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Research Article

TENSION QUASI-QUINTIC TRIGONOMETRIC B´EZIER CURVE WITH TWO SHAPE PARAMETERS

Mridula Dube[1] and Urvashi Mishra[2]

[1]Department of Mathematics and Computer Science, R.D. University, Jabalpur, Madhya Pradesh, India
[2]Department of Mathematics, Mata Gujri Mahila Mahavidyalaya, Jabalpur, Madhya Pradesh, India

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ABSTRACT

In this paper, a class of quasi-quintic trigonometric Bezier´ curve with two shape parameters, with a tension parameter, are presented. The new basis functions share the properties with Bernstein basis functions, so the generated curves inherit many properties of traditional Bezier´ curves. The presence of shape parameters provides a local control on the shape of the curve which enables the designer to control the curve more than the ordinary Bezier´ curve. These type of functions are useful for constructing trigonometric Bezier curves and surfaces, they can be applied to construct continuous shape preserving interpolation spline curves with shape parameters. To better visualize objects and graphics a tension parameter is included. In this work we constructed the Trigonometric Bezier curves followed by a construction of the shape preserving interpolation spline curves with local shape parameters and a tension parameter.

INTRODUCTION

In computer aided geometric design (CAGD), it is often necessary to generate curves and surfaces that approximate shapes with some desired shape features. Designing free form curves and surfaces is a prevalent feature of CAGD. The key problem, simply stated, is to enable the designer to generate curves and surfaces which behave as he wants them to. The parametric cubic is a powerful tool which, when properly defined, is capable of representing most geometric entities of practical interest. In recent years, the trigonometric spline with shape parameters has gained more interest in CAGD, in particular curve design. Han [1] presented a class of quadratic trigonometric polynomial curves with a shape parameter. The shape of the curve was more easily controlled by altering the values of shape parameter than the ordinary quadratic B-Spline curves. Han [2] introduced piecewise quadratic trigonometric polynomial curves with C^1 continuity analogous to the quadratic B-Spline curves which have C^1 continuity. Cubic trigonometric polynomial curves with a shape parameter were discussed by Han [3]. In these papers the authors described the trigonometric polynomial with global shape parameter. Single parameter does not provide local control on the curves. To remedy this, Wu *et al* presented quadratic trigonometric polynomial curves with multiple shape parameters. Bezier´ technique is one of the methods of analytic representation of curves and surfaces that has won wide acceptance as a valuable tool in CAD/CAM system. They are used to produce curves which appear reasonably smooth at all scales. Today, many CAGD systems feature Bezier´ curves as their major building block, since they are very efficient and attain a number of mathematical properties. Both rational and non-rational forms of Bezier´ curves have been studied by many authors. A cubic trigonometric Bezier´ curve with two shape parameters was presented by Han *et al*. It enjoyed all the geometric properties of the ordinary cubic Bezier´ curve and was used for spur gear tooth design with S-shaped transition curve Abbas, *et al*. Liu, *et. al* presented a study on class of TC-Bezier´ curve with shape parameters. Yang, *et. al* discussed trigonometric extension of quartic Bezier´ curves attaining G^2 and C^2 continuity. A class of general quartic spline curves with shape parameters were introduced by Han. Yang, *et al* studied a class of quasi-quartic trigonometric Bezier´ curves and surfaces. It is important to study the spline curve representations that provide local control, that is, the capability of modifying one portion of the curve without altering the remainder. From a practical standpoint, we are interested in constructing the trigonometric polynomial representations which can manipulate a curve e ectually.

*Corresponding author: **Mridula Dube and Urvashi Mishra**
Department of Mathematics and Computer Science, R.D. University, Jabalpur, Madhya Pradesh, India

The purpose of this paper is to present quintic trigonometric polynomial blending functions where we include a tension parameter, the latter is mainly important for object visualization. These blending functions are useful for constructing trigonometric Bezier curves and can be applied to construct continuous shape preserving interpolation spline curves with shape parameters. Using tensor product, we can construct Bezier-type surfaces, which have properties similar to polynomial Bezier surfaces. A newly constructed quasi-quintic trigonometric Bezier´ curve with two shape parameters is presented in this paper. The proposed curve inherits all trigonometric properties of the traditional Bezier´ curve and is used to construct open and closed curves.

The remainder of this paper is organized as follows. In section 2, the quintic Trigonometric polynomial blending functions with shape and tension parameter are described and the properties of these functions are shown. In the same section, Trigonometric Bezier curves are constructed. Shape control is discussed in section 3. The Trigonometric polynomial approximation is discussed in Section 4. Conclusion is given in section 5.

