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# **Research Article**

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# **ON QUASI NANO P-NORMAL SPACES**

# Adinatha C. Upadhya

Department of Mathematics, Sangolli Rayanna First Grade Constituent College, Belagavi, Karnataka, India

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#### ABSTRACT

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The aim this paper is to study a weaker version of nano p-normality called quasi nano p-normality. We obtain some properties, various characterizations and also, we establish various preservation Received 10<sup>th</sup> March, 2017 theorems of quasi nano p-normality under nano continuous. Received in revised form 14th April, 2017 Accepted 08th May, 2017

Key Words:

Nanotopology, nano-open, nano  $\pi$ -closed, nano p-closed, nano p- normal, nanocontinuous and nano  $\pi$ -continuous.

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## 1.Introduction

L.Thivagar [4,5] introduced the concept of nano topological spaces with respect to a subset X of a universe which is defined in terms of lower and upper approximations of X. The elements of a nano topological space are called the nano- open sets. He has also studied nano closure and nano interior of a set. Later he was introduced and studied the certain weak forms of nano-open sets namely nano  $\alpha$ -open sets, nano semi-open sets etc. In 2014, K. Bhuvaneswari et. al.[2] defined and studied the class of nano gp-closed and nano pg- closed sets. In this paper, we introduced a weaker version of nano pnormality called quasi nano p-normality and obtained some properties, various characterizations and also, we established various preservation theorems of quasi nano p-normality under nano continuous

## 2. Preliminaries

Definition 2.1/6]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be approximation space. Let  $X \subseteq U$ .

The lower approximation of X with respect to R is the 1. set of all objects, which can be for certain classified as X with respect to R and its is denoted by LR(X). That

is  $LR(X) = \bigcup_{x \in u} \{R(x): R(x) \subseteq X\}$ , where R(x) denotes

the equivalence class determined by x.

2 The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and its is denoted by UR(X). That

is UR(X) =  $\bigcup_{x \in u} R(x)$ : R(x)  $\cap X \neq \emptyset$ }, where R(x) denotes the equivalence class determined by x.

The boundary region of X with respect to R is the set 3. of all subjects, which can be classified neither as X nor as not X with respect to R and it is denoted by BR(X). That is BR(X) = UR(X) - LR(X).

Property 2.2 [6]: If (U, R) is an approximation space and X,  $Y \subseteq U$ , then

- a.  $L_{\mathbb{R}}(X) \subseteq X \subseteq U_{\mathbb{R}}(X)$ .
- b.  $L_{R}(\emptyset) = U_{R}(\emptyset) = \emptyset$  and  $L_{R}(U) = U_{R}(U) = U$ .
- c.  $U_{R}(X \cup Y) = U_{R}(X) \cup U_{R}(Y)$
- d.  $U_{R}(X \cap Y) \subseteq U_{R}(X) \cap U_{R}(Y)$

<sup>\*</sup>Corresponding author: Adinatha C. Upadhya

Department of Mathematics, Sangolli Rayanna First Grade Constituent College, Belagavi, Karnataka, India

- e.  $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
- f.  $L_{R}(X \cap Y) \subseteq L_{R}(X) \cap L_{R}(Y)$
- g.  $L_R(X) \subseteq L_R(Y)$  and  $U_{R(X)} \subseteq U_R(Y)$ , where  $X \subseteq Y$
- h.  $U_R(X^c) \subseteq [L_R(X)]^c$  and  $L_R(X^c) \subseteq [U_R(X)]^c$ .
- i.  $U_R U_R(X) = L_R U_R(X) = U_R(X)$
- j.  $L_R L_R(X) = U_R L_R(X) = L_R(X)$

**Definition 2.3 [4]:** Let U be the universe, R be an equivalence relation on U and  $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ Where  $X \subseteq U$ . Then by property 2.3  $\tau_R(X)$  satisfies the following axioms:

- a. U and  $\emptyset \in \tau_{R}(X)$ ,
- b. The union of the elements of any sub collection of  $\tau_{R}(X)$  is in  $\tau_{R}(X)$ .
- c. The intersection of the elements of any finite sub collection of  $\tau_R(X)$  is in  $\tau_R(X)$ . That is,  $\tau_R(X)$  is a topology on U called the nano topology on U with respect to X. We call  $(U, \tau_R(X))$  as the nano topological space. The elements  $\tau_R(X)$  are called as nano- open sets.

