ISSN: 0976-3031

Available Online at http://www.recentscientific.com

International Journal of Recent Scientific Research
Vol. 7, Issue, 7, pp. 12614-12624, July, 2016

International Journal of Recent Scientific Research

Research Article

FREE CONVECTION IN A VERTICAL ANNULUR CYLINDER IN POROUS MEDIA WITH EFFECT OF VISCOUS DISSIPATION

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ARTICLE INFO

Article History:

Received 17th April, 2016
Received in revised form 12th May, 2016
Accepted 04th June, 2016
Published online 28th July, 2016

Keywords:

Free convection, Heat transfer, Viscous dissipation, Porous medium

ABSTRACT

We study the effect of viscous dissipation with the varying hot wall temperature in a vertical annular cylinder at four different locations. The effect of non-isothermal temperature and viscous dissipation on the heat transfer behavior in a saturated porous medium embedded in a vertical annular cylinder. The partial differential equations can be solved iteratively with the help of the Galerkin Finite Element Method of three nodded triangular elements Influence of aspect ratio, radius ratio, viscous dissipation parameter and Power law exponent temperature presented. The fluid flow and heat transfer is presented in terms of streamlines and isotherms.

INTRODUCTION

There is an increasing interesting in the study of natural convection in fluid-saturated porous media as proved by the explosive growth in the literature on the subject and also and increasing interest in the consideration of the viscous dissipation effects on the flow and temperature fields as well as on the heat transfer performance of the involved devices. From an order of magnitude analysis it can be concluded that the viscous dissipation can be neglected in many situations of practical interest both for domains filled with a clear fluid or for domains filled with fluid-saturated porous media. This is however, a subject that attracts many research workers and in particular special attention is being devoted to the natural convection in vertical annular cylinder filled with porous medium including the viscous dissipation effects. Natural convection in porous media has recently received considerable attention because of numerous applications in Geophysics and Energy related engineering problems. These problems arise in Geothermal Energy conservation, use of fibrous materials in the thermal insulation of buildings, Enhanced recovery of petroleum resources, Geophysical flows and Packed-bed chemical reactors.

Excellent reviews of existing theoretical and experimental results have been presented by Chen [1], Nield [2] and Bejan [3]. In all these theoretical approaches however, the viscous dissipation effects have been neglected from the governing equations. The object of this research note is therefore to present an approximate solution of the steady free convection boundary layer over a non-isothermal body of arbitrary shape embedded in a fluid-saturated porous medium when the viscous dissipation term is considered in the energy equation. Similar to Ref. [4], The Karman-pohlhausen integral relation is applied for a general solution procedure.

Nakayama and Pop [5] studied the effects of the viscous dissipation on heat transfer rates by considering the viscous dissipation term in the energy equation. It is shown that the viscous dissipation results in lowering the level of the heat transfer rate. Richardson [6] solved the problem of heat transfer of a power law fluid in laminar flow including the effect of heat generation by viscous dissipation. A similar analysis with the same effect of high Prandtl number of the fluid was performed by Basu and Roy [7]. Al-Hadhrami *et. al.*, [8] extended the analysis to cases where the Darcy-Brinkman model apply. The same qualitative results appear here too, but they also show that multiple solutions arise in general. More applications and a

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good insight into the subject are given by Nield and Bejan [9], Vafai [10], Pop and Ingham [11].

In this paper, we concentrate on the effect of non-isothermal temperature and viscous dissipation on the heat transfer behavior in a saturated porous medium embedded in a vertical annular cylinder. We study the effect of viscous dissipation with the varying hot wall temperature in a vertical annular cylinder at different locations i.e., the vertical annulus is supplied with heat at Inner wall heated at three different locations as shown in the figures. The partial differential equations can be solved iteratively with the help of a computer code. The Galerkin Finite Element Method of three nodded triangular elements is used to divide the physical domain into smaller segments, which is a pre-requisite for finite element method. Influence of aspect ratio (A_r), radius ratio (R_r), viscous dissipation parameter (ε), Power law exponent temperature (λ) and Rayleigh number on Nusselt number is presented. The fluid flow and heat transfer is presented in terms of streamlines and isotherms.

MATHEMATICAL FORMULATION

A vertical annular cylinder of inner radius r_i and outer radius r_o is considered to investigate the heat transfer behavior. The co-ordinate system is chosen such that the r-axis points towards the width and z-axis towards the height of the cylinder respectively. Because of the annular nature, two important parameters emerge which is aspect ratio, (A_r) and radius ratio (R_r) of the annulus. They are defined as

$$A_r = \frac{H}{r_o - r_i}, R_r = \frac{r_o - r_i}{r_i}$$

Where H_t is the height of the cylinder.

The inner surface of the cylinder is assumed to be power law function and it varies in the vertical direction along the height of the inner wall of the vertical annular cylinder $T_h = T_\infty + B(z^\lambda)$ and the outer surface at an ambient temperature T_∞ respectively, where λ and B are the constants responsible for temperature variations along the length of the vertical annular cylinder. The top and bottom surfaces of the vertical annular cylinder are adiabatic. It may be noted, that due to ax symmetry only half of the annulus is sufficient for analysis purpose, since other half is mirror image of the first half.

We assume that the flow inside the porous medium is assumed to obey Darcy law and there is no phase change of fluid. The porous medium is saturated with fluid, the convective fluid and the porous medium are everywhere in local thermal equilibrium in the domain. The properties of the fluid and of the porous medium are homogeneous, isotropic constant except variation of fluid density with temperature. Under these assumptions the equations governing the flow, heat transfer are given by

Continuity Equation:

$$\frac{\partial(ru)}{\partial r} + \frac{\partial(rw)}{\partial z} = 0 \tag{1}$$

The velocity in r and z direction can be described by Darcy law as velocity in horizontal direction

$$u = \frac{-K}{\mu}\frac{\partial p}{\partial r}$$

Velocity in vertical direction

$$w = \frac{-K}{\mu}\left(\frac{\partial p}{\partial z} + \rho g\right)$$

The permeability K of porous medium can be expressed as Bejan [3]

$$K = \frac{D_P^2 \phi^3}{180(1-\phi)^2}$$

Momentum Equation:

$$\frac{\partial w}{\partial r} - \frac{\partial u}{\partial z} = \frac{gK\beta}{\nu}\frac{\partial T}{\partial r} \tag{2}$$

Energy Equation

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \alpha\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right) + \frac{\mu}{K(\rho C_P)_f}(u^2 + w^2) \tag{3}$$

The continuity equation can be satisfied by introducing the stream function ψ as

$$u = \frac{1}{r}\frac{\partial \psi}{\partial z} \quad w = \frac{1}{r}\frac{\partial \psi}{\partial r} \tag{4}$$

The variation of density with respect to temperature can be described by Boussinesq approximation as

$$\rho = \rho_\infty ([1 - \beta_T(T - T_\infty)) \tag{5}$$

The corresponding boundary conditions are

When heat is supplied at three different locations at the inner wall of the Vertical Annular Cylinder.

at $r = r_i$ and $0 \le Z \le \frac{H}{6}, \frac{5H}{12} \le Z \le \frac{7H}{12}, \frac{5H}{6} \le Z \le$, $T_w = T_\infty + B(z)^\lambda$, $\psi = 0$ (6)

at $r = r_o$ $T - T_\infty$, $\psi = 0$

The new parameters arising due to cylindrical co-ordinates system are

Non-dimensional Radius $\bar{r} = \frac{r}{L}$

Non-dimensional Height $\bar{z} = \frac{z}{L}$

Non-dimensional Stream function $\bar{\psi} = \frac{\psi}{\alpha L}$ (7)

