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A MULTI SERVER MARKOVIAN QUEUEING SYSTEM WITH DISCOURAGED ARRIVALS, RETENTION OF RENEGED CUSTOMERS, CONTROLLABLE ARRIVAL RATES, LOSS AND DELAY AND NO PASSING

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ABSTRACT

In this Paper, M/M/c/K queueing model with loss and delay, no passing finite capacity c-sever discouraged arrivals, retention of reneged customers and controllable arrival rates is considered. The steady state probabilities of system size are derived explicitly. The effect of the probability of customer's retention on the expected system size has been studied. The analytical results are numerically illustrated and relevant conclusions are presented.

INTRODUCTION

Queues with discouraged arrivals have applications in computer with batch job processing where job submissions are discouraged when the system is used frequently and arrivals are modelled as a Poisson process with state dependent arrival rate. The discouragement affects the arrival rate of the queueing system. Customer arrive in a Poisson fashion with rate depends on the number of customers present in the system with faster arrival rate $\frac{a_0}{n+1}$ and with slower arrival rate $\frac{a_1}{n+1}$. The service times and reneging times follow exponential distribution with parameters μ and ¶ .

Due to restriction of no passing, the customers are allowed to depart from the system in the chronological order of their arrival. In the loss and delay queueing system, the customers are classified into two classes. They are (i) Elective customers and (ii) Emergency customers. The elective customers have patience to form a queue and wait while the emergency customers find the server busy on their arrival, leave the system and are lost. The arrival and service processes are taken to be independent in most of these models.

Many models on loss and delay queueing system with no passing have been studied. Queueing with impatience finds its origin during the early 1950's Haight [2] studies a single server Markovian Queueing system with reneging. Srinivasa Rao *et al*[6] have discussed M/M/1/∞ interdependent queueing model with controllable arrival rates. A.Srinivasan and M.Thiagarajan [7, 8] have analysed M/M/1/K interdependent queueing model with controllable arrival rates balking, reneging and spares.

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Choudhury and Medhi [1] have studied customer impatience in multi server queues, Kapodistria [3] has studied a single server Markovian queue with impatient customers and considered the situations where customers abandon the system with retention of reneged customers. Recently S.Premalatha and M.Thiagarajan [5] have studied interdependent discouraged arrivals and retention of reneged customers with controllable arrival rates. An attempt is made in this paper to obtain the relevant results of the M/M/c/K interdependent discouraged arrivals and retention of reneged customers with arrival rates is considered.

Description of the Model

Consider a c- server finite capacity loss and delay and no passing queuing system with the following assumptions.

It is assumed that the arrival process [X_1(t)] and the service process [X_2(t)] of the system are correlated and follows a bivariate Poisson process having the joint probability mass function of the form

$$P(X_1(t)=x_1, X_2(t)=x_2)=e^{-(ª_{ij}+«_n-\in)t}\sum_{K=0}^{\min(x_1,x_2)}\frac{(\in t)^K[(ª_{ij}-\in)t]^{x_1-K}[(«-\in)t]^{(x_2-K)}}{K!(x_1-K)!(x_2-K)!} \tag{2.1}$$

where x_1, x_2 = 0,1, 2,..., $ª_{01,}$ $ª_{02,}$ $ª_{11}$, $«_{n.}$ >0, n=0,1,2,...,c-1,c,,c+1,...,r-1,r,r+1,..., R-1,R,R+1,K-1,K

0< $ª_i$, « ; $0 \le \in < \min(ª_{ij}, «_n), i = 0,1$

with parameters $ª_{01}$, $ª_{02,}$ $ª_{11}$, $«_n$ and $\in$ as mean arrival rate of elective customer when the system is in the faster rate of arrivals, mean arrival rate of emergency customers when the system is in the faster rate of arrivals, mean arrival rate of elective customers when the system is in the slower rate of arrivals, mean service rate of customers of type B and the covariance between arrival and service process respectively. Also the mean arrival rate and mean service rate when the system size is n is defined as

$$ª_n = \begin{cases} ª_{0j}p & ; \quad 0 \le n < c, j = 1,2 \\ ª_{01}p & ; \quad c \le n \le R-1 \\ ª_{11}p & ; \quad r+1 \le n \le K \end{cases}$$

$$«_n = \begin{cases} n« & ; \quad 0 \le n < c \\ c« & ; \quad c \le n \le K \end{cases}$$

