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Research Article

EXTREMES IN FUNCTION SPACES FOR SOLUTION OF OPTIMIZATION

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ABSTRACT

Authors divided the exploration extreme in functions and functional for the purposes of optimization solutions. Shows that the search extremes in functionals consists in algorithmization calculations. Whereas the search for the extremes of the function is a static optimization consisting mainly of linear programming and nonlinear.

INTRODUCTION

Preliminaries

Theorem 1.1 For the function $f(z) = X(x,y) + iY(x,y)$ wherein "z" contained in R, obtained derivative (Szu-Hoa Min 1944):

$$f'(z) = \lim_{\delta\to 0}\frac{f(z+\delta) - f(z)}{\delta}$$

Theorem 1.2 However, when $X(x,y) + iY(x,y)$ have a continuous partial derivatives of the first row then: $z_0 = x_0 + iy_0$, where the substitution is obtained:

$$\lim_{\delta\to 0}\frac{f(z_0+\delta) - f(z_0)}{\delta}$$

Szu-Hoa Min (1944) assuming that for $\delta = k + ?$ pointed that the result of this action is as follows:

$$X_x^{\theta} = X_x(x_0 + \theta ?, y_0 + \theta k), \quad Y_x^{\theta'} = Y_x(x_0 + \theta' ?, y_0 + \theta' k)$$

To sum up:

- Theorem 1.1 assuming the existence of the form f(z) as a function of non-analytical, its derivative can not exist.
- Theorem 1.2 gives new information if k and h.

Urban (1908), Ban and Gal (2002) same as Szu-Hoa Min (1944) concluded that it is wrong to combination of the function non-analytical and analytical in one form analytic function. Urban (1908) concluded that the analyticity function is a "basis for an infinite number of claims creating a finite number of observations." In any normed spaces suited assumption (series) Taylor for the function of the real-valued or vector.

Theorem 2.1 Taylor's theorem means that if a function f transforms a closed set [a, b] in a normed space Y ($f: [a,b] \to Y$) will become (n+1) times differentiation in a continuous manner, and then each point x of the interval (a, b) will be expressed by the formula of Taylor (Kreminski 2010, Domański 2016):

$$f(x) = f(a) + \frac{(x-a)}{1!}f^{(1)}(a) + \frac{(x-a)^2}{2!}f^{(2)}(a) + \cdots + \frac{(x-a)^n}{n!}f^{(n)}(a) + R_n(x,a)$$

$R_n(x,a)$ is the rest (peano), estimated for $x \in [a,b]$:

$$\|R_n(x,a)\| \le \frac{M}{(n+1)!}|x-a|^{n+1}$$

assuming that $M \ge 0$:

$$\|f^{(n+1)}(x)\| \le M$$

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In the above formula to the right of the symbol „=" occurs a polynomial (algebraic sum) Taylor. Depending on the subsequent derived differentiation the function ensue at a sufficiently low rest (Deadman and Relton, 2016).

Theorem 2.2 Conversely, where a=0, can obtain formulas McLaurin (Pogány and Tomovski 2006):

$$f(x) = f(0) + \frac{f^{(1)}(0)}{1!}x^1 + \dots + \frac{f^{(n-1)}(0)}{(n-1)!}x^{n-1} + \frac{f^{(n)}(c)}{n!}x^n$$

Theorem 2.3 In cases where function f transforms a set D which is a subset or equal to R ($D \subseteq R$) in a normed space Y ($f: D \to Y$) having derivatives of any a row in point $x_0 \in D$ appears a Taylor series (Domanski 2016), where it is assumed $f^{(0)}(x_0) = f(x_0)$:

$$\sum_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(x_0)(x - x_0)^n$$

Functions and Their Extremes

Theorem 3.1 The function f:

$f: A \to R$
where:
$A \subseteq R^n$

Optimization procedure seeks a value such that $x^* \in A$. However, for each $x \in A \setminus \{x^*\}$, wherein finding the maximum of function occurs:

$$f(x) < f(x^*)$$

Finding the minimum of the function occurs:

$$f(x) > f(x^*)$$

Theorem 3.2. In search extreme can be used a mathematical model describing the optimization. Let X - state space (any finite set), $f: X \to R$ - objective function:

$min\ F(x)\ , x \in X_0 \subseteq X$
or
$max\ F(x)\ , x \in X_0 \subseteq X$

To sum up:

- Theorem 3.1 is simple, but the function at some real problems is complicated, making it difficult to find the optimum (extreme).
- On the basis of Theorem 3.1. the mathematical description of the problem to the search for extremes requires bring to the model of optimize the functional spaces. Extreme value depending on the formulation of the task will be the largest and the smallest, but always extreme (extreme definition of a function - optimization of static).
- Theorem 3.2 by optimization task is understood as finding a point X_0 from set X which is the the largest or the smallest.
- Find the optimum complex functions it is possible using optimization algorithms.