Non-dimensional Temperature

$$\bar{T} = \frac{(T - T_\infty)}{(T_w - T_\infty)}$$

Rayleigh Number

$$Ra = \frac{g\beta_T \Delta TKL}{v\alpha}$$

Viscous dissipation parameter

$$\varepsilon = \frac{\alpha\mu}{\Delta TK\rho C_P}$$

The non-dimensional equations for the heat transfer in vertical cylinder are Momentum equation:

$$\frac{\partial^2 \overline{\psi}}{\partial \overline{z}} + \overline{r}\left(\frac{1}{r}\frac{\partial \overline{\psi}}{\partial \overline{r}}\right) = \overline{r}Ra\frac{\partial \overline{T}}{\partial \overline{r}} \qquad (8)$$

Energy Equation:

$$\frac{1}{r}\left[\frac{\partial \overline{\psi}}{\partial \overline{r}}\frac{\partial \overline{T}}{\partial \overline{z}} - \frac{\partial \overline{\psi}}{\partial \overline{z}}\frac{\partial \overline{T}}{\partial \overline{r}}\right] = \left(\frac{1}{r}\frac{\partial}{\partial \overline{r}}\left(\overline{r}\frac{\partial \overline{T}}{\partial \overline{r}}\right) + \frac{\partial^2 \overline{T}}{\partial \overline{z}^2}\right) + \varepsilon\left[\left(\frac{1}{r}\frac{\partial \overline{\psi}}{\partial \overline{r}}\right)^2 + \left(\frac{1}{r}\frac{\partial \overline{\psi}}{\partial \overline{z}}\right)^2\right] \qquad (9)$$

The corresponding non-dimensional boundary conditions are when heat is supplied at three different locations at the inner wall of the Vertical Annular Cylinder

at $r = r_i$ and

$$0 \le Z \le \frac{H}{6}, \frac{5H}{12} \le Z \le \frac{7H}{12}, \frac{5H}{6} \le Z \le H, \overline{T} = T_\infty + B(z)^\lambda, \overline{\psi} = 0$$

at $r = r_o$ $\qquad \overline{T} = 0, \quad \overline{\psi} = 0$

SOLUTION OF THE GOVERNING EQUATIONS

Applying Galerkin method to Momentum equation (8) yields

$$\{R^e\} = -\int_v N^T\left[\frac{\partial^2 \overline{\psi}}{\partial \overline{z}^2} + \overline{r}\frac{\partial}{\partial \overline{r}}\left(\frac{1}{r}\frac{\partial \overline{\psi}}{\partial \overline{r}}\right) - \overline{r}Ra\frac{\partial \overline{T}}{\partial \overline{r}}\right]dv \qquad (11)$$

$$\{R^e\} = -\int_v N^T\left[\frac{\partial^2 \overline{\psi}}{\partial \overline{z}^2} + \overline{r}\frac{\partial}{\partial \overline{r}}\left(\frac{1}{r}\frac{\partial \overline{\psi}}{\partial \overline{r}}\right) - \overline{r}Ra\frac{\partial \overline{T}}{\partial \overline{r}}\right]2\pi\overline{r}dv \qquad (12)$$

where R^e is the residue. Considering the individual terms of equation (12)

The differentiation of following term results into

$$\frac{\partial}{\partial \overline{r}}\left(\left[N^T\right]\frac{\partial \overline{\psi}}{\partial \overline{r}}\right) = \left[N^T\right]\frac{\partial^2 \overline{\psi}}{\partial \overline{r}^2} + \frac{\partial [N]^T}{\partial \overline{r}}\frac{\partial \overline{\psi}}{\partial \overline{r}} \qquad (13)$$

Thus

$$\int_A N^T\frac{\partial^2 \overline{\psi}}{\partial \overline{r}^2}dA \int_A \frac{\partial}{\partial \overline{r}}\left([N]^T\frac{\partial^2 \overline{\psi}}{\partial \overline{r}^2}\right)2\pi\overline{r}dA - \int_A \frac{\partial [N]^T}{\partial \overline{r}}\frac{\partial \overline{\psi}}{\partial \overline{r}} \qquad (14)$$

The first term on right hand side of equation (14) can be transformed into surface integral by the application of Greens theorem and leads to inter-element requirement at boundaries of an element. The boundary conditions are incorporated in the force vector.

Let us consider that the variable to be determined in the triangular area is "T". The polynomial function for "T" can be expressed as

$T = \alpha_1 + \alpha_2 r + \alpha_3 r$ (15)

The Variable T has the value T_i, T_j and T_k at the nodal position i, j and k of the element. The r and z co-ordinates at these points are r_i, r_j, r_k and z_i, z_j, z_k respectively.

Since $T = N_iT_i + N_jT_j + N_kT_k$ (16)

where N_i, N_j & N_k are shape functions give by

$$N_m = \frac{a_m + b_m r + c_m z}{2A} \qquad (17)$$

Making use of (18) gives

$$\int_A N^T\frac{\partial^2 \overline{\psi}}{\partial \overline{r}}2\pi\overline{r}dA\int_A \frac{\partial N^T}{\partial \overline{r}}\frac{\partial N}{\partial \overline{r}}\begin{Bmatrix}\overline{\psi}_1 \\ \overline{\psi}_2 \\ \overline{\psi}_3\end{Bmatrix}dA \qquad (18)$$

Substitution (7) into (8) gives:

$$= -\frac{1}{(2A)^2}\begin{bmatrix}b_1 \\ b_2 \\ b_3\end{bmatrix}[b_1 b_2 b_3]\begin{Bmatrix}\overline{\psi}_1 \\ \overline{\psi}_2 \\ \overline{\psi}_3\end{Bmatrix}2\pi\overline{r}dA$$

$$= -\frac{2\pi\overline{R}}{4A}\begin{bmatrix}b_1^2 & b_1b_2 & b_1b_3 \\ b_1b_2 & b_2^2 & b_2b_3 \\ b_1b_3 & b_2b_3 & b_3^2\end{bmatrix}\begin{Bmatrix}\overline{\psi}_1 \\ \overline{\psi}_2 \\ \overline{\psi}_3\end{Bmatrix} \qquad (19)$$

Similarly

$$\int_A N^T\frac{\partial^2 \overline{\psi}}{\partial \overline{z}}2\pi\overline{r}dA = -\frac{2\pi\overline{R}}{4A}\begin{bmatrix}c_1^2 & c_1c_2 & c_1c_3 \\ c_1c_2 & c_2^2 & c_2c_3 \\ c_1c_3 & c_2c_3 & c_3^2\end{bmatrix}\begin{Bmatrix}\overline{\psi}_1 \\ \overline{\psi}_2 \\ \overline{\psi}_3\end{Bmatrix} \qquad (20)$$

The third term of equation (12) is

$$\int_A N^T\overline{r}Ra\frac{\partial \overline{T}}{\partial \overline{r}}2\pi\overline{r}dA = Ra\int_A N^T\overline{r}\frac{\partial \overline{T}}{\partial \overline{r}}2\pi\overline{r}dA \qquad (21)$$

since M_1, $M_2 = N_2$, $M_3 = N_3$

Where M_1, M_2 and M_3 are the area ratios of the triangle and N_1, N_2 and N_3 are the shape functions.

