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Research Article

BOUNDEDNESS PROPERTY FOR DISTRIBUTIONAL FST & 2DFMT

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ABSTRACT

The Boundedness Theorem which states that a continuous function *f* in the closed interval $[a, b]$ is bounded on that interval i.e. there exist real numbers m and M such that:

$m \leq f\ x \leq M, \quad for\ all\ x \in [a, b]$

The continuous function f is said to be bounded as domain I is closed & bounded interval $I = [a, b]$. Every transformation is linear and bounded.

In the present paper, the Fourier-Stieltjes Transform and Two Dimensional Fourier-Mellin Transform is extended in the distributional generalized sense. The Boundedness theorem for the Fourier-Stieltjes Transform (FST) and Two Dimensional Fourier-Mellin Transform(2DFMT) are proved.

INTRODUCTION

Transform methods providing unifying mathematical approach to the study of electrical, electronics, network, devices for energy conversion and control, antennas and other component of electrical system. These are equally applied to the subject of electrical communication by wire or optical fibers, to wireless radio propagation [1]. Other theoretical techniques are used in handling the basic fields of engineering, but integral transform method are virtually indispensible in all of them. Integral transforms are most powerful technique for different fields of sciences and their applications are more useful. Fourier Transform plays an important part in the theory of many branches of science.

The Fourier Transform is a tool for solving physical problems. It is applied to optics, crystallography, solving science problems. Fourier Transform can be strongly used in acoustics. It is used to understand how different musical instruments create their different sounds. The Fourier Transform occurs naturally all throughout physics [5].

Stephane Derrode addresses the gray-level image representation ability of the Fourier–Mellin transform (FMT) for pattern recognition, reconstruction and image database retrieval. It is also used in digital signal and image processing [2].

Stieltjes transforms can be used to express more intuitive performance measures of communication systems such as signal-to-interference-and-noise ratios and channel capacity [6]. Stieltjes-transform method is considered today as one of the most practical and powerful tools for handling large random matrices in wireless communications research [7].

In the present paper, we generalized Fourier-Stieltjes Transform and Two dimensional Fourier-Mellin transform in distributional sense. The distributional Fourier-Stieltjes Transform is defined in [8, 11, 12] as-

$$F\ s,p = FS\ f\ t,x = \langle f\ t,x\ , e^{-ist}(x+y)^{-p}\rangle,$$

Where, for some $s > 0, k > Re\ p$ and each fixed t $0 < t < \infty$ $, x(0 < x < \infty)$ and $f\ t,x \in FS^*_\alpha$ to $e^{-ist}(x+y)^{-p} \in FS_\alpha$.

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Also, Distributional Two Dimensional Fourier-Mellin Transform is defined in [9, 10, 13] as-

$F(s,u,p,v) = FM\{f(t,l,x,y)\} = \langle f(t,l,x,y), e^{-i(st+ul)}x^{p-1}y^{v-1}\rangle$

Where, for some $s > 0, u > 0$, $a < Re\, p < b, c < Re\, v < d$ and each fixed $t\ (0 < t < \infty), l\ (0 < l < \infty), x\ (0 < x < \infty), y\ (0 < y < \infty)$ and $f(t,l,x,y) \in FM_{a,b,c,d,\alpha}$ to $e^{-i(st+ul)}x^{p-1}y^{v-1} \in FM_{a,b,c,d,\alpha}$.

In this paper, we have proved the Boundedness theorem for the Distributional Fourier-Stieltjes Transform and Two Dimensional Fourier-Mellin Transform.

Outline of this Paper as follows

Section 2 gives the Boundedness theorem for the Distributional Two Dimensional Fourier-Mellin transform. The Boundednes theorem for Distributional Fourier-Stieljes Transform is proved in section 3. Lastly we conclude the present paper. Notation and terminology is given as per A.H. Zemanian [3,4].

Boundedness Theorem for Distributional Two Dimensional Fourier-Mellin Transform

Theorem: Let $f(t,l,x,y) \in FM^{*}_{a,b,c,d,\alpha}$ and $F(s,u,p,v) = FM\{f(t,l,x,y)\} = \langle f(t,l,x,y), e^{-i(st+ul)}x^{p-1}y^{v-1}\rangle$, $a < Re\, p < b, c < Re\, v < d, s > 0, u > 0$. Let $supp\, f(t,l,x,y) \in S_A \cap S_B$ such that- $S_A = \{t,l: t,l \in R^n, |t|,|l| \le A, A > 0\}$ and $S_B = \{x,y: x,y \in R^n, |x|,|y| \le B, B > 0\}$, then for each $\epsilon > 0, \delta > 0$ there exist a constant $C > 0$ and non-negative n such that-

$$|F(s,u,p,v)| \le C(1+|s|^q)(1+|u|^j)\, e^{(|s|+|u|)(A+\epsilon)} \max_{\substack{0\le h\le n\\ 0\le g\le n}} (B+\delta)^{p+v}$$

Proof

Suppose that $supp\, f(t,l,x,y) \in S_A \cap S_B$ and let $\epsilon > 0, \delta > 0$. Choose ρ_1 and $\rho_2 \in \mathcal{D}(R^n)$ such that $\rho_1(t).\rho_2(l) = 1$ on a neighborhood of S_A and supp $\rho_1 \subset S_{A+\epsilon}$ and supp $\rho_2 \subset U_{A+\epsilon}$.