**Remark 2.4 [4]:** If  $\tau_R$  (X) is the nano topology on U with respect to X, then the set  $B = \{U, L_R(X), B_R(X)\}$  is the basis for  $\tau_R(X)$ .

**Definition 2.5 [4]:** If  $(U, \tau_R(X))$  is an nano topological space with respect to X where  $X \subseteq U$  and if  $A \subseteq U$ , then then nano interior of A is defined as the union of all nano -open subsets of A and it is denoted by NInt(A).

**Definition 2.6[4]:** If(U,  $\tau_R(X)$ ) is an nano topological space with respect to X where  $X \subseteq U$  and if  $A \subseteq U$ , then the nano closure of A is defined as the intersection of all nano -closed sets containing A and it is denoted by NCl(A).

**Definition 2.7[5]:** If  $(U, \tau_R(X))$  is a nano topological space and  $A \subseteq U$ . Then A is said to be

- a. Nano semi -open if  $A \subseteq NCl(NInt(A))$
- b. Nano pre- open (briefly nano p-open) if A⊆ NInt(NCl(A))
- c. Nano  $\alpha$  open if A  $\subseteq$  NInt(NCl(NInt(A)))

NSO(U, X), NPO(U, X) and N $\alpha$ O(U, X) respectively we denote the families of all nano semi-open, nano pre-open and nano  $\alpha$ -open subsets of (U,  $\tau_R(X)$ ).

Let  $(U, \tau_R(X))$  be a nano topological space and  $A \subseteq U$ . A is said to be nano semi- closed, nano pre-closed and nano  $\alpha$ -closed if its complement is respectively nano semi-open, nano pre-open and nano  $\alpha$ -open.

**Definition 2.8[1]:** A subset A of  $(U, \tau_R(X))$  is called nano generalized closed set (briefly Ng-closed) if Ncl(A)  $\subseteq$  V whenever A  $\subseteq$  V and V is nano open in(U,  $\tau_R(X)$ ).

**Definition 2.9[2]:** A subset A of (U,  $\tau_R(X)$ ) is called nano generalized pre closed set (briefly Ngp-closed) if Npcl(A)  $\subseteq$  V whenever  $A \subseteq V$  and V is nano open in (U,  $\tau_R(X)$ ).

**Definition 2.10:** A subset A of  $(U, \tau_R(X))$  is called nano  $\pi$ -generalized pre-closed set (briefly N $\pi$ gp-closed) if Npcl(A)  $\subseteq$  V whenever A  $\subseteq$  V and V is nano  $\pi$ -open in (U,  $\tau_R(X)$ ).

**Definition 2.11:** A subset A of  $(U, \tau_R(X))$  is called an nano  $\pi$ -closed if it is a finite intersection of nano closed domain sets.

**Definition 2.12:** A subset A of  $(U, \tau_R(X))$  is called an nano  $\pi$ -open if its complement is nano  $\pi$ -closed.

**Definition 2.13:** A subset A of  $(U, \tau_R(X))$  is said to be an nano p-neighborhood of u, if there exists a nano-open set V such that  $u \in V \subseteq A$ .

**Definition 2.14:** A function f:  $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is said to be

- almost nano continuous (resp. nano rc-continuous) [3] if f<sup>1</sup>(F) nano closed (resp. nano closed domain) set in U for each nano closed domain subset Fof V.
- 2. almost nano closed (resp. nano rc-preserving) [3] function on if f(F) is nano closed (resp. nano closed domain) set in V for each nano closed domain subset F of U.
- weakly nano open if for each nano open subset F of U, f(F) ⊆ NInt(f(F))
- a nano R-map (resp. completely nano continuous) [3] if f<sup>-1</sup>(F) is nano open domain in U for every nano open domain (resp. nano open) subset F ofV
- 5. a nano  $\pi$  -continuous (resp. nano rc-continuous) [3] if f<sup>1</sup>(A)is nano  $\pi$  -closed (resp. nano closed domain) set in U for each nano closed set (resp. nano closed domain) set A in V.
- 6. nano pre gp closed if f(F) is nano gp-closed set in V for every nano p-closed subset F of U.
- nano pre gp-continuous if f<sup>1</sup>(F)is nano gp-closed set U for every nano p- closed subset F of V.