Replacing the shape functions in the above equation (21) gives

$$\int_A N^T\overline{r}Ra\frac{\partial \overline{T}}{\partial \overline{r}}2\pi\overline{r}dA = \overline{r}Ra\int_A \begin{bmatrix}M_1 \\ M_2 \\ M_3\end{bmatrix}\frac{\partial [N]}{\partial \overline{r}}\begin{bmatrix}\overline{T}_1 \\ \overline{T}_2 \\ \overline{T}_3\end{bmatrix}2\pi\overline{r}dA \qquad (22)$$

$$Ra = \frac{A}{3}\begin{bmatrix}1 \\ 1 \\ 1\end{bmatrix}\frac{2\pi R^2}{2A}\left[b_1 + b_2 + b_3\right]\begin{bmatrix}\overline{T}_1 \\ \overline{T}_2 \\ \overline{T}_3\end{bmatrix}$$

$$= \frac{2\pi\overline{R}^2 Ra}{6}\begin{Bmatrix}b_1\overline{T}_1 + b_2\overline{T}_2 + b_3\overline{T}_3 \\ b_1\overline{T}_1 + b_2\overline{T}_2 + b_3\overline{T}_3 \\ b_1\overline{T}_1 + b_2\overline{T}_2 + b_3\overline{T}_3\end{Bmatrix}$$

Now the Momentum equation leads to

$$\frac{2\pi\overline{R}}{4A}\left[\begin{bmatrix} b_1^2 & b_1b_2 & b_1b_3 \\ b_1b_2 & b_2^2 & b_2b_3 \\ b_1b_3 & b_2b_3 & b_3^2 \end{bmatrix}+\begin{bmatrix} c_1^2 & c_1c_2 & c_1c_3 \\ c_1c_2 & c_2^2 & c_2c_3 \\ c_1c_3 & c_2c_3 & c_3^2 \end{bmatrix}\begin{Bmatrix}\overline{\psi}_1 \\ \overline{\psi}_2 \\ \overline{\psi}_3\end{Bmatrix}\right]$$

$$=\frac{2\pi\overline{R}^2 Ra}{6}\begin{Bmatrix} b_1\overline{T}_1+b_2\overline{T}_2+b_3\overline{T}_3 \\ b_1\overline{T}_1+b_2\overline{T}_2+b_3\overline{T}_3 \\ b_1\overline{T}_1+b_2\overline{T}_2+b_3\overline{T}_3 \end{Bmatrix} \tag{23}$$

This is in the form of the stiffness matrix

$[K_s]\{\overline{\psi}\}$ = {f}

Similarly application of Galerkin method to Energy equation gives

$$[R^e]=-\int_A N^T\left[\frac{1}{r}\left[\frac{\partial\overline{\psi}}{\partial\overline{r}}\frac{\partial\overline{T}}{\partial\overline{z}}-\frac{\partial\overline{\psi}}{\partial\overline{z}}\frac{\partial\overline{T}}{\partial\overline{r}}\right]-\left(\frac{1}{r}\frac{\partial}{\partial\overline{r}}\left(\overline{r}\frac{\partial\overline{T}}{\partial\overline{r}}\right)+\frac{\partial^2\overline{T}}{\partial\overline{z}^2}\right)-\varepsilon\left[\left(\frac{1}{r}\frac{\partial\overline{\psi}}{\partial\overline{r}}\right)^2+\left(\frac{1}{r}\frac{\partial\overline{\psi}}{\partial\overline{z}}\right)^2\right]\right]2\pi\overline{e}dA \tag{24}$$

Considering the terms individually of the above equation (24)

$$\int_A N^T\frac{\partial\overline{\psi}}{\partial\overline{z}}\frac{\partial\overline{T}}{\partial\overline{r}}2\pi dA=\int_A\begin{bmatrix}M_1\\M_2\\M_3\end{bmatrix}\frac{\partial[N]}{\partial\overline{z}}\{\overline{\psi}\}\frac{\partial[N]}{\partial\overline{r}}\{\overline{T}\}2\pi dA \tag{25}$$

$$=\frac{2\pi A}{3}\times\frac{1}{4A^2}[c_1\overline{\psi}_1+c_2\overline{\psi}_2+c_3\overline{\psi}_3][b_1b_2b_3]\begin{Bmatrix}\overline{T}_1\\\overline{T}_2\\\overline{T}_3\end{Bmatrix} \tag{26}$$

$$=\frac{2\pi}{12A}\begin{bmatrix} c_1\overline{\psi}_1+ & c_2\overline{\psi}_2+ & c_3\overline{\psi}_3 \\ c_1\overline{\psi}_1+ & c_2\overline{\psi}_2 & c_3\overline{\psi}_3 \\ c_1\overline{\psi}_1 & c_2\overline{\psi}_2 & c_3\overline{\psi}_3 \end{bmatrix}[b_1b_2b_3]\begin{Bmatrix}\overline{T}_1\\\overline{T}_2\\\overline{T}_3\end{Bmatrix} \tag{27}$$

$$\int_A N^T\frac{\partial\overline{\psi}}{\partial\overline{r}}\frac{\partial\overline{T}}{\partial\overline{z}}2\pi dA=\int_A\begin{bmatrix}M_1\\M_2\\M_3\end{bmatrix}\frac{\partial[N]}{\partial\overline{r}}\{\overline{\psi}\}\frac{\partial[N]}{\partial\overline{z}}\{\overline{T}\}2\pi dA$$

$$\int_A N^T\frac{\partial\overline{\psi}}{\partial\overline{r}}\frac{\partial\overline{T}}{\partial\overline{z}}2\pi dA=\frac{2\pi}{12A}\begin{bmatrix} b_1\overline{\psi}_1+ & b_2\overline{\psi}_2+ & b_3\overline{\psi}_3 \\ b_1\overline{\psi}_1+ & b_2\overline{\psi}_2 & b_3\overline{\psi}_3 \\ b_1\overline{\psi}_1 & b_2\overline{\psi}_2 & b_3\overline{\psi}_3 \end{bmatrix}[c_1c_2c_3]\begin{Bmatrix}\overline{T}_1\\\overline{T}_2\\\overline{T}_3\end{Bmatrix}$$

The remaining terms of Energy equation can be evaluated in similar fashion of equation (24)

$$\int_A N^T\frac{1}{r}\frac{\partial}{\partial\overline{r}}\left(\overline{r}\frac{\partial\overline{T}}{\partial\overline{r}}\right)2\pi\overline{r}dA=-\frac{2\pi\overline{R}}{4A}\begin{bmatrix} b_1^2 & b_1b_2 & b_1b_3 \\ b_1b_2 & b_2^2 & b_2b_3 \\ b_1b_3 & b_2b_3 & b_3^2 \end{bmatrix}\begin{Bmatrix}\overline{T}_1\\\overline{T}_2\\\overline{T}_3\end{Bmatrix}$$

$$\int_A N^T\frac{\partial^2\overline{T}}{\partial\overline{z}}2\pi\overline{r}dA=-\frac{2\pi\overline{R}}{4A}\begin{bmatrix} c_1^2 & c_1c_2 & c_1c_3 \\ c_1c_2 & c_2^2 & c_2c_3 \\ c_1c_3 & c_2c_3 & c_3^2 \end{bmatrix}\begin{Bmatrix}\overline{T}_1\\\overline{T}_2\\\overline{T}_3\end{Bmatrix}$$

$$\int_A N^T\varepsilon\left(\frac{\partial\overline{\psi}}{\partial\overline{r}}\right)^2dA=\varepsilon\int_A\begin{bmatrix}M_1\\M_2\\M_3\end{bmatrix}\left(\left(\frac{\partial N}{\partial\overline{x}}\right)\{\overline{\psi}\}\right)^2dA$$

$$=\frac{2\pi A\varepsilon}{12\overline{r}}\begin{bmatrix}1\\1\\1\end{bmatrix}[b_1\overline{\psi}_1+b_2\overline{\psi}_2+b_3\overline{\psi}_3]^2$$

$$\int_A N^T\varepsilon\left(\frac{\partial\overline{\psi}}{\partial\overline{z}}\right)=\frac{2\pi A\varepsilon}{12\overline{r}}\begin{bmatrix}1\\1\\1\end{bmatrix}[c_1\overline{\psi}_1+c_2\overline{\psi}_2+c_3\overline{\psi}_3]^2$$