Trigonometric Bezier Curves

Firstly we can define the generalized trigonometric polynomial blending functions with tension parameter and the corresponding generalized trigonometric Bezier curves.

Trigonometric Polynomial Blending Functions with Tension Parameter

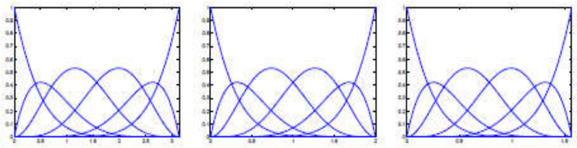
The trigonometric polynomial blending functions are given as follows:

Definition 2.1 Let β be the tension parameter and $t \in \left[0, \frac{\pi}{2\beta}\right]$. The generalized trigonometric polynomial blending functions with tension parameter and two shape parameters m and n where $3 \leq m, n \leq 1$; $B_{i,\beta}$; i = 0;1, . . ., 5 are defined as:

$$
\begin{aligned}
B_{0,\beta}(t) &= (1 - \sin(\beta t))^3 (1 - m \sin(\beta t)); \\
B_{1,\beta}(t) &= \sin(\beta t)(1 - \sin(\beta t))^2 (3 + m(1 - \sin(\beta t))); \\
B_{2,\beta}(t) &= (-1 + \cos(\beta t) + \sin(\beta t))^2 (1 + \sin(\beta t)); \\
B_{3,\beta}(t) &= (-1 + \cos(\beta t) + \sin(\beta t))^2 (1 + \cos(\beta t)); \\
B_{4,\beta}(t) &= \cos(\beta t)(1 - \cos(\beta t))^2 (3 + n(1 - \cos(\beta t))); \\
B_{5,\beta}(t) &= (1 - \cos(\beta t))^3 (1 - n \cos(\beta t));
\end{aligned}
\qquad (1)
$$

The blending functions studied in the present work have the following properties which are analogous to those found for the quintic trigonometric Bezier basis functions generalized trigonometric polynomial blending functions.

Figure 1, plots these basis functions for different values of the tension parameter β in the interval $[0, \frac{\pi}{2\beta}]$.



(a) $\beta = 0.5$. (b) $\beta = \pi/4$. (c) $\beta = 1$.

Figure 1. The curves of the blending functions basis (for $t \in [0, \frac{\pi}{2\beta}]$).

Properties of Trigonometric Polynomial Blending Functions

Theorem1. The basis functions $B_{i,\beta}(t), (i=0,1,...,5)$ (1) have the following properties:

1. **Nonnegativity**. When $t \in \left[0, \frac{\pi}{2\beta}\right]$ there are $B_{i,\beta}(t) \geq 0, (i=0,1,...,5)$.
2. **Partition of Unity**. One has $\sum_{i=0}^{5} B_{i,\beta}(t) = 1$;
3. **Symmetry**. $B_{i,\beta}(t) = B_{5-i,\beta}(\frac{\pi}{2\beta} - t)$; for i = 0,1,2;
4. **Maximum**: Each $B_{i,\beta}(t)$ has one maximum value in $t \in \left[0, \frac{\pi}{2\beta}\right]$

Trigonometric Bezier Curves with Tension Parameter

This section describes the theory and method of using the tension parameter β to control the form of the interpolating trigonometric Bezier curve $B_{i,\beta}(t)$. Note that changing the tension factor β does not affect the form of $B_{i,\beta}(t)$ and the interpolation features at the data points.

Let V = (V_0,…,V_5) be a set of points $V_i \in R^2$ or R^3. The Trigonometric Bezier curves with tension parameter $\beta > 0$ associated with the set V is defined by:

$$B_\beta(t) = \sum_{i=0}^{5} (B_{i,\beta}(t))\, V_i ; \text{ where } t \in \left[0, \frac{\pi}{2\beta}\right] \qquad (2)$$

The points V_i; (i = 0, . . .,., 5) are called quintic trigonometric Bezier control points.

Figure 1 shows the quintic Trigonometric Bezier curves with different tension parameter values. Keeping the same control polygon, as β varies we are not simply changing the domain of a single curve, but defining different curves. It can be seen that quintic trigonometric Bezier curves are close to the control polygon. Therefore, quinrtic trigonometric Bezier curves can nicely preserve the feature of the control polygon. Control polygons provide an important tool in geometric modeling.

The tension like effect of this tension factor β is illustrated in Figures 1 and 2 where the interval changes as a function of β keeping all the properties of the blending functions verified. It is an advantage if the curve being modeled tends to preserve the shape of its control polygon.