Since $f(t,l,x,y) \in FM^{*}_{a,b,c,d,\alpha}$ and in view of boundedness property of generalized functions, there exist a constant C and a non-negative integer n such that-

$$|F(s,u,p,v)| = |\langle f(t,l,x,y), e^{-i(st+ul)}x^{p-1}y^{v-1}\rangle|$$
$$= |\langle f(t,l,x,y), \rho_1(t).\rho_2(l)\, e^{-i(st+ul)}x^{p-1}y^{v-1}\rangle|$$

$$\le C_1 \max_{\substack{0\le q\le n\\ 0\le h\le n\\ 0\le j\le n\\ 0\le g\le n}} \gamma_{a,b,c,d,k,r,h,g,q,j}[\rho_1(t).\rho_2(l)\, e^{-i(st+ul)}x^{p-1}y^{v-1}]$$

$$\le C_1 \max_{\substack{0\le q\le n\\ 0\le h\le n\\ 0\le j\le n\\ 0\le g\le n}} \sup_{I_1} |t^k l^r \xi_{a,b}(x)\, x^{h+1}\eta_{c,d}(y)\, y^{g+1} D_t^q D_x^h D_l^j D_y^g \rho_1(t).\rho_2(l)\, e^{-i(st+ul)}x^{p-1}y^{v-1}|$$

$$\le C_1 \max_{\substack{0\le q\le n\\ 0\le h\le n\\ 0\le j\le n\\ 0\le g\le n}} \sup_{I_1} |t^k D_t^q[\rho_1(t)\, e^{-ist}]\, l^r D_l^j[\rho_2(l)\, e^{-iul}]\, \xi_{a,b}(x)\, x^{h+1}\eta_{c,d}(y)\, y^{g+1} D_x^h D_y^g\, x^{p-1}y^{v-1}|$$

$$\le C_1 \max_{\substack{0\le q\le n\\ 0\le h\le n\\ 0\le j\le n\\ 0\le g\le n}} \sup_{I_1} |t^k \sum_v \binom{q}{v} D_t^{q-v}\rho_1(t)\, D_t^v e^{-ist}\, l^r \sum_w \binom{j}{w} D_l^{j-w}\rho_2(l)\, D_l^w e^{-iul}\xi_{a,b}(x)\, x^{h+1}\eta_{c,d}(y)\, y^{g+1} D_x^h x^{p-1} D_y^g y^{v-1}|$$

$$\le C_1 t^k e^{|s|(A+\epsilon)} C_2 l^r e^{|u|(A+\epsilon)} \max_{\substack{0\le q\le n\\ 0\le h\le n\\ 0\le j\le n\\ 0\le g\le n}} \sum_v \binom{q}{v} |s|^v \sum_w \binom{j}{w} |u|^w\, |\xi_{a,b}(x) P(p)\, x^p \eta_{c,d}(y) V(v) y^v|$$

$$\le C_1(1+|s|^q)t^k e^{|s|(A+\epsilon)} C_2(1+|u|^j) l^r e^{|u|(A+\epsilon)} \max_{\substack{0\le h\le n\\ 0\le g\le n}} |\xi_{a,b}(x) P(p)\, x^p\eta_{c,d}(y) V(v) y^v|$$

$$\le C_1(1+|s|^q)\, e^{|s|(A+\epsilon)} C_2(1+|u|^j)\, e^{|u|(A+\epsilon)} t^k \max_{0\le h\le n} |\xi_{a,b}(x) P(p)\, x^p|\, l^r \max_{0\le g\le n} |\eta_{c,d}(y) V(v) y^v|$$

$$\le C_1(1+|s|^q)\, e^{|s|(A+\epsilon)} C_2(1+|u|^j)\, e^{|u|(A+\epsilon)} C_3 \max_{0\le h\le n}(B+\delta)^p\, C_4 \max_{0\le g\le n}(B+\delta)^v$$

$$\{\text{Since}, C_3 = t^k|\xi_{a,b}(x) P(p)|, C_4 = l^r|\eta_{c,d}(y) V(v)|\}$$

$$\le C(1+|s|^q)\, e^{|s|(A+\epsilon)}(1+|u|^j)\, e^{|u|(A+\epsilon)} \max_{\substack{0\le h\le n\\ 0\le g\le n}}(B+\delta)^{p+v}$$

$$|F(s,u,p,v)| \le C(1+|s|^q)(1+|u|^j)\, e^{(|s|+|u|)(A+\epsilon)} \max_{\substack{0\le h\le n\\ 0\le g\le n}}(B+\delta)^{p+v}$$

Hence Proved.