Thus the Stiffness matrix of Energy equation is given by

$$\left[\frac{2\pi}{12A}\begin{Bmatrix} c_1\overline{\psi}_1+ & c_2\overline{\psi}_2+ & c_3\overline{\psi}_3 \\ c_1\overline{\psi}_1+ & c_2\overline{\psi}_2 & c_3\overline{\psi}_3 \\ c_1\overline{\psi}_1 & c_2\overline{\psi}_2 & c_3\overline{\psi}_3 \end{Bmatrix}[b_1b_2b_3]-\frac{2\pi}{12A}\begin{Bmatrix} b_1\overline{\psi}_1+ & b_2\overline{\psi}_2+ & b_3\overline{\psi}_3 \\ b_1\overline{\psi}_1+ & b_2\overline{\psi}_2 & b_3\overline{\psi}_3 \\ b_1\overline{\psi}_1 & b_2\overline{\psi}_2 & b_3\overline{\psi}_3 \end{Bmatrix}[c_1c_2c_3]\begin{Bmatrix}\overline{T}_1\\\overline{T}_2\\\overline{T}_3\end{Bmatrix}\right]$$

$$+\frac{2\pi\overline{R}}{4A}\left\{\begin{bmatrix} b_1^2 & b_1b_2 & b_1b_3 \\ b_1b_2 & b_2^2 & b_2b_3 \\ b_1b_3 & b_2b_3 & b_3^2 \end{bmatrix}\begin{Bmatrix}\overline{T}_1\\\overline{T}_2\\\overline{T}_3\end{Bmatrix}+\begin{bmatrix} c_1^2 & c_1c_2 & c_1c_3 \\ c_1c_2 & c_2^2 & c_2c_3 \\ c_1c_3 & c_2c_3 & c_3^2 \end{bmatrix}\begin{Bmatrix}\overline{T}_1\\\overline{T}_2\\\overline{T}_3\end{Bmatrix}\right\}$$

$$+\frac{2\pi A\varepsilon}{12\overline{r}}\left[\begin{bmatrix}1\\1\\1\end{bmatrix}[b_1\overline{\psi}_1+b_2\overline{\psi}_2+b_3\overline{\psi}_3]^2+\begin{bmatrix}1\\1\\1\end{bmatrix}[c_1\overline{\psi}_1+c_2\overline{\psi}_2+c_3\overline{\psi}_3]^2\right]=0 \tag{28}$$

RESULTS AND DISCUSSIONS

Results are obtained in terms of Nusselt number at hot wall for various parameters such as aspect ration (A_r), radius ration (R_r) and Rayleigh number (Ra), when heat is supplied to the vertical annular cylinder for four different cases i.e., when heat is supplied to the vertical annulus at Inner wall heated at three different portions. The average Nusselt number $(\overline{N}u)$, when heat is supplied at the three different locations of the inner wall of the vertical annular cylinder is given as

$$\overline{N}u=-\frac{1}{L}\int_0^L\left(\frac{\partial\overline{T}}{\partial\overline{r}}\right)_{\overline{r}=r_i,r_0}$$

since L = L_1 + L_2 + L_3, where L_1, L_2 and L_3 are the length of the heated wall portions and L is the total length of the heated wall.

DISCUSSION

Figure 1 shows the evolution of streamlines and isothermal lines inside the porous medium for various values of Aspect ration (A_r). It is clear from the streamlines and isothermal lines that the thermal boundary layer thickness decreases as the Aspect ratio (A_r) increases. The magnitude of the streamlines increases as A_r increases and tends to move towards the cold wall of the vertical annular cylinder. At low Aspect ratio i.e., Fig. 1a the streamlines occupy the whole domain of the vertical annular cylinder. By observing the Fig. 1c we came to know that the streamlines doesn't occupy the whole domain. This is due to the reason that more convection takes place at the upper portion of the vertical annular cylinder.

Figure 2 shows the streamlines and isothermal lines distribution inside the porous medium for various values of Radius ratio (R_r). It can be seen that the thermal boundary layer thickness decreases at Radius ratio (R_r) increases. The streamlines and isothermal lines move away from the cold wall and reach nearer to hot wall as Radius ratio (R_r) increases. The streamlines tend to occupy the whole domain of the vertical annular cylinder at low values of Radius ratio (R_r).

When heat is supplied at the three different locations of the inner wall of the Vertical Annulur Cylinder