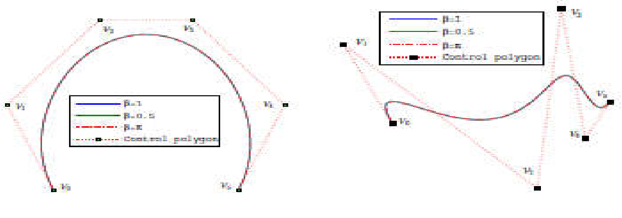


Figure 2. The curves of the blending functions basis (for $t \in [0, \frac{\pi}{2\beta}]$).

Theorem 2 The curve (2) upholds the following properties:

End point properties

$$B_\beta(0)=V_0; B_\beta(1)=V_5;$$

$$B'_\beta(0)=(3+m)\beta(V_1-V_0);$$

$$B'_\beta\left(\frac{\pi}{2}\right)=(3+n)\beta(V_5-V_4);$$

$$B''_\beta(0)=2\beta^2[3(1+m)V_0-3(2+m)V_1+V_2+2V_3];$$

$$B''_\beta\left(\frac{\pi}{2}\right)=2\beta^2[3(1+n)V_5-3(2+n)V_4+V_3+2V_2];$$

***Symmetry**:* The control points V_i and V_{5-i}; i =0;1;2;3;4;5 define the same quasi-quintic trigonometric B´ezier curve in different parameterizations, i.e.for i = 0; 1; 2; 3; 4; 5

$$B_\beta(u;\ m;\ n; V_i)=B_\beta\left(\frac{\pi}{2\beta}-u, n, m, V_{5-i}\right)$$

$$B_\beta(t;V_0;V_1;V_2;V_3;V_4;V_5)=B_\beta\left(\frac{\pi}{2\beta}-t;V_5;V_4;V_3;V_2;V_1\right);$$

***Geometric invariance**:* The shape of the curve (2) is independent of the choice of its control points. Equivalently the curve (2) satisfies the following two equations for i = 0; 1; 2; 3; 4; 5.

$$B_\beta(u;m;n;V_i+q)=B_\beta(u;m;n;V_i)+q;$$

$$B_\beta(u;m;n;V_i*q)=B_\beta(u;m;n;V_i)+q;$$

where q is any arbitrary vector in R^2 and T is an arbitrary 2 _ 2 matrix.

***Convex hull**:* The entire curve is contained within the convex hull of its defining control points Pi.

***Variation diminishing property**:* no straight line intersects a Bezier curve more times than it intersects its control polygon.

***Convexity- preserving property**:* the variation diminishing property means the convexity preserving property holds.

Shape Control of the Quintic Trigonometric Bezier Curve

For control points $V = (V_0,\ldots,V_5)$ be a set of points $V_i \in R^2$ or R^3. The Trigonometric Bezier curves with tension-parameter $\beta > 0$ associated with the set V is defined by:

$$B_\beta(t)=\sum_{i=0}^{5}(B_{i,\beta}(t))\,V_i;$$

where $t \in \left[0, \frac{\pi}{2\beta}\right]$ (3)

We rewrite this equation as follows;

$$B_\beta(t)=\sum_{i=0}^{5}(C_{i,\beta}(t))\,V_i+m\bullet\sin(\beta t)(1-\sin(\beta t))^3(V_1-V_0)+n\bullet\cos(\beta t)(1-\cos(\beta t))^3(V_5-V_4);$$

where

$$C_{0,\beta}(t)=(1-\sin(\beta t))^3;$$

$$C_{1,\beta}(t)=3[\sin(\beta t)(1-\sin(\beta t))^2];$$

$$C_{2,\beta}(t)=(-1+\cos(\beta t)+\sin(\beta t))^2(1+\cos(\beta t));$$

$$C_{3,\beta}(t)=(-1+\cos(\beta t)+\sin(\beta t))^2(1+\cos(\beta t));$$

$$C_{4,\beta}(t)=3[\cos(\beta t)(1-\cos(\beta t))^2];$$

$$C_{5,\beta}(t)=(1-\cos(\beta t))^3;$$

Obviously, shape parameter m and n affects the curve on the control edges $(V_1\text{-}V_0)$ and $(V_5\text{-}V_4)$ .The shape parameter m and n serve to effect local control in the curve: m and n as increases, the curve moves in the direction of edges $(V_1\text{-}V_0)$ and $(V_5\text{-}V_4)$ and as m and n decreases, the curve moves in the opposite direction to the edges $(V_1\text{-}V_0)$ and $(V_5\text{-}V_4)$.