### 3. Quasi Nano p - Normal Spaces

**Definition 3.1:** A nano topological  $(U, \tau_R(X))$  is said to be a nano p-normal if for any pair of disjoint nano closed sets A and B of U, there exist disjoint nano p-open sets V and W of U such that  $A \subseteq V$  and  $B \subseteq W$ .

We now introduce the following definition.

**Definition 3.2:** A nano topological  $(U, \tau_R(X))$  is said to be quasi nano p-normal if for every pair of disjoint nano  $\pi$ -closed subsets A and B of U, there exist disjoint nano p-open subsets V and W of U such that  $A \subseteq V$  and  $B \subseteq W$ .

**Theorem 3.3:** For a nano topological (U,  $\tau_R(X)$ ), the following are equivalent.

- 1. U is quasi nano p- normal space.
- 2. for every pair of nano  $\pi$ -open subsets V and W of U whose union is U, there exist nano p-closed subsets G and H of U such that  $G \subseteq V$  and  $H \subseteq W$  and  $G \cup H=U$ .
- 3. For any nano  $\pi$ -closed set A and each nano  $\pi$ -open set B such that  $A \subseteq B$ , there exists an nano p- open set V such that  $A \subseteq V \subseteq NpCl(V) \subseteq B$ .

4. For every pair of disjoint nano π-closed subsets A and B of U there exist nano p- open subsets V and W of U such that A ⊆ V, B ⊆ W and NpCl(V) ∩ NpCl(W) = φ.

**Proof:** (1) $\Rightarrow$ (2): Let V and W be any nano  $\pi$ -open subsets of a quasi nano p- normal space U such that  $V \cup W = U$ . Then, U\V and U \W are nano  $\pi$ -closed subsets of U. By quasi nano  $\pi$ -normality of U, there exist disjoint nano  $\pi$ -open subsets V<sub>1</sub> and  $W_1$  of U such that  $U \setminus V \subseteq V_1$  and  $U \setminus W \subseteq W_1$ . Let G =  $U \setminus V_1$  and H= U \setminus W\_1. Then G and H are nano p-closed subsets of U such that  $G \subseteq V$  and  $H \subseteq W$  and  $G \cup H = U$ . (2) $\Rightarrow$ (3): Let A be a nano  $\pi$ -closed set and B be an nano  $\pi$ -open subset of U such that A  $\subseteq$  B. Then, U \A and B are nano  $\pi$ -open subsets of U whose union is U. Then by (2), there exist nano p-closed sets G and H of U such that  $G \subseteq U \setminus A$  and  $H \subseteq B$  and  $G \cup H = U$ . Then  $A \subseteq U \setminus G$ ,  $B \subseteq U \setminus H$  and  $(U\backslash G) \cap (U\backslash H) = \phi$ . Let V= U\G and W= U\H. Then V and W are disjoint nano p-open sets such that  $A \subseteq V \subseteq U \setminus W \subseteq B$ . Since U\W is nano p-closed, then we have  $NpCl(V) \subseteq U \setminus W$ . Thus,  $A \subset V \subset NpCl(V) \subset B$ .

 $(3) \Rightarrow (4)$ : Let A and B are any two disjoint nano  $\pi$ -closed sets of U. Then  $A \subseteq U \setminus B$  where U\B is nano  $\pi$ -open. Then, by (3), there exists a nano p- open subset V of U such that  $A \subseteq V \subseteq NpCl(V) \subseteq U \setminus B$ . Let  $W = U \setminus NpCl(V)$ . Then W is nano p-open subset of U. Thus, we obtain,  $A \subseteq V$ ,  $B \subseteq W$  and  $NpCl(V) \cap NpCl(W) = \phi$ .

 $(4) \Rightarrow (1)$ : Obvious.

**Proposition 3.4:** Let f: (U,  $\tau_R(X)$ )  $\rightarrow$  (V,  $\tau_R(Y)$ ) be a function then

- 1. The image of nano p-open subset under nano open nano continuous function is nano p-open.
- 2. The inverse image of nano p-open (resply. nano p-closed) subset under nano open nano continuous function is nano p-open.
- 3. The image of nano p-closed subset under nano open and an nano closed nano continuous surjective function is nano p-open.