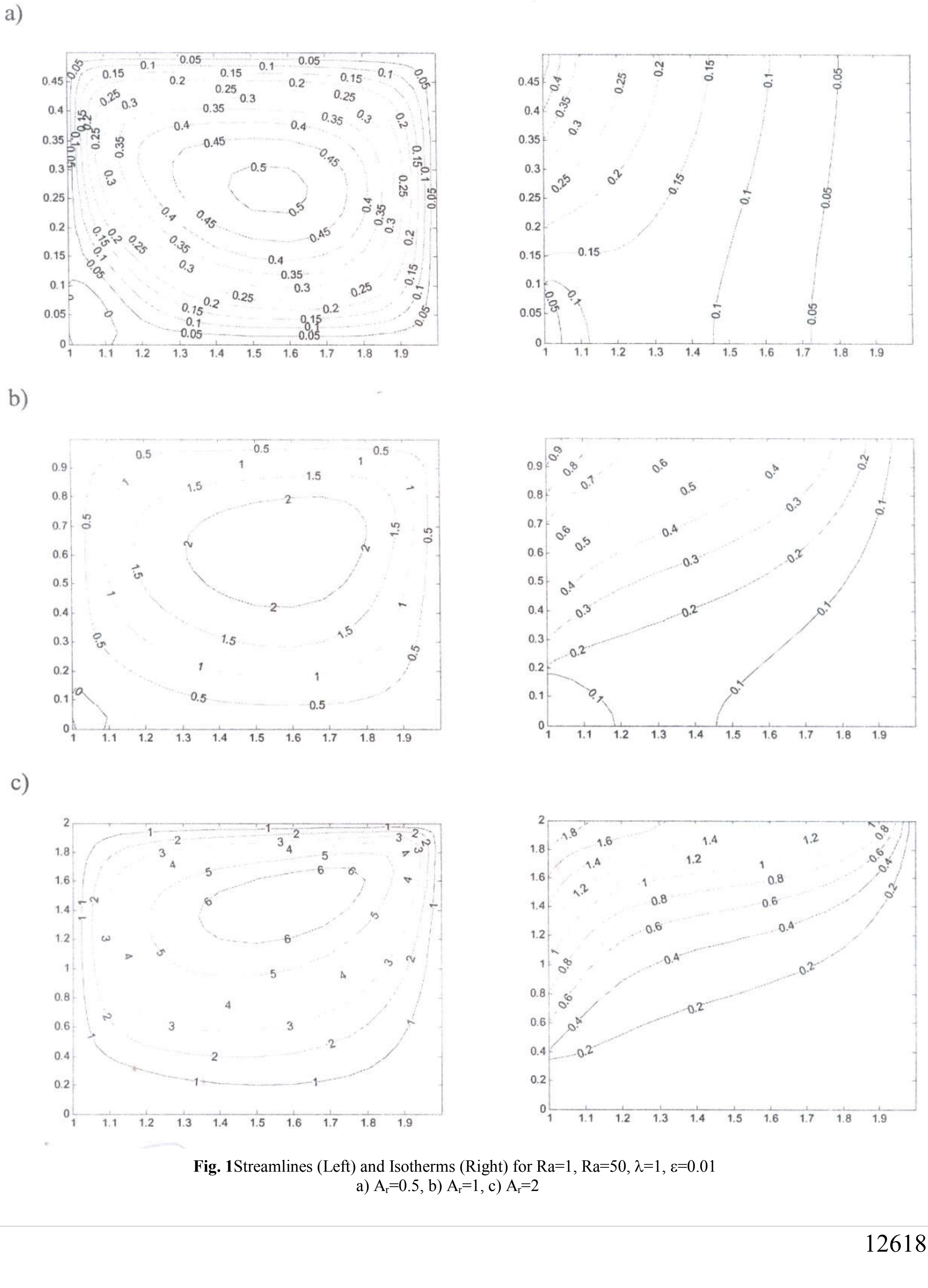

Fig. 1 Streamlines (Left) and Isotherms (Right) for Ra=1, Ra=50, λ=1, ε=0.01
a) A_r=0.5, b) A_r=1, c) A_r=2

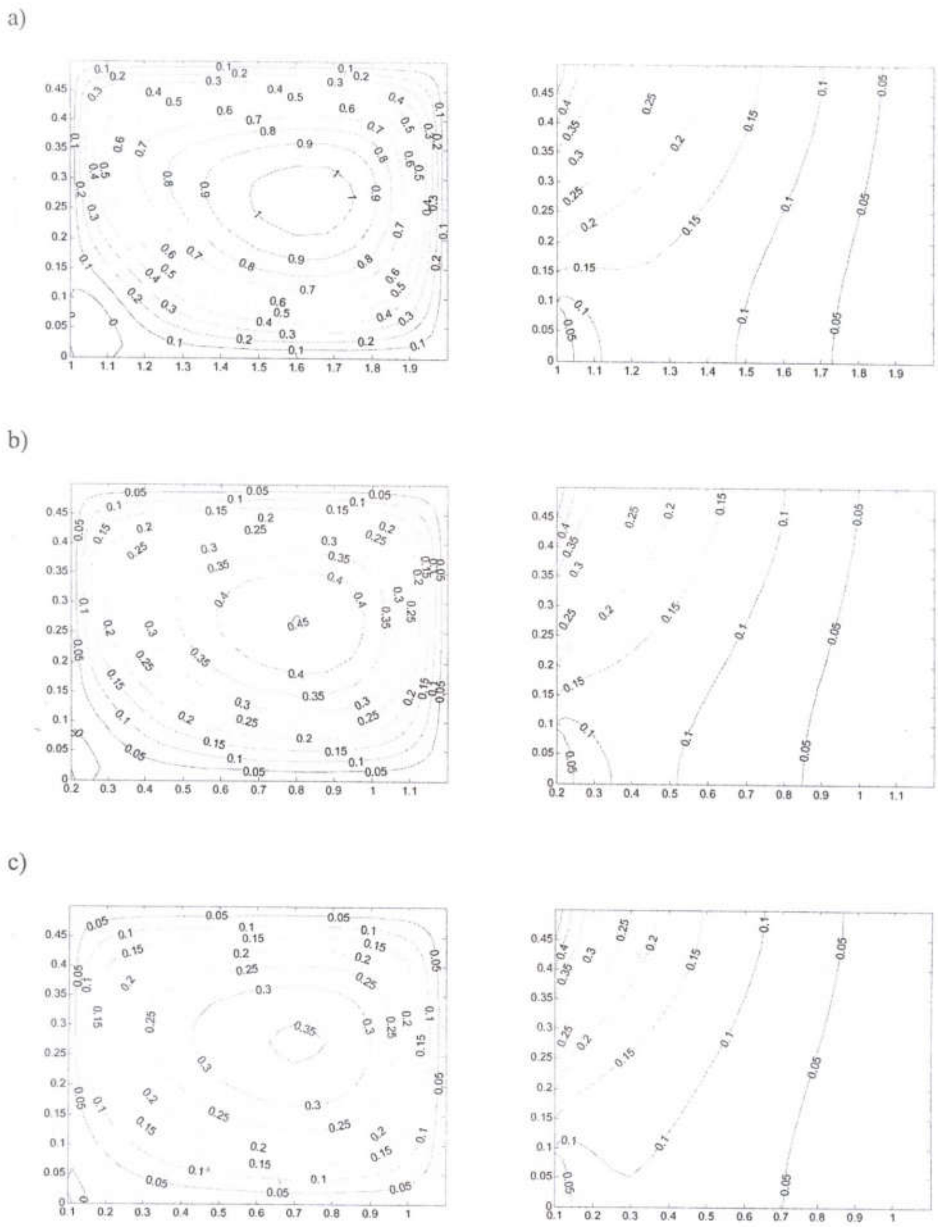

Fig. 2 Streamlines (Left) and Isotherms (Right) for ε=0.01, λ=0.5, Ra=100
a) R_r=1, b) R_r=5, c) R_r=10

It is obvious from the Fig. 2c that the heat transfer rate is higher at the upper portion of the annular cylinder at higher values of Radius ratio (R_r), which is vindicated by crowding of isothermal lines in the vicinity of upper side of hot wall as shown in Fig. 2. When Radius ratio (R_r) is increased, the streamlines and isothermal lines move from inner radius towards outer radius as shown in Fig. 2b.

Figure 3 shows the streamlines and isothermal lines distribution inside the porous medium of the vertical annular cylinder for various values of viscous dissipation parameter (ε). As the Viscous dissipation parameter (ε) increases, the streamlines and isothermal lines move away from the hot wall and reaches nearer to the cold wall of the vertical annular cylinder.

This is due to the reasons that the Viscous dissipation parameter (ε) is basically production of heat due to local friction between moving fluid and the solid matrix of the porous medium. The generation of heat due to Viscous dissipation parameter (ε) effect increases the temperature inside the medium occupied by increased temperature lines at the upper portion of the vertical annular cylinder. The circulation of the fluid increases as the Viscous dissipation parameter (ε) increases.

Figure 4 depicts the streamlines and isothermal lines inside the porous medium for various values of Power law exponent (λ). The fluid gets heated up near hot wall and moves up towards the cold wall due to buoyancy force and then returns back to hot wall.