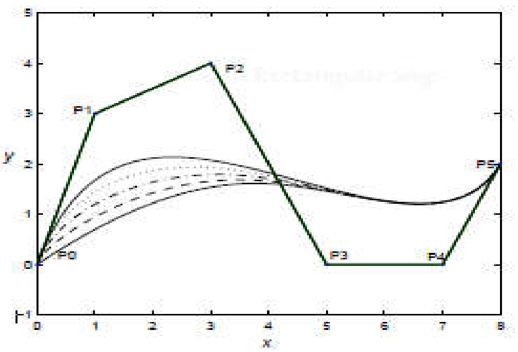


Fig. 3 Effect on the shape of the curve with altering the values of shape parameter *m*.

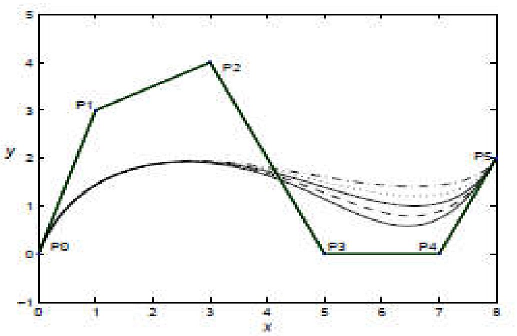


Fig. 4 Effect on the shape of the curve with altering the values of shape parameter *n*.

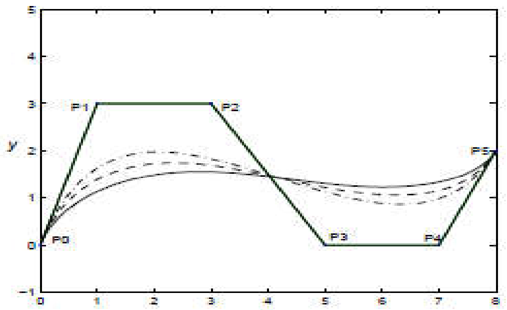


Fig. 5 Effect on the shape of the curve with altering the values of shape parameters m and n simultaneously.

Approximability

Control polygon provides an important tool in geometric modeling. It is an advantage if the curve being modeled tends to preserve the shape of its control polygon. Now we show the relations of the cubic trigonometric Polynomial B-spline curve and cubic Bézier curves corresponding to their control polygon.

Suppose V_0,V_1,V_2,V_3,$V_4$and V_5 are not collinear; the relationship between quintic trigonometric Polynomial B-spline curve $C_\beta(t)$ and the quintic Bézier curve $B(t)=\sum_{j=0}^{5} V_i(5,j)(1-t)^{5-j}t^{j}$; $t \in [0, 1]$ with the same control points V_i (i=0,1,2,3,4,5) are given by

$$C_\beta\left(\frac{\pi}{4}\right)-aV^*=\frac{1}{2\sqrt{2}}\begin{bmatrix}\left(\sqrt{2}-1\right)^3(1-m)V_0+\left(\sqrt{2}-1\right)^2\left(3+\left(\sqrt{2}-1\right)m\right)V_1+\\ \left(\sqrt{2}-1\right)^2\left(3+\left(\sqrt{2}-1\right)n\right)V_4+\left(\sqrt{2}-1\right)^3(1-m)V_5\end{bmatrix}$$

$$\text{where}V^*=\left(V_2+V_3\right);a=\frac{\left(\sqrt{2}-1\right)}{2\sqrt{2}};b=-\left(\frac{5}{16}\right)$$

and

$$B\left(\frac{1}{2}\right)-bV^*=\frac{1}{32}\left(V_0+5V_1+5V_4+V_5\right)$$

These equations shows that quintic Bézier Curves can be made closer to the control polygon by altering the values of shape parameters.

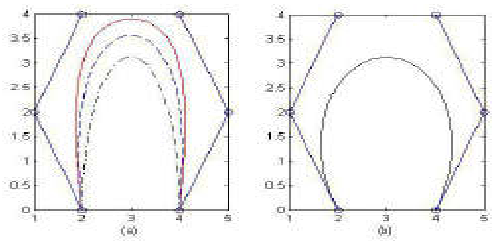


Figure 6: The relationship between the quintic trigonometric Bezier curve and quintic Bezier curve.

CONCLUSION

The trigonometric polynomial blending functions constructed in this paper have the properties analogous to those of the quintic Bernstein basis functions and the trigonometric Bezier curves are also analogous to the quintic Bezier ones. In this basis we included the tension parameter which is mainly important for object visualization. The trigonometric Bezier curves are close to the control polygon. Therefore, these trigonometric Bezier curves can preserve the shape of the control polygon. For any shape parameters satisfying the shape preserving conditions, the obtained shape preserving trigonometric interpolation spline curves are all continuous. There is no need to solve a linear system and the changes of a local shape parameter will only affect two curve segments. The shape of the curve can be adjusted using the values of the shape parameters.. Further, it can be extended to tensor product surfaces generalizing this idea to quasi-interpolation with trigonometric spline curve.

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