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**Research Article**

# PROPERTIES OF IRREGULAR INTUITIONISTIC FUZZY HYPERGRAPHS

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**ABSTRACT**

Intuitionistic fuzzy hypergraphs, Irregular Intuitionistic fuzzy graphs, Degree, order and size in Intuitionistic fuzzy graphs, Properties of irregular Intuitionistic fuzzy graphs are discussed in the papers[12],[9],[10],[3]. In this paper, we define irregular intuitionistic fuzzy hypergraphs and we extend the result to various classifications and also study the properties of neighbourly irregular IFHG and totally irregular IFHG. Some results on totally irregular IFHG are established.



## 1. INTRODUCTION

Hypergraph is a graph in which an edge can connect more than two vertices. Hypergraphs can be applied to analyze architecture structures and to represent system partitions. Hypergraphtheory, developed by C. Berge [2] in 1960, is a generalization of graph theory. The notion of hypergraphs has been extended in fuzzy theory and the concept of fuzzy hyper graphs was proposed by Lee-Kwang and S.M.Chen. The concept of an Intutionistic fuzzy graph was introduced by Atanassov[1]. Parvathy. R and M.G. Karunambigai's paper [12] introduced the concepts of IFH and analyzed its components. M. Akram [5] applied the concepts in to a real-life problem with a numerical example.

NagoorGani. A and SajithBegum. S [10] defined degree, order and size in Intuitionistic fuzzy graphs and extend the properties. NagoorGani A and Radha[8] introduced regular fuzzy graphs, total degree and totally regular fuzzy graphs. NagoorGani. Aand Latha. R [9] also introduced irregular fuzzy graphs and discussed some of its properties. GnaanaBhragsam and Ayyaswamy suggested a method to construct a neighborly irregular graph of order and also discussed some properties on neighborly irregular graph. Also Jahir Hussain. R and Yahya Mohamed. S[3] established properties on irregular IFG. Yousef Alavi, F.R.K. Chung, *et.al* [13] introduced k-path irregular graph and studied some properties on k-path irregular graphs.

In this paper, we define some types of irregular IFHG. The discussion on the relation between neighbourly irregular IFHG and highly irregular IFHG are established.

### 2. *Preliminaries*

***Definition2.1:*** A hyper graph $H$ is an ordered pair $H=(X,E)$ where

i) $X=\{x_1,x_2,...,x_n\}$ a finite set of vertices.

ii) $E=\{E_1,E_2,...E_m\}$ a family of subsets of $V$ .

iii) $E_j\neq\phi, j=1,2,...m$ and $\underset{j}{\cup}E_j=X$.

The set X is called the set of vertices and E is the set of edges (or hyper edges).

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In a hypergraph two or more vertices $x_1, x_2, .........x_n$ are said to be adjacent if there exist an edge $E_j$ which contains those vertices. In a hypergraph two edges $E_i \& E_j \quad i \neq j$ is said to be adjacent if their intersection is not empty. The size of a hyper graph depends on the cardinality of its hyperedges. we define the size of H as the sum of the cardinalities of its hyperedges.

A regular hyper graph is one in which every vertex is contained in k edges for some constant k.

***Definition 2.2:*** Let $X$ be a finite set and let $\xi$ be a finite family of non-trivial fuzzy subsets of $X$ such that $X = \bigcup_{\mu \in E} Supp(\mu)$. The pair $H = (X, \xi)$ is called a fuzzy hypergraph (on $X$) and $\xi$ is called the edge set of H which is sometimes denoted $\xi(H)$. The members of $\xi$ are called the fuzzy edges of H.

***Definition 2.3:*** The IFHG H is an ordered pair $H = \{X, E\}$ where H satisfies the following conditions

i) $X = \{x_1, x_2 ... x_n\}$ is a finite set of vertices.

ii) $E = \{E_1, E_2, ...., E_m\}$ is a family on intuitionistic fuzzy subsets of $X$.

iii) $E_j = \{(x_i, \mu_j(x_i), \gamma_j(x_i)); \mu_j(x_i), \gamma_j(x_i) \geq 0 \ \& \ \mu_j(x_i) + \gamma_j(x_i) \leq 1\} \quad j = 1, 2 ... m$

iv) $E_j \neq \phi, \ j = 1, 2, ..., m$

v) $\bigcup_j Supp(E_j) = X, \quad j = 1, 2, ... m$

The edges $E_j$ are an IFSs of vertices, $\mu_j(x_i)$ and $\gamma_j(x_i)$ denote the degree of membership and non-membership of vertex $x_i$ to edge $E_j$.