The Steady State Equations

We observe that $P_n(0)$ exists when n = 0,1,2,...,c-1,c,c+1,...,r -1,r both $P_n(0)$ & $P_n(1)$ exist when n = r+1, r+2, ...,R-1 ; only $P_n(1)$ exists when n=R, R+1,...,K. Further $P_n(0) = P_n(1) = 0$ if n>K.

Then the steady state Probability difference equations are

$$-(ª_0 - 2\in)pP_0(0) + («-\in)P_1(0) = 0 \tag{3.1}$$

$$-[(ª_0 - 2\in)p + n(«-\in)]P_n(0) + [(n+1)(«-\in)]P_{n+1}(0) + (ª_0 - 2\in)pP_{n-1}(0) = 0 \qquad 1 \le n \le c-1 \tag{3.2}$$

$$-\left[\frac{(ª_0 - \in)p}{2} + c(« - \in)\right]P_c(0) + [c(« - \in) + ¶p]P_{c+1}(0) + (ª_0 - 2\in)P_{c-1}(0) = 0 \tag{3.3}$$

$$-\left[\left(\frac{(\text{ª}_{01}-\in)p}{(n-c)+2}\right)+c(\text{«}-\in)+(n-c)\P p\right]P_n(0)$$

$$+\left[c(\text{«}-\in)+(n+1-c)\P p\right]\mathrm{P}_{n+1}(0)+\left(\frac{(\text{ª}_{01}-\in)}{(n-c)+1}\right)P_{n-1}(0)=0 \quad c+1\le n\le r-1 \tag{3.4}$$

$$-\left[\left(\frac{(\text{ª}_{01}-\in)p}{(r-c)+2}\right)+c(\text{«}-\in)+(n-c)\P p\right]P_r(0)+\left[c(\text{«}-\in)+(r+1-c)\P p\right]\mathrm{P}_{r+1}(0)$$

$$\left[c(\text{«}-\in)+(r+1-c)\P p\right]\mathrm{P}_{r+1}(1)+\left(\frac{(\text{ª}_{01}-\in)}{(r-c)+1}\right)P_{r-1}(0)=0 \tag{3.5}$$

$$-\left[\left(\frac{(\text{ª}_{01}-\in)p}{(n-c)+2}\right)+c(\text{«}-\in)+(n-c)\P p\right]P_n(0)+\left[c(\text{«}-\in)+(n+1-c)\P p\right]P_{n+1}(1)$$

$$+\left(\frac{(\text{ª}_{01}-\in)}{(n-c)+1}\right)P_{n-1}(0)=0 \qquad r+1\le n\le R-2 \tag{3.6}$$

$$-\left[\left(\frac{(\text{ª}_{01}-\in)p}{(R-c)+1}\right)+c(\text{«}-\in)+(R-1-c)\P p\right]P_{R-1}(0)$$

$$+\left(\frac{(\text{ª}_{01}-\in)}{(R-c)}\right)\mathrm{P}_{R-2}(0)=0 \tag{3.7}$$

$$-\left[\left(\frac{(\text{ª}_{11}-\in)p}{(r-c)+3}\right)+c(\text{«}-\in)+(r+1-c)\P p\right]P_{r+1}(1)+\left[c(\text{«}-\in)+(r+2-c)\P p\right]\mathrm{P}_{r+2}(1)=0 \tag{3.8}$$