Now, we prove the following result

**Theorem 3.5:** The image of quasi nano p-normal space under nano open nano continuous injective function is quasi nano p-normal.

**Proof:** Let U be a quasi nano p-normal space and let  $f:(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  be an nano open nano continuous injective function. We need to show that f(U) is quasi nano p-normal. Let A and B be two disjoint nano  $\pi$ -closed sets in f(U). Since the inverse image of an nano  $\pi$ -closed set under nano open, nano continuous function is nano  $\pi$ -closed, we have  $f^1(A)$  and  $f^1(B)$  are disjoint nano  $\pi$ -closed subsets in U. Since U is a quasi nano p-normal, there exists nano p-open subsets V and W of U such that  $f^1(A) \subseteq V$ ,  $f^1(B) \subseteq W$  and  $V \cap W = \phi$ . Since f is nano open nano continuous injective function, we have  $A \subseteq f(V)$ ,  $B \subseteq f(W)$  and  $f(V) \cap f(W) = \phi$ . By Proposition 3.3, f(V) and f(W) are disjoint nano p-open sets in f(U) such that  $A \subseteq f(V)$  and  $B \subseteq f(W)$ . Hence, f(U) is quasi nano p-open normal space.

From the above theorem, we obtain the following corollary.

*Corollary 3.6:* Quasi nano p-normality is a topological property.

#### Characterizations of quasi nano p-normal spaces

**Theorem 3.7:** For a space (U,  $\tau_R(X)$ ), the following are equivalent.

- 1. U is quasi nano p- normal.
- 2. For any disjoint nano  $\pi$ -closed subsets A and B of U there exist nano gp-open subsets V and W of U such that  $A \subseteq V$  and  $B \subseteq W$ .
- 3. For any disjoint nano  $\pi$ -closed subsets A and B of U there exist nano  $\pi$ gp-open subsets V and W of U such that  $A \subseteq V$  and  $B \subseteq W$ .
- 4. For every nano  $\pi$ -closed set A and every nano  $\pi$ -open set B such that A  $\subseteq$  B, there exists nano gp-open subset of V of U such that A  $\subseteq$  V  $\subseteq$  NpCl(V)  $\subseteq$  B.
- 5. For every nano  $\pi$ -closed set A and every nano  $\pi$ -open set B such that  $A \subseteq B$ , there exists nano  $\pi$ gp-open subset of V of U such that  $A \subseteq V \subseteq NpCl(V) \subseteq B$ .

**Proof:** (1)  $\Rightarrow$  (2): Let U be a quasi nano p-normal space. Let A and B be any disjoint nano  $\pi$ -closed subsets of U. By quasi nano p-normality of U, there exist disjoint nano p-open subsets V and W of U such that A  $\subseteq$  V and B  $\subseteq$  W. Thus, V and W are disjoint nano gp-open subsets U such that A  $\subseteq$  V and B  $\subseteq$  W.

(2) ⇒ (3): Suppose (2) holds. Let A and B be disjoint nano  $\pi$ -closed subsets of U. By (2), there exist disjoint nano gp-open subsets V and W of U such that A ⊆ V and B ⊆ W. Since every nano gp-open set is nano  $\pi$ gp-open, then V and W are disjoint  $\pi$ gp-open subsets of U such that A ⊆ V and B ⊆ W (3)⇒(4): Suppose (c) holds. Let A be an nano  $\pi$ -closed and B be an nano  $\pi$ -open subset of U such that A ⊆ B. Then, A ∩ U \B =  $\phi$ . Thus A and U\B are disjoint nano  $\pi$ gp- open subsets of U such that A ⊆ V and U\B ⊆ W. Therefore, we have A ⊆ NpInt(V), U\B ⊆ NpInt(W) and NpInt(V) ∩ NpInt(W) =  $\phi$ . Let G = NpInt(V). Then, G is a nano p-open subset of U and hence nano gp-open such that A ⊆ G ⊆ NpCl(G) ⊆ B.