The sets X and E are crisp sets.

***Notations***

The triple $\langle x_i, \mu_{1i}, \gamma_{1i} \rangle$ denotes the degree of membership and non-membership of the vertex $x_i$. The triple $\langle e_{ij}, \mu_{2ij}, \gamma_{2ij} \rangle$ denote the degree of membership and non- membership of the edge $e_{ij} = (x_i, x_j)$ on X.

That is $\mu_{1i} = \mu_1(x_i), \gamma_{1i} = \gamma_1(x_i) \ \ and \ \ \mu_{2ij} = \mu_2(x_i, x_j), \gamma_{2ij} = \gamma_2(x_i, x_j)$.

***Definition 2.4***

In an Intuitionistic Fuzzy Hypergraph H, the degree of a vertex $x$ is denoted by $d(x)$ and defined by $d(x) = \left(d^{\mu}(x), d^{\gamma}(x)\right)$

$$\text{Where } d^{\mu}(x) = \sum_{x \varepsilon e_j} \mu_E(e_j)$$

$$d^{\gamma}(x) = \sum_{x \varepsilon e_j} \gamma_E(e_j)$$

***Example2.5***

Consider an IFHG H =(V,E). Let $X = \{x_1, x_2, x_3 x_4\} \ and \ E = \{e_1, e_2\}$ where $\mu_1(x_1) = 0.1, \mu_1(x_2) = 0.3, \mu_1(x_3) = 0.4, \mu_1(x_4) = 0.2 \ \ and \ \ \gamma_1(x_1) = 0.2, \gamma_1(x_2) = 0.5, \gamma_1(x_3) = 0.6, \gamma_1(x_4) = 0.2$ $e_1 = \{x_1, x_2, x_3\} \ \mu_E(e_1) = 0.01, \gamma_E(e_1) = 0.06 \ and \ e_2 = \{x_3, x_4\} \ \mu_E(e_2) = 0.08, \gamma_E(e_2) = 0.12$. Then the degree of vertices is $d(x_1) = (0.01, 0.06)$ $d(x_2) = (0.01, 0.06)$ $d(x_3) = (0.09, 0.18)$ $d(x_4) = (0.08, 0.12)$.

***Definition 2.6***

In an Intuitionistic Fuzzy Hypergraph H, the total degree of a vertex $x$ is denoted by td(x) and defined by t $d(x) = \left(td^{\mu}(x), td^{\gamma}(x)\right)$

$$\text{Where } td^{\mu}(x) = d(x) + \mu_1(x)$$

$$td^{\gamma}(x) = d(x) + \gamma_1(x)$$

***Example 2.7***

Consider an IFHG H =(V,E). Let $X=\{x_1,x_2,x_3x_4\}$ *and* $E=\{e_1,e_2\}$ where $\mu_1(x_1)=0.1,\mu_1(x_2)=0.3,\mu_1(x_3)=0.4,\mu_1(x_4)=0.2$ *and* $\gamma_1(x_1)=0.2,\gamma_1(x_2)=0.5,\gamma_1(x_3)=0.6,\gamma_1(x_4)=0.2$ $e_1=\{x_1,x_2,x_3\}$ $\mu_E(e_1)=0.01,\gamma_E(e_1)=0.06$ *and* $e_2=\{x_3,x_4\}$ $\mu_E(e_2)=0.08,\gamma_E(e_2)=0.12$. Then the total degree of vertices is $d(x_1)=(0.11,0.0.26)$ $d(x_2)=(0.31,0.56)$ $d(x_3)=(0.49,0.78)$ $d(x_4)=(0.28,0.32)$.

***3 Irregular Intuitionistic fuzzy hypergraphs***

***Definition 3.1***

Let H =(V,E) be IFHG. Then H is irregular if there is a vertex which is adjacent to vertices with distinct degrees.

