given control points P_i $(\mathrm{i} = 0,1,\dots,\mathrm{n})$ in R^2 or R^3, then Bézier-type cubic trigonometric curve with a shape parameters λ is defined by

$$r(t) = \sum_{i=0}^{\mathrm{n}} \mathrm{B}_{i,n}(t)P_i \qquad (2)$$

where $\mathrm{n} = 3$, $t \in [0,1]$, $\lambda \in [-9,0]$ and $\mathrm{B}_{0,3}(t)$, $\mathrm{B}_{1,3}(t)$, $\mathrm{B}_{2,3}(t)$, $\mathrm{B}_{3,3}(t)$ are the Bernstein-type cubic trigonometric basis functions.

The Bézier-type cubic trigonometric curves (2) have the following properties:

Terminal Properties

$$r(0) = P_0, \qquad r(1) = P_3,$$
$$r'(0) = 3(P_1 - P_0), \qquad r'(1) = 3(P_3 - P_2)$$

Symmetry

P_0, P_1, P_2, P_3 and P_3, P_2, P_1, P_0 define the same Bézier-type cubic trigonometric curve in different parametrizations, i.e.,
$r(t; \lambda; P_0, P_1, P_2, P_3) = r(1-t; \lambda; P_3, P_2, P_1, P_0)$;
for $t \in [0,1]$, $\lambda \in [-9,\ 0]$

Bernstein-type cubic trigonometric basis functions

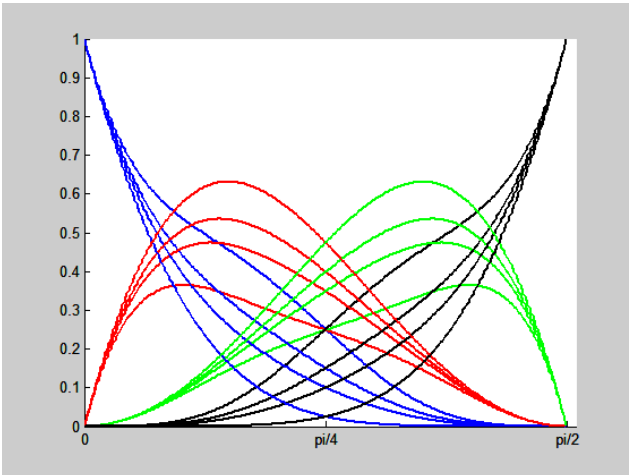

Figure 1 Bernstein-type cubic trigonometric basis functions for $\lambda = -9,-5,-2,0$

Geometric invariance

The shape of a Bézier-type cubic trigonometric curve is independent of the choice of coordinates, i.e. (2) satisfies the following two equations:

$$r(t; \lambda; P_0 + q, P_1 + q, P_2 + q, P_3 + q) = r(t; \lambda; P_0, P_1, P_2, P_3) + q$$

$r(t;\lambda;P_0 * T, P_1 * T, P_2 * T, P_3 * T) = r(t;\lambda;P_0, P_1, P_2, P_3) * T$ for $t \in [0,1], \lambda \in [-9,0]$

where q is arbitrary vector in R^2 or R^3, and T is an arbitrary $d \times d$ matrix, d = 2 or 3.

Convex hull property

Since the Bernstein-type cubic trigonometric basis functions have the properties of non-negativity and partition of unity, the entire Bézier-type cubic trigonometric curve segment must lie inside the control polygon spanned by P_0, P_1, P_2, P_3.

Shape Control of The Bézier-Type Cubic Trigonometric Curves

For $t \in [0,1]$, we rewrite (2) as follows:

$$r(t) = \sum_{i=0}^{3} P_i c_i(t) + \lambda\left(1-\sin\tfrac{\pi}{2}t\right)\left(1-\cos\tfrac{\pi}{2}t\right)\left[\left(1-\sin\tfrac{\pi}{2}t\right)(P_1-P_0) + \left(1-\cos\tfrac{\pi}{2}t\right)(P_2-P_3)\right] \quad (7)$$

where $c_0(t) = \left(1-\sin\frac{\pi}{2}t\right)^3$, $c_1(t) = \sin\frac{\pi}{2}t\left(3-4\sin\frac{\pi}{2}t+\sin^2\frac{\pi}{2}t\right)$,

$c_2(t) = \cos\frac{\pi}{2}t\left(3-4\cos\frac{\pi}{2}t+\cos^2\frac{\pi}{2}t\right)$, $c_3(t) = \left(1-\cos\frac{\pi}{2}t\right)^3$.