 $(4) \Rightarrow (5)$ : Suppose (4) holds. Let A be an nano  $\pi$ -closed and B be an nano  $\pi$ -open subset of U such that  $A \subseteq B$ . By (4), there exists disjoint nano gp- open subset V of U such that  $A \subseteq V \subseteq \text{NpCl}(V) \subseteq B$ . Therefore, V is a nano  $\pi$ gp-open subset of V of U such that  $A \subseteq V \subseteq \text{NpCl}(V) \subseteq B$ .

 $(5) \Rightarrow (1)$ : Suppose (5) holds. Let A and B be disjoint nano  $\pi$ closed subsets of U. Then, we have  $A \subseteq U \setminus B$  where  $U \setminus B$  is nano  $\pi$ -open. By (5) there exists disjoint nano  $\pi$ gp- open subset V of U such that  $A \subseteq V \subseteq \text{NpCl}(V) \subseteq U \setminus B$ . Then, We obtain  $A \subseteq \text{NpInt}(V) \subseteq V \subseteq \text{NpCl}(V) \subseteq U \setminus B$ . Let G = NpInt(V) and  $H = U \setminus \text{NpCl}(V)$ . Then, G and H are disjoint nano p-open subsets of U such that  $A \subseteq G$ ,  $B \subseteq H$ . Hence, U is quasi nano p-normal.

**Definition 3.8:** A function f:  $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is said to be

1. almost nano p-irresolute if for each u in U and each nano p-neigibourhood W of f(u) in V, NpCl( $f^{1}(V)$ ) is a nano p-neighbourhood of u in U.

2. nano Mp-closed (nano Mp-open), if f(A) is nano p-closed (resp. nano p-open) set in V for each nano p-closed (resp. nano p-open) set A in U.

*Lemma 3.9:* A function f:  $(U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$  is weakly nano open nano continuous function, then f is nano Mp-open and nano R-map.

Next we prove the invariance of quasi nano p-normality in the following.

**Theorem 3.10:** If f:  $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is nano Mp-open nano rc- continuous and almost nano pre-irresolute function from a quasi nano p-normal space  $(U, \tau_R(X))$  on to a  $(V, \tau_{R'}(Y))$ , then  $(V, \tau_{R'}(Y))$  is quasi nano p-normal.

**Proof:** Let A be an nano  $\pi$ -closed and B be an nano  $\pi$ -open subsets of  $(V, \tau_R(Y))$  such that  $A \subseteq B$ . Then, by nano rc-continuity of f,  $f^1(A)$  be nano  $\pi$ -closed and  $f^1(B)$  be nano  $\pi$ -open subsets of  $(U, \tau_R(X))$  such that  $f^1(A) \subseteq f^1(B)$ .Since  $(U, \tau_R(X))$  is quasi nano p-normal, then by Theorm 3.2. There exists an nano p-open subset V of  $(U, \tau_R(X))$  such that  $f^1(A) \subseteq V \subseteq NpCl(V) \subseteq f^1(B)$ . Since f is nano Mp- open and an almost nano pre-irresolute surjection, it follows that f(V) is nano p-open subset of  $(V, \tau_R'(Y))$  and  $A \subseteq f(V) \subseteq NpCl(V) \subseteq B$ . Hence by Theorem 3. 2  $(V, \tau_{R'}(Y))$  is quasi nano p-normal.

**Theorem 3.11:** If f: (U,  $\tau_R(X)$ )  $\rightarrow$  (V,  $\tau_{R'}(Y)$ ) is weakly nano open nano  $\pi$ -continuous almost nano pre- irresolute surjection and (U,  $\tau_R(X)$ ) is quasi nano p-normal, then (V,  $\tau_{R'}(Y)$ ) is quasi nano p-normal.