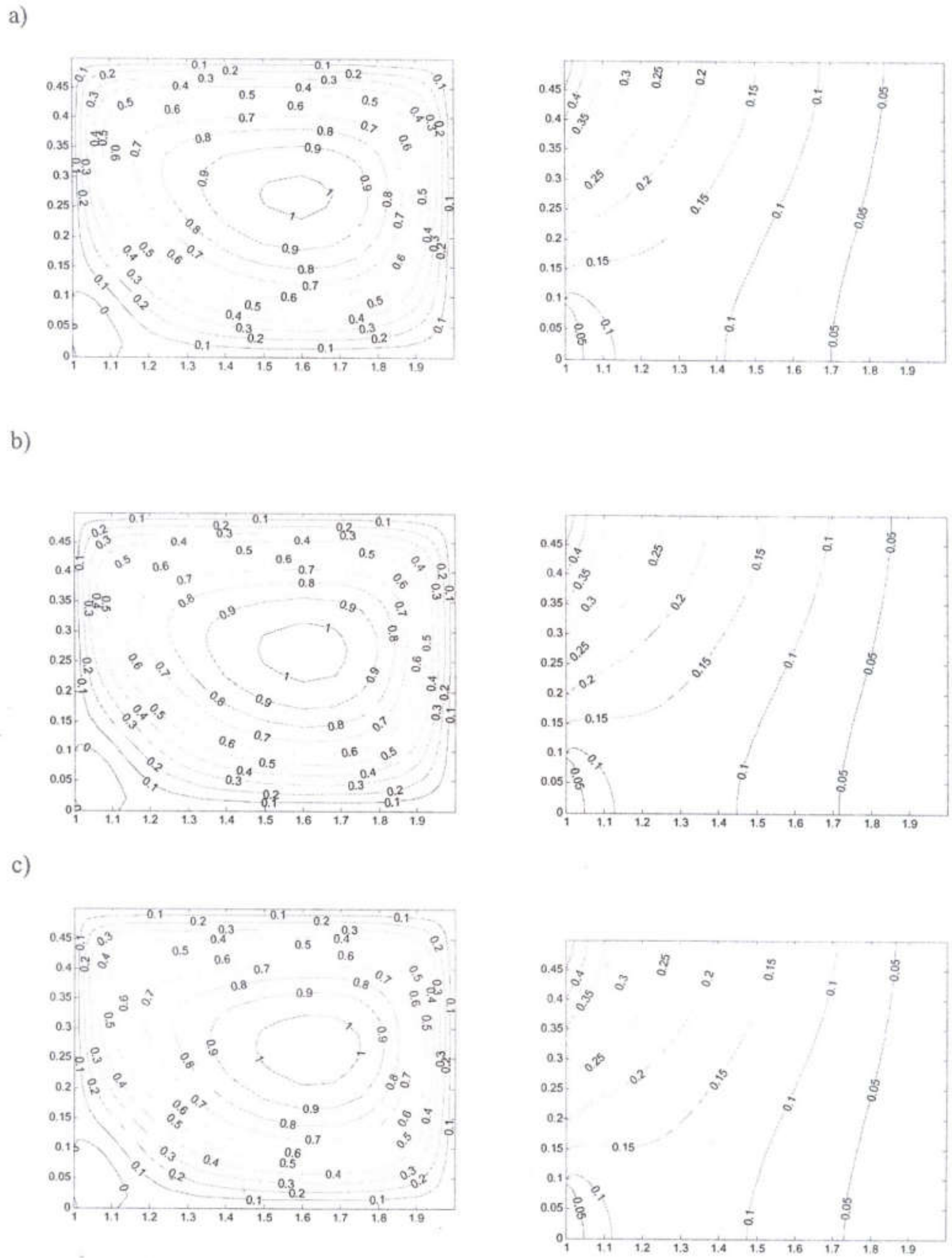

Fig. 3 Streamlines (Left) and Isotherms (Right) for A_r=0.5, R_r=1, Ra=100, λ=1
a) ε_r=0, b) ε=0.005, c) ε=0.01

For the case of isothermal temperature (λ = 0), the magnitude of the streamlines is high is compared to the no-isothermal temperature i.e., (λ>0). The thermal boundary layer thickness increases as the Power law exponent (λ) increases. It can be seen from the streamlines and isothermal lines that high convection heat transfer occurs mainly in upper portion of the vertical annular cylinder with the increase in Power law exponent (λ). The magnitude of the streamlines decreases with the increase in Power law exponent (λ). The fluid circulation decreases with the increase in Power law exponent (λ).

Figure 5 illustrates the streamlines and isothermal lines distribution inside the porous medium for various values of Rayleigh number. The magnitude of the streamlines increases as the Rayleigh number increases. It is clearly seen by observing the figure 5c. This is due to the reasons that the increased Rayleigh number promotes the fluid movement due to higher buoyancy force, which in turn allows the convection heat transfer to take dominant position. The increased Rayleigh number particularly enhances the heat transfer rate at upper portion of hot and cold walls of vertical annular cylinder respectively.

Figure 6 depicts the effect of Aspect ratio (A_r) and Viscous dissipation parameter (ε) on the average Nusselt number $(\overline{Nu})$ at hot wall of the vertical annular cylinder. This figure corresponds to the values Ra = 50, R_r = 1 and λ = 1.

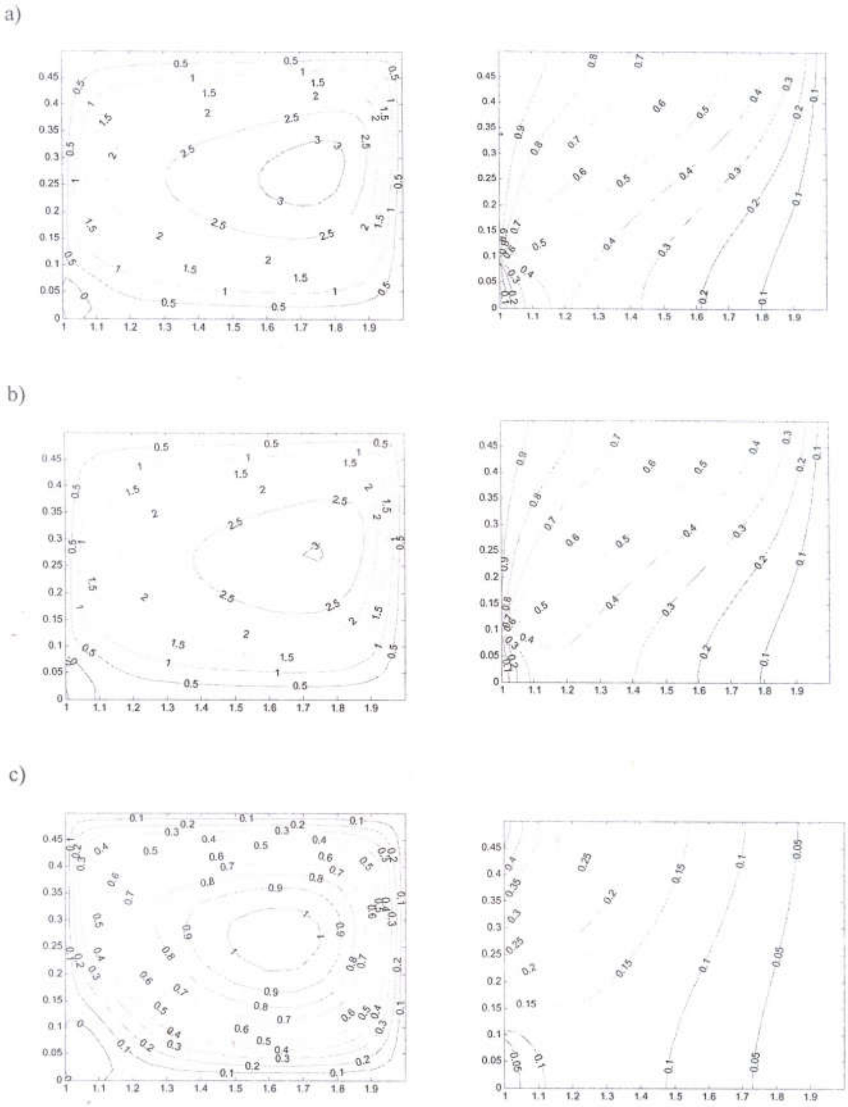

Fig. 4 Streamlines (Left) and Isotherms (Right) for A_r=0.5, R_r=1, Ra=100, ε=0.01
a) λ=0, b) λ=0.5, c) λ=1

It is seen that the average Nusselt number $(\overline{Nu})$ increases with the increase in Aspect ratio (A_r). The average Nusselt number $(\overline{Nu})$ decreases by 3.2% at A_r = 1, when Viscous dissipation parameter (ε) is increased from 0 to 0.01. The corresponding decrease in the average Nusselt number $(\overline{Nu})$ at A_r = 10 is found to be 13.13%. It is seen that there is a sharp increase in the average Nusselt number $(\overline{Nu})$ beyond A_r = 1 and then gradually increases for higher values of the Viscous dissipation parameter (ε). This happens due to the reasons that the viscous dissipation leads to local heat generation, which increases the temperature in the porous medium. As the temperature of hot wall T_w is (λ = 1), the increased temperature of porous medium increases the temperature difference between hot wall and the nearby region of the vertical annular cylinder. Due to this reasons the heat transfer from hot wall to the porous medium increases which results in increasing the average Nusselt number $(\overline{Nu})$.