***Example 3.2***

Consider an IFHG H =(X,E). Let $X=\{x_1,x_2,x_3x_4\}$ *and* $E=\{e_1,e_2,e_3,e_4\}$ where

$\mu_1(x_1)=0.2,\mu_1(x_2)=0.3,\mu_1(x_3)=0.4,\mu_1(x_4)=0.5$ *and*
$\gamma_1(x_1)=0.6,\gamma_1(x_2)=0.5,\gamma_1(x_3)=0.2,\gamma_1(x_4)=0.2$

$e_1=\{x_1,x_2\},\mu_E(e_1)=0.02,\gamma_E(e_1)=0.1$
$e_2=\{x_2,x_3\},\mu_E(e_2)=0.05,\gamma_E(e_2)=0.04$
$e_3=\{x_4\},\mu_E(e_3)=0.02,\gamma_E(e_3)=0.1$
$\&\ e_4=\{x_1,x_4\},\ \mu_E(e_4)=0.05,\gamma_E(e_4)=0.04.$

Then the degrees of vertices are $d(x_1)=(0.07,0.14)$ $d(x_2)=(0.07,0.14)$ $d(x_3)=(0.05,0.04)$ $d(x_4)=(0.07,0.14)$

Here $x_2$ adjacent to $x_1$ and $x_3$ have distinct degrees.

***Definition 3.3***

Let H = (X, E) be connected IFHG, Then H is said to be a highly irregular IFHG if every vertex of H is adjacent to vertices with distinct degrees.

***Example 3.4***

Consider an IFHG H =(X, E). Let $X=\{x_1,x_2,x_3x_4\}$ *and* $E=\{e_1,e_2,e_3\}$ where

$\mu_1(x_1)=0.2,\mu_1(x_2)=0.3,\mu_1(x_3)=0.4,\mu_1(x_4)=0.5$ *and*
$\gamma_1(x_1)=0.6,\gamma_1(x_2)=0.5,\gamma_1(x_3)=0.2,\gamma_1(x_4)=0.2$

$e_1=\{x_1,x_2,x_3\},\mu_E(e_1)=0.01,\gamma_E(e_1)=0.06$
$e_2=\{x_2,x_4\},\mu_E(e_2)=0.06,\gamma_E(e_2)=0.1$
$e_3=\{x_1,\ x_4\},\mu_E(e_3)=0.01,\gamma_E(e_3)=0.04.$

Then the degrees of vertices are $d(x_1)=(0.07,0.14)$ $d(x_2)=(0.07,0.14)$ $d(x_3)=(0.05,0.04)$ $d(x_4)=(0.07,0.14)$

Here $d(x_1)\neq d(x_2)\neq d(x_3)\neq d(x_4)$

***Definition 3.5***

Let H = (X, E) be connected IFHG, Then H is said to be a neighbourly irregular IFHG if every two adjacent vertices of H have distinct degrees.

***Proposition3.6***

A highly irregular IFHG need not be neighbourly irregular IFHG.As for example we consider an IFHG H =(X,E) such that

$X=\{x_1,x_2,x_3x_4\}$ *and* $E=\{e_1,e_2\}$ where

$\mu_1(x_1)=0.1,\mu_1(x_2)=0.3,\mu_1(x_3)=0.4,\mu_1(x_4)=0.2$ *and*
$\gamma_1(x_1)=0.2,\gamma_1(x_2)=0.5,\gamma_1(x_3)=0.6,\gamma_1(x_4)=0.3$

$e_1=\{x_1,x_2,x_3\},\mu_E(e_1)=0.01,\gamma_E(e_1)=0.06$
$e_2=\{x_3,x_4\},\mu_E(e_2)=0.08,\gamma_E(e_2)=0.12$
$d(x_1)=(0.01,0.06)$ $d(x_2)=(0.01,0.06)$ $d(x_3)=(0.09,0.18)$ $d(x_4)=(0.08,0.12)$

Here the IFHG is highly irregular but not neighbourlyirregular as $d(x_1)=d(x_2)$.

***Definition 3.7***

Let H = (V,E) be IFHG. Then H is totally irregular if there is a vertex which is adjacent to vertices with distinct total degrees.

***Example 3.8***

Consider an IFHG H =(X,E) such that $X=\{x_1,x_2,x_3x_4\}$ *and* $E=\{e_1,e_2\}$ where

$$\mu_1(x_1)=0.1,\mu_1(x_2)=0.1,\mu_1(x_3)=0.4,\mu_1(x_4)=0.03 \;\; and \;\; e_1=\{x_1,x_2,x_3\},\mu_E(e_1)=0.01,\gamma_E(e_1)=0.06$$

$$\gamma_1(x_1)=0.2,\gamma_1(x_2)=0.5,\gamma_1(x_3)=0.6,\gamma_1(x_4)=0.6 \qquad e_2=\{x_3,x_4\},\mu_E(e_2)=0.08,\gamma_E(e_2)=0.12$$

Then the total degrees of vertices are

$$td(x_1)=(0.11,0.26)\;\; td(x_2)=(0.11,0.56)\;\; td(x_3)=(0.49,0.78)\;\; td(x_4)=(0.11,0.72)$$

Here $x_2$ adjacent to $x_1$ and $x_3$ have distinct total degrees.