**Proof:** Let A be an nano  $\pi$ -closed (V,  $\tau_{R'}(Y)$ ) and B be an nano  $\pi$ -open subsets of (V,  $\tau_{R'}(Y)$ ) such that  $A \subseteq B$ . By nano  $\pi$ -continuity of f, we have  $f^{1}(A)$  be a nano  $\pi$ -closed and  $f^{1}(B)$ is nano  $\pi$ -open subset of (U,  $\tau_R(X)$ ) such that  $f^{1}(A) \subseteq f^{1}(B)$ . Since  $(U, \tau_{R}(X))$  is quasi nano p-normal, there exists an nano p-open subset of V of  $(U, \tau_R(X))$  $f^{1}(A) \subseteq V \subseteq NpCl(V) \subseteq f^{1}(B).$ such that Then  $f(f^{1}(A)) \subseteq f(U) \subseteq f(NpCl(V)) \subseteq f^{1}(B)$ . Since f is weakly nano-open nano continuous and almost nano pre-irresolute surjection, then by Lemma 3.9 we have f is nano Mp-open and nano R-map. Thus, we have f(V) is nano p-open subset of  $(U, \tau_{R'}(Y))$  such that  $A \subseteq f(V) \subseteq NpCl(V) \subseteq B$ . Hence by Theorem 3. 2 (V,  $\tau_{R'}(Y)$ ) is quasi nano p- normal.

**Theorem 3.12:** If f:  $(U, \tau_R(X)) \rightarrow (V, \tau_R'(Y))$  is an nano  $\pi$ -continuous, weakly nano-open nano pre gp-closed surjection and  $(U, \tau_R(X))$  is quasi nano p-normal, then  $(V, \tau_{R'}(Y))$  is nano  $\pi$ p-normal.

**Proof:** Let A and B be any disjoint nano closed subsets of  $(V, \tau_R^{,*}(Y))$  such that A is nano  $\pi$ -closed. Since f is nano  $\pi$ -continuous surjection, then  $f^1(A)$  and  $f^1(B)$  are disjoint nano  $\pi$ -closed subsets of  $(U, \tau_R(X))$ . Since  $(U, \tau_R(X))$  is quasi nano p- normal, then there exists disjoint nano p-open subsets V and W of  $(U, \tau_R(X))$  such that  $f^1(A) \subseteq V$  and  $f^1(B) \subseteq W$ . Since f is weakly nano-open nano continuous, surjection, then by Lemma 3.9 we have f is nano Mp-open and nano R-map. Thus, f(V) and f(W) are disjoint nano p-open subsets of  $(V, \tau_{R'}(Y))$  such that  $A \subseteq f(V)$  and  $B \subseteq f(W)$ . Hence, $(V, \tau_{R'}(Y))$  is nano

 $\pi$ p-normal. The following theorems can be proved easily by using arguments similar to those in the Theorem 3.10 and Theorem 3.11.

Theorem 3.13: The following statements are true:

- 1. If:  $(U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$  is nano rc-continuous, nano Mp-closed map from a quasi nano p-normal space  $(U, \tau_R(X))$  on to a space  $(V, \tau_R(Y))$ , then  $(V, \tau_R(Y))$  is quasi nano p-normal.
- 2. If:  $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is nano R-map, nano pre gpclosed surjection and  $(U, \tau_R(X))$  quasi nano p-normal, then  $(V, \tau_{R'}(Y))$  is quasi nano p-normal.
- 3. If:  $(U, \tau_R(X)) \rightarrow (U, \tau_R(Y))$  is a completely nano continuous, nano pre gp-closed surjection and  $(U, \tau_R(X))$  quasi nano p-normal, then  $(V, \tau_R(Y))$  is nano p-normal.
- 4. If:  $(U, \tau_R(X)) \rightarrow (V, \tau_R(Y))$  is almost nano continuous nano pre gp-closed surjection and  $(U, \tau_R(X))$  nano p-normal, then  $(V, \tau_R(Y))$  is quasi nano p- normal.
- 5. If:  $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is nano  $\pi$ -continuous weakly nano -open nano pre gp-closed surjection and  $(U, \tau_R(X))$  quasi nano p-normal, then  $(V, \tau_{R'}(Y))$  is nano p- normal.
- 6. If:  $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is nano pre gp-continuous nano rc-preserving injection and  $(V, \tau_{R'}(Y))$  is nano p- normal, then  $(U, \tau_R(X))$  quasi nano p- normal.
- 7. If:  $(U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$  is nano pre gp-continuous almost nano closed injection and  $(V, \tau_{R'}(Y))$  is nano p-normal, then  $(U, \tau_R(X))$  quasi nano p-normal.

## 4. Conclusion

We used nano generalized closed (open) sets to obtain various characterizations and preservation theorems of quasi nano p-normality. Some properties and results on quasi nano pnormal spaces were given.

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