Figure 7 illustrates the effect of Aspect ratio (A_r) and Power law exponent (λ) on the average Nusselt number $(\overline{Nu})$ at hot wall of the vertical annular cylinder for the values Ra = 50, R_r = 1 and ε = 0.01. It can be seen that the average Nusselt number $(\overline{Nu})$ is higher for the case of isothermal wall temperature. For a given value of Aspect ratio (A_r), average Nusselt number $(\overline{Nu})$ decreases with increase in Power law exponent (λ). This happens due to the reason that the heat content of the wall is more at (λ = 0), as compared to other values of λ>0. This leads to increased fluid movement near the hot wall, which in turn increases the average Nusselt number $(\overline{Nu})$ decreased by 71% at A_r = 0.5, when Power law exponent (λ) is increased from 0 to 1. It can be seen that the average Nusselt number $(\overline{Nu})$ decreases with increase in Aspect ratio (A_r). The decrease in average Nusselt number $(\overline{Nu})$ is sharp at higher values of Aspect ratio (A_r).

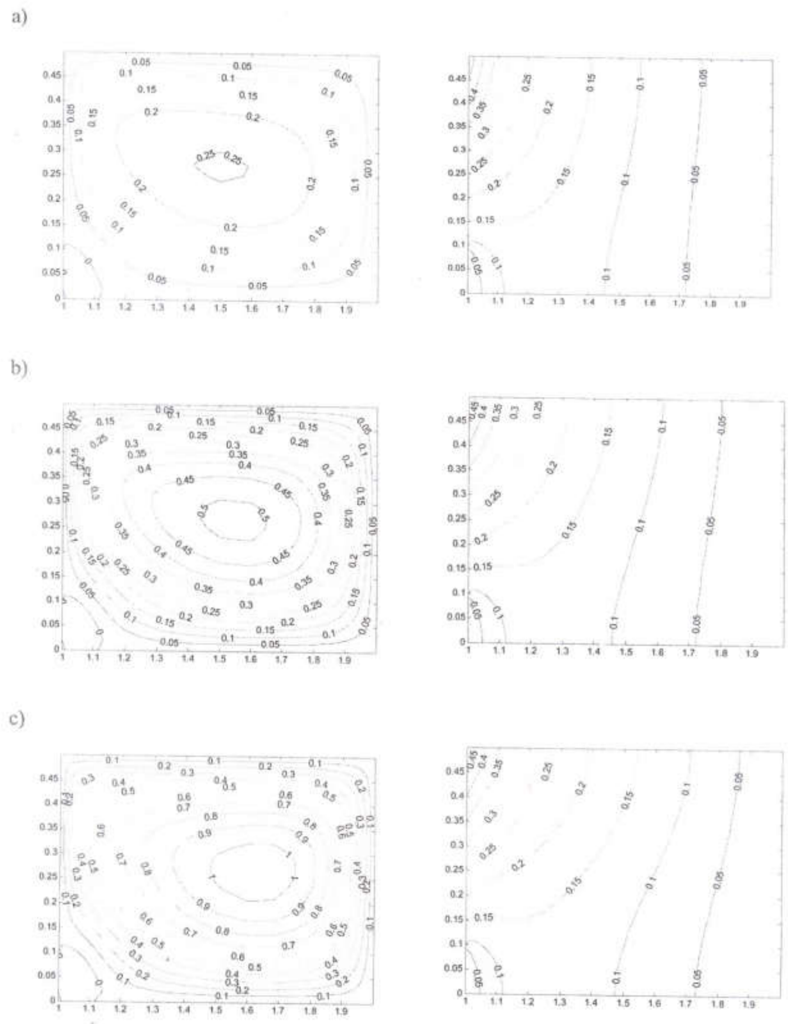

Fig. 5 Streamlines (Left) and Isotherms (Right) for A_r=0.5, R_r=1, λ=1
a) Ra=25, b) Ra=50, c) Ra=100

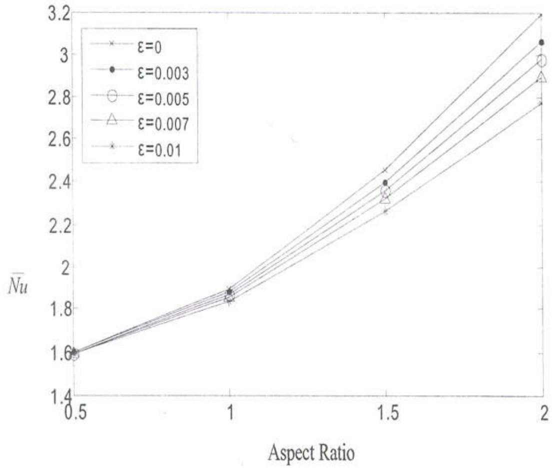

Fig.6 $\overline{Nu}$ Variation with A_r with at Hot wall for Different values of ε at R_r=1, R_a=50, λ=1

Figure 8 demonstrates the effect of Radius ratio (R_r) and Viscous dissipation parameter (ε) on the average Nusselt number $(\overline{Nu})$.

This figure corresponds to the values Ra = 100, A_r = 0.5, λ = 1. The average Nusselt number $(\overline{Nu})$ at hot wall of the vertical annular cylinder increases with the increase in Radius ratio (R_r). The Viscous dissipation leads to local heat generation, which increases the temperature in the porous medium.

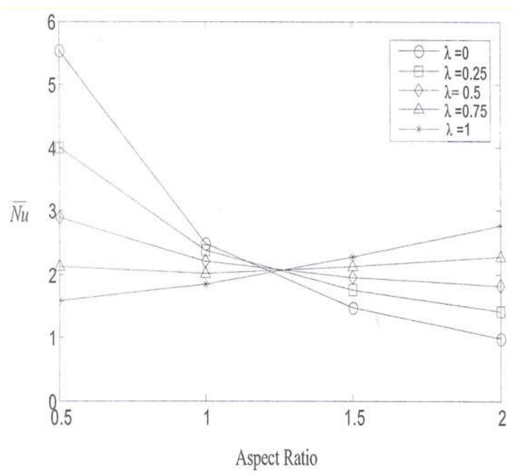

Fig.7 $\overline{Nu}$ Variation with A_r at Hot wall for different values of ε at R_r=1, R_a=50,