$$-\left[\left(\frac{(\text{ª}_{11}-\in)p}{(n-c)+2}\right)+c(\text{«}-\in)+(n-c)\P p\right]P_n(1)+\left(\frac{(\text{ª}_{11}-\in)p}{(n-c)+1}\right)P_{n-1}(1)$$

$$+\left[c(\text{«}-\in)+((n+1)-c)\P p\right]P_{n+1}(1)=0 \tag{3.9}$$

$$-\left[\left(\frac{(\text{ª}_{11}-\in)p}{(R-c)+2}\right)+c(\text{«}-\in)+(R-c)\P p\right]P_R(1)+\left(\frac{(\text{ª}_{11}-\in)p}{(R-c)+1}\right)P_{R-1}(1)+\left(\frac{(\text{ª}_{01}-\in)p}{(R-c)+1}\right)P_{R-1}(0)$$

$$+\left[c(\text{«}-\in)+(R+1-c)\P p\right]\mathrm{P}_{R+1}(1)=0 \tag{3.10}$$

$$-\left[\left(\frac{(\text{ª}_{11}-\in)p}{(n-c)+2}\right)+c(\text{«}-\in)+(n-c)\P p\right]P_n(1)+\left[c(\text{«}-\in)+((n+1)-c)\P p\right]\mathrm{P}_{n+1}(1)$$

$$+\left(\frac{(\text{ª}_{11}-\in)p}{(n-c)+1}\right)P_{n-1}(1)=0 \tag{3.11}$$

$$\left[c(\text{«}-\in)+(K-c)\P p\right]P_K(1)+\left(\frac{(\text{ª}_{11}-\in)p}{(K-c)+1}\right)P_{K-1}(1)=0 \tag{3.12}$$

From (3.1) to (3.2) we get

$$P_n(0)=\frac{1}{n!}\left(\frac{(\text{ª}_0-2\in)p}{(\text{«}-\in)}\right)^n P_0(0) \qquad n=0,1,2,\dots,c-1 \tag{3.13}$$

$$P_n(0) = \frac{(ª_0 - 2\in)^c}{(«-\in)^c}\frac{p^c}{c!}\frac{\left[(ª_{01}-\in)p\right]^{n-c}}{(n-c+1)!}\frac{1}{\prod_{l=c+1}^{n}\left[c(«-\in)+(l-c)¶p\right]}P_0(0) \qquad \text{n} = \text{c}, \text{c}+1, ..., \text{r} \tag{3.14}$$

From (3.5) to (3.6) we get

$$P_n(0) = \frac{(ª_0 - 2\in)^c}{(«-\in)^c}\frac{p^c}{c!}\frac{\left[(ª_{01}-\in)p\right]^{n-c}}{(n-c+1)!}\frac{1}{\prod_{l=c+1}^{n}\left[c(«-\in)+(l-c)¶p\right]}P_0(0)$$
$$-\frac{P_{r+1}(1)}{\prod_{l=r+2}^{n}\left[c(«-\in)+(l-c)¶p\right]}\left[\frac{\left[(ª_{01}-\in)p\right]^{n-r-1}}{(n+1-c)P_{n-r-1}}+\frac{\left[(ª_{01}-\in)p\right]^{n-r-2}}{(n+1-c)P_{n-r-2}}\left[c(«-\in)+(r+1-c)¶p\right]\right. \tag{3.15}$$
$$\left.+...+\left[c(«-\in)+(r+1-c)¶p\right]..\left[c(«-\in)+((n-1)-c)¶p\right]\right] \qquad \text{n} = \text{r}+1, \text{r}+2, ...,, \text{R -1}$$