***Definition 3.9***

Let H = (X,E) be connected IFHG ,Then H is said to be a neighbourly totally irregular IFHG if every two adjacent vertices of H Have distinct total degrees.

***Theorem 3.10***

Let H be a IFHG .Then H is highly irregular IFHG and neighbourly irregular IFHG iff the degrees of all vertices of H are distinct.

***Proof***

Let, G be the IFHG with n vertices $x_1$, $x_2$,…,$x_n$. Now, Suppose H is highly irregular and neighbourly irregular IFHG. Let the adjacent vertices of $x_1$ be $x_2$, $x_3$,…,$x_n$ with degrees ($m_2$, $n_2$), ($m_3$,$n_3$),…,($m_n$,$n_n$) respectively.

Since H is highly irregular, $d(x_1)\neq d(x_2)\neq d(x_3)\neq d(x_4)\neq \ldots\ldots\ldots\ldots \neq d(x_n)$. So it is obvious that all vertices of H are of distinct degrees.

**Conversely,** Assume that the degrees of all vertices of H are distinct. This means that, every two adjacent vertices have distinct degrees and to every vertex, the adjacent vertices have distinct degrees. Hence, H is neighbourly irregular and highly irregular IFHG.

***Theorem 3.11***

Let H = <X, E> be IFHG. If H is neighbourly irregular and $\mu_1$ and $\gamma_1$ are constant functions, then H is a neighbourly totally irregular IFHG.

***Proof:***

Let H = <X, E> be IFHG. Assume thatH = <X, E> is neighbourly Irregular IFHG.(i.e) the degrees of every two adjacent vertices are distinct.

Consider two adjacent vertices $x_1$ and $x_2$ with distinct degrees say ($m_1$,$n_1$) and ($m_2$, $n_2$) respectively.

Also assume $\mu_1(x)$ = a for all $x \in X$

and$\gamma_1(x)$ = b for all $x \in X$ ; where a, b are constant and in [0,1]

Therefore, $$td(x_1)=(d^{\mu}(x_1)+\mu_1(x_1),d^{\gamma}(x_1)+\gamma_1(x_1))=(m_1+a,n_1+b)$$ $$td(x_2)=(d^{\mu}(x_2)+\mu_1(x_2),d^{\gamma}(x_2)+\gamma_1(x_2))=(m_2+a,n_2+b)$$ clearly $td(x_1)\neq td(x_2)$

Therefore the two adjacent vertices $x_1$ and $x_2$ with distinct degrees, its total degrees are also distinct, provided $\mu_1,\gamma_1$ are constant functions. The above argument is true for every pair of adjacent vertices in H.

***Theorem 3.12***

Let H = <X, E> be a neighbourly total irregular IFHG and $\mu_1$ and $\gamma_1$ are constant functions, thenH is a neighbourly irregular IFHG.
Proof:
Let H = <X, E> be IFHG. Assume that H = <X, E> is neighbourly total irregular IFHG.
(i.e) the total degree of every two adjacent vertices are distinct.
Consider two adjacent vertices $x_1$ and $x_2$ with distinct degrees say ($m_1$,$n_1$) and ($m_2$, $n_2$) respectively.

Also assume $\mu_1(x)$ = a for all $x \in X$

and $\gamma_1(x)$ = b for all $x \in X$ ; where a, b $\in [0,1]$are constants.

Also $td(x_1)\neq td(x_2)$.

We are to prove that $d(x_1) \neq d(x_2)$

$m_1 + a \neq m_2 + a$ *and* $n_1 + b \neq n_2 + b$.

So $m_1 \neq m_2$ *and* $n_1 \neq n_2$

Hence the degrees of adjacent vertices of H are distinct.
This is true for every pair of adjacent vertices in H. Hence the result.

## CONCLUSION

Here we derived and discussed some types of irregular IFHGs. We also discussed the relationship between the irregularities of IFHGs. Further we are planned to extend the work to different types of IFHGs.

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