Boundedness Theorem for Distributional Fourier-Stieltjes Transform

Theorem: Let $f(t,x) \in FS_{\alpha}^{\prime}$ and $F(s,p) = FS\{f(t,x)\} = \langle f(t,x), e^{-ist}(x+y)^{-p}\rangle, \boldsymbol{k} > Re\, \boldsymbol{p},\ s > 0$. Let supp $f(t,x) \subset S_A \cap S_B$ such that-
$S_A = \{t: t \in R^n, |t| \leq A, A > 0\}$ and $S_B = \{x: x \in R^n, |x+y| \leq B, B > 0, y\ is\ some\ constant\}$, then for each $\epsilon > 0, \delta > 0$ there exist a constant $C > 0$ and non-negative n such that-$|F(s,p)| \leq C(1+|s|)^l\, e^{|s|(A+\epsilon)} \max_{0\leq q\leq n}(B+\delta)^{-(p+q)}$

Proof

Suppose that $supp\, f(t,x) \subset S_A \cap S_B$ and let$\epsilon > 0, \delta > 0$. Choose $\rho \in \mathcal{D}(R^n)$ such that $\rho(t) = 1$ on a neighborhood of S_A and supp $\rho \subset S_{A+\epsilon}$.
Since $f(t,x) \in FS_{\alpha}^{\prime}$ and in view of boundedness property of generalized functions, there exist a constant C and a non-negative integer n such that-

$$|F(s,p)| = \langle f(t,x), e^{-ist}(x+y)^{-p}\rangle$$
$$= \langle f(t,x), \rho(t)\, e^{-ist}(x+y)^{-p}\rangle$$
$$\leq C_1 \max_{\substack{0\leq l\leq n\\0\leq q\leq n}} \gamma_{k,p,l,q}\, \rho(t)\, e^{-ist}(x+y)^{-p}$$
$$\leq C_1 \max_{\substack{0\leq l\leq n\\0\leq q\leq n}} \sup_{I_1} t^k (1+x)^p D_t^l (xD_x)^q \rho(t)\, e^{-ist}(x+y)^{-p} \leq C_1 \max_{\substack{0\leq l\leq n\\0\leq q\leq n}} \sup_{I_1} \left| t^k \sum_v \binom{l}{v} D_t^{l-v}\rho(t)\, D_t^v\, e^{-ist}(1+x)^p x^q (D_x)^q (x+y)^{-p}\right|$$
$$\leq C_1 \max_{\substack{0\leq l\leq n\\0\leq q\leq n}} \sup_{I_1} \left| t^k \sum_v \binom{l}{v} D_t^{l-v}\rho(t)\,(-is)^v\, e^{-ist}\,(1+x)^p x^q P(p)(x+y)^{-p-q}\right|$$
$$\leq C_1 t^k e^{|s|(A+\epsilon)} \max_{\substack{0\leq l\leq n\\0\leq q\leq n}} \sum_v \binom{l}{v} |s|^v \left|(1+x)^p x^q P(p)(x+y)^{-(p+q)}\right|$$
$$\leq C_1 (1+|s|)^l\, t^k e^{|s|(A+\epsilon)} \max_{0\leq q\leq n} \left|(1+x)^p x^q P(p)(x+y)^{-(p+q)}\right|$$
$$\leq C_1 (1+|s|)^l\, e^{|s|(A+\epsilon)} t^k \max_{0\leq q\leq n} \left|(1+x)^p x^q P(p)(x+y)^{-(p+q)}\right|$$
$$\leq C_1 (1+|s|)^l\, e^{|s|(A+\epsilon)} C_2 \max_{0\leq q\leq n}(B+\delta)^{-(p+q)} \qquad \{\text{Since}, C_2 = t^k|(1+x)^p x^q P(p)|\}$$
$$|F(s,p)| \leq C(1+|s|)^l\, e^{|s|(A+\epsilon)} \max_{0\leq q\leq n}(B+\delta)^{-(p+q)} \qquad \{C_1 . C_2 = C\}$$

Hence Proved.

CONCLUSION

In the present paper we have proved the Boundedness theorem for Distributional Two dimensional Fourier-Mellin transform and for Distributional FourierStieltjes Transform.

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