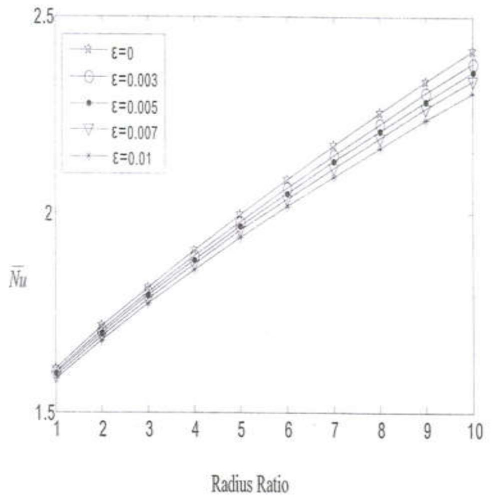

Fig8 $\overline{Nu}$ Variation with R_r at Hot wall for different values of ε at A_r=0.5, R_a=100, λ=1

As the temperature of hot wall T_w is λ = 1, the increased temperature of porous medium increases the temperature difference between the hot wall and the nearby region. Due to this reasons the heat transfer from hot wall to the porous medium increases which results in increasing the average Nusselt number $(\overline{Nu})$. The effect of Viscous dissipation is higher at the higher values of Radius ratio (R_r) as compared to the lower values of Radius ratio (R_r). The average Nusselt number $(\overline{Nu})$ is decreased by 1.58%, at R_r = 1. The corresponding decrease at R_r = 10, is found to be 4.58%, when Viscous dissipation parameter (ε) is increased from 0 to 0.01. This reduction in the average Nusselt number $(\overline{Nu})$ at hot wall is m ore pronounced at higher values of Radius ratio (R_r) when Viscous dissipation parameter (ε) is increased.

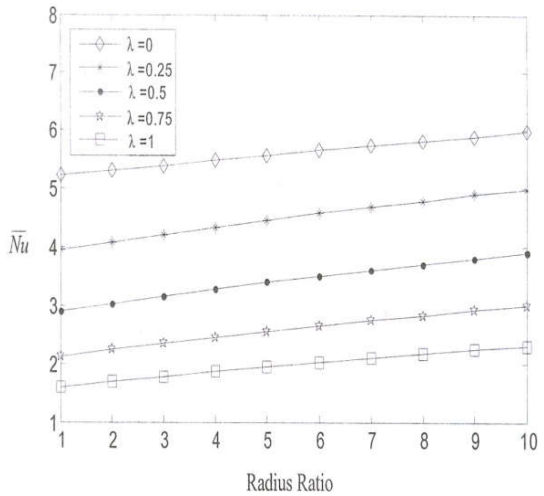

Fig. 9 $\overline{Nu}$ Variation with R_r at Hot wall for different values of λ at A_r=0.5, R_a=100, ε=0.01

Figure 9 shows the variation of average Nusselt number $(\overline{Nu})$ at hot wall, with respect to Radius ratio (R_r) of the vertical annular cylinder for various values of Power law exponent (λ) at A_r = 0.5, Ra = 100, ε = 0.01. It is found that the average Nusselt number $(\overline{Nu})$ increases with increase in Radius ratio (R_r). It can be seen that the average Nusselt number $(\overline{Nu})$ decreases with increase in Power law exponent (λ). For a given Radius given (R_r), the difference between the Nusselt number at two different values of Power law exponent (λ) increases with increase in Power law exponent (λ). For instance, the average Nusselt number $(\overline{Nu})$ decreased by 61%, When Power law exponent (λ) is increased from 0 to 1, at R_r = 1. However the average Nusselt number $(\overline{Nu})$ decreased by 61% when Power law exponent (λ) is increased from 0 to 1 at R_r = 10. This shows that the average Nusselt number $(\overline{Nu})$ increase linearly with the increase in Radius ratio (R_r).

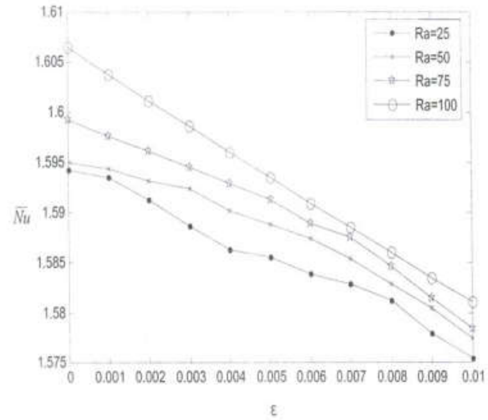

Fig. 10 $\overline{Nu}$ Variation with λ at Hot wall for different values of Ra at A_r=0.5, R_a=1, λ=1

Figure 10 illustrates the effect of Viscous dissipation parameter (ε) on the average Nusselt number $(\overline{Nu})$ for various values of Reyleigh number. This figure is obtained for A_r = 0.5 and R_r=1, λ=1. It can be seen that the average Nusselt number $(\overline{Nu})$ decreases with the increase in viscous dissipation parameter (ε). When there is no viscous dissipation then the average Nusselt number $(\overline{Nu})$ at hot wall always increases with increase in Rayleigh number. At ε=0.005, the average Nusselt number $(\overline{Nu})$ always decrease with increase in Rayleigh number. This happens due to the reason that higher Rayleigh number leads to high buoyancy force and thus faster fluid movement. This faster fluid movement enhances the local friction between fluid and solid matrix thus increasing the local heat generation, which in turn reduces the average Nusselt number $(\overline{Nu})$. When there is no Viscous dissipation parameter (ε) i.e., ε=0, the decrease in the average Nusselt number $(\overline{Nu})$ is found to be 0.29%, when Rayleigh number increases from 25 to 100. The decrease in the average Nusselt number $(\overline{Nu})$, when Viscous dissipation parameter (ε) is present i.e., at ε=0.01 is found to be 0.011%. This shows that there is a much decrease in the average Nusselt number $(\overline{Nu})$ as the Viscous dissipation parameter (ε) increases.

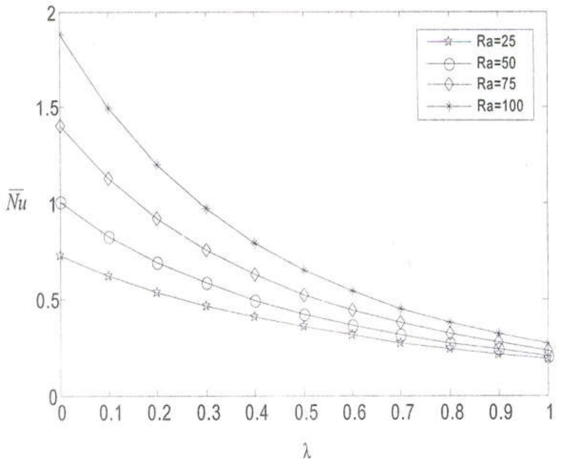

Fig. 11 $\overline{Nu}$ Variation with λ at Hot wall for different values of Ra at A_r=0.5, R_a=1, ε=0.01

Figure 11 demonstrates the variation of average Nusselt number $(\overline{Nu})$ at hot wall, with respect to Power law exponent (λ) of the vertical annular cylinder for various values of Rayleigh number at A_r=0.5, R_r=1, ε=0.01. It is found that the average Nusselt number $(\overline{Nu})$ decreases with increase in Power law exponent (λ). It can be seen that the average Nusselt number decreases with decrease in Rayleigh number. At λ=0, the average Nusselt number decreased by 61.5% when Rayleigh number is increased from 25 to 100.

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How to cite this article:

Nagaradhika V and Subbha Rao A.2016, Free Convection In A Vertical Annulur Cylinder In Porous Media With Effect of Viscous Dissipation. *Int J Recent Sci Res.* 7(7), pp. 12614-12624.