Obviously, shape parameter λ affects the curve on the control edges $(P_1 - P_0)$ and $(P_2 - P_3)$. As λ increases, the curve moves in the direction of edges $(P_1 - P_0)$ and $(P_2 - P_3)$ and as λ decreases, the curve moves in the opposite direction to the edges $(P_1 - P_0)$ and $(P_2 - P_3)$.

Figure 2 shows a computed example of Bézier-type cubic trigonometric curve with different values of shape parameter λ. These curves are generated by setting $\lambda = -9, -8, -6, -4, -2, 0$ (blue lines). The corresponding classical cubic Bézier curve with Bernstein basis is shown by red line. Thus the Bézier-type cubic trigonometric curve can be made closer to the control polygon than the classical cubic Bézier curve with Bernstein basis. Hence the shape parameter $\lambda \in [-9,0]$ provides an efficient tool for obtaining various smooth shapes in geometric designing.

Bézier-type cubic trigonometric curve

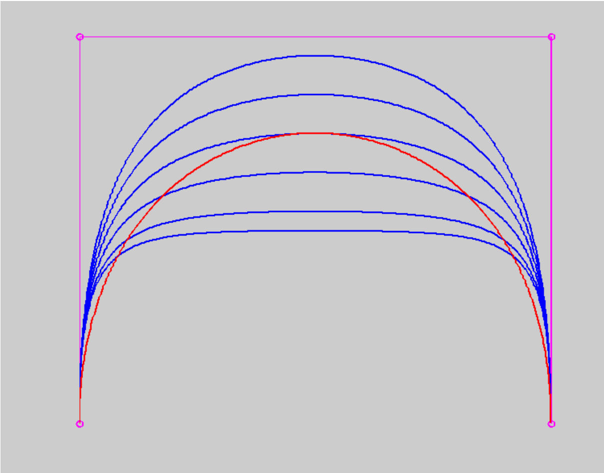

Figure 2 Bézier-type cubic trigonometric curve with different values of shape parameter λ.

Bézier -Type Cubic Trigonometric Surface (6)

Definition 2. Given control points $P_{i,j}$ $(i = 0,1,\dots,3, j = 0,1,\dots,3)$ in R^2 or R^3, then using the tensor product method, the Bézier-type cubic trigonometric surface with a shape parameter λ is defined by

$$T(u,v) = \sum_{i=0}^{3}\sum_{j=0}^{3} \mathrm{B}_{i,n}(\lambda_1,t_1)\,\mathrm{B}_{i,m}(\lambda_2,t_2)\,P_{i,j}\,; \qquad (t_1,t_2) \in [0,1]\times[0,1]$$

where $\mathrm{B}_{i,n}(\lambda_1,t_1)$ and $\mathrm{B}_{i,m}(\lambda_2,t_2)$ are Bernstein-type cubic trigonometric basis functions with shape parameter $\lambda_1 \in [-9,0]$ and $\lambda_2 \in [-9,0]$ respectively. Obviously these surfaces have properties similar to the corresponding cubic Bézier curves having Bernstein basis. In addition, since the surfaces have two shape parameters λ_1 and λ_2, the shape of the surfaces can be adjusted from two direction (in each direction using one shape parameter λ_1 and λ_2 respectively), so they can more conveniently be used in the outline design.

CONCLUSION

As mentioned above Bézier-type cubic trigonometric curve have all the properties that cubic Bézier curves have. Since these curves are closer to the control polygon than the cubic Bézier curves. Therefore, the Bézier-type cubic trigonometric curves can better preserve the shape of the control polygon. Since there is nearly no difference in structure between a Bézier-type cubic trigonometric curve and a cubic Bézier curve, it is not difficult to adapt a Bézier-type cubic trigonometric curve to a CAD/CAM system that already uses the cubic Bézier curves.

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