Prospecting Extreme of Function

Extremum of the function is the optimal point at which the value of the objective function is the best. It may be the largest and the smallest value. Tasks the search for extreme of function is called static optimization. Can be distinguished groups such as: continuous programming, linear programming and nonlinear programming.

Theorem 4.1 The definition of continuous programming describes the model:

$F(x): R^n \Rightarrow R^1$
$x \in X_0 \subseteq X$

It can be carried out without restrictions and with restrictions (Pichler and Tomasgard 2016).

Theorem 3.2 The task without restrictions amounts to form:

$$X_0 = X = R^n$$

The exception is the search for one-dimensional (n=1):

$X = R^1, Y = R^1$
$F(x): R^1 \to R^1$

Theorem 3.3 The tasks with restrictions calculated according to the formula:

$X_0 = \{x: g(x) = 0, h(x) \leq 0\}$
$g(x): R^n \to R^m, h(x): R^n \to R^p$

The theorem of linear programming form of the objective function is always a linear (Renegar 1995):

$$f = c_0 + c_1x_1 + c_2x_2 + \dots + c_nx_n$$

where: x_i – real number

The task of maximizing the objective function:

$$F(x) = C^T x$$

The task of linear programming can exist of discrete character in the objective function form (Renegar 1994, 1995):

$F(x): Z^n \Rightarrow R^1$
where:
$X = Z^n \subseteq R^n$
$Y = R^1$

$X_0 = X$ (without restrictions) , $X_0 \subset X$ (with restrictions)
For binary programming:

$$X = B^n \subset Z^n$$

Prospecting Extreme of Functionals

Knapsack problem. With the definition that is searched for such a subset in which the total value (cj) was the largest and also the sum of weights (wj) is not greater, and possibly equal from the listed capacity backpack.

Theorem 5.1 The value of cj and the size (weight) wj at each element in the set of N is specified. Maximization problem takes the following form:

$$\sum_{j=1}^{N} c_j x_j$$

$$\sum_{j=1}^{N} w_j x_j \leq B, \qquad x_j = 0\ lub\ 1, \quad j = 1, \dots, n$$

gdzie: *B* – the maximum capacity of the backpack

When the number of components is reduced, the assumption of maximize consists:

$$\sum_{j=1}^{N} w_j x_j \le B, \quad 0 \le x_j \le b_j, \quad j = 1, \dots, n$$

The calculus of variations. The search for extremes (functional) in functional spaces can be achieved in accordance with the question of the calculus of variations. Functionals in this space reflect the number of real, measurable, which usually constitute the definite integral, stationary, sometimes imported to a differential form. However, the task of variational calculus is a solution in given area and differential equations searching the area of given point.

Theorem 5.2 The basic equation of the calculus of variations is an equation Eulera-Lagrange'a, in which the solution takes the form:

$$\frac{d}{dt}\frac{\partial L}{\partial x'} - \frac{\partial L}{\partial x} = 0$$

Whereby the functional is defined as follows:

$$S = \int_{t_1}^{t_2} L\left(x(t), x'(t), t\right) dt$$

were:

x(t) - functions
S - stationarity

For the function x (t), S must take extreme. The function with extreme values of definite integral is solutions of the equation calculus of variations. Eulera-Lagrange'a method used in the calculus of variations, looking for the road, which will be the shortest, minimize costs (outlays).

Assuming the existence of a few independent variables analyzed process or phenomenon, which is only one dependent variable in the equation, the problem of variations rely to find extreme functional of the form:

$$J = \int f(v, v_x, v_y, v_z, x, y, z)\,dx\,dy\,dz$$

where:
$v \equiv v(x, y, z)$,
$v(x, y, z)$ - searched function, in which the value of functional is extreme

Whereas, the variance assumes the form:

$$\delta v = \delta v(x, y, z) = V(x, y, z) - v(x, y, z)$$

where:
$V(x, y, z)$ - comparison function

The solution of calculus of variations equation results with finding a function representing examined phenomenon in such a way that its functional, or integral depending has been minimized to a suitable degree.

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