From (3.5) and (3.7), we get

$$P_{r+1}(1) = \frac{\dfrac{(ª_0 - 2\in)^c}{(«-\in)^c}\dfrac{p^c}{c!}\dfrac{\left[(ª_{01}-\in)p\right]^{R-c}}{(R-c+1)!}\dfrac{1}{\prod_{l=c+1}^{r+1}\left[c(«-\in)+(l-c)¶p\right]}P_0(0)}{\left[\dfrac{\left[(ª_{01}-\in)p\right]^{R-r-1}}{(R+1-c)P_{R-r-1}}+\dfrac{\left[(ª_{01}-\in)p\right]^{R-r-2}}{(R+1-c)P_{R-r-2}}\left[c(«-\in)+(r+1-c)¶p\right]+...+\right.}$$
$$\left.+\left[c(«-\in)+(r+1-c)¶p\right]..\left[c(«-\in)+((R-1)-c)¶p\right]\right] \tag{3.16}$$

From (3.8) and (3.9), we get

$$P_n(1) = \frac{P_{r+1}(1)}{\prod_{l=r+2}^{n}\left[c(«-\in)+(l-c)¶p\right]}\left[\frac{\left[(ª_{11}-\in)p\right]^{n-r-1}}{(n+1-c)P_{n-r-1}}+\frac{\left[(ª_{11}-\in)p\right]^{n-r-2}}{(n+1-c)P_{n-r-2}}\left[c(«-\in)+(r+1-c)¶p\right]\right. \tag{3.17}$$
$$\left.+\left[c(«-\in)+(r+1-c)¶p\right]..\left[c(«-\in)+((n-1)-c)¶p\right]\right]$$

where $P_{r+1}(1)$ is given by (3.16)

From (3.10) ,(3.11) and (3.12), we recursively derive

$$P_n(1) = \frac{P_{r+1}(1)}{\prod_{l=r+2}^{n}\left[c(«-\in)+(l-c)¶p\right]}\left[\frac{\left[(ª_{11}-\in)p\right]^{n-r-1}}{(n+1-c)P_{n-r-1}}+\frac{\left[(ª_{11}-\in)p\right]^{n-r-2}}{(n+1-c)P_{n-r-2}}\left[c(«-\in)+(r+1-c)¶p\right]\right. \tag{3.18}$$
$$\left.+..+\frac{\left[(ª_{11}-\in)p\right]^{n-R}}{(n+1-c)P_{n-R}}\left[c(«-\in)+(r+1-c)¶p\right]..\left[c(«-\in)+((R-1)-c)¶p\right]\right]$$

Characteristics of the Model

The following system characteristics are considered and their analytical results are derived in this section

(i) The probability P(0) that the system is in faster rate of arrivals

(ii) The probability P(1) that the system is in slower rate of arrivals

(iii) The probability $P_0(0)$ that the system is empty

(iv) The expected number of customers in the system L_{s_0} , when the system is in the faster rate of arrivals.

(v) The expected number of customers in the system L_{s_1} , when the system is in the slower rate of arrivals.

(vi) The expected waiting time of the customer in the system W_s

The Probability that the system is in faster rate of arrivals is

$$P(0)=\sum_{n=0}^{c-1}P_n(0)+\sum_{n=r}^{r}P_n(0)+\sum_{n=r+1}^{R-1}P_n(0) \qquad (4.1)$$

From (3.13) ,(3.14) ,(3.15) ,(3.16) and (4.1), we get

$$P(0)=P_0(0)+\sum_{n=1}^{c}\left\{\frac{1}{n!}\left(\frac{(ª_0-2\in)p}{«-\in}\right)^n\right\}P_0(0)$$
$$+\sum_{n=c+1}^{R-1}\frac{(ª_0-2\in)^c}{(«-\in)^c}\frac{p^c}{c!}\frac{\left[(ª_{01}-\in)p\right]^{n-c}}{(n-c+1)!}\frac{1}{\prod_{l=c+1}^{n}\left[c(«-\in)+(l-c)¶p\right]}P_0(0)$$
$$-\frac{\sum_{n=r+1}^{R-1}}{\prod_{l=c+1}^{n}\left[c(«-\in)+(l-c)¶p\right]}\left[\frac{A}{B}\frac{\left[(ª_{01}-\in)p\right]^{R-c}}{(«-\in)^c}\frac{p^c}{c!}\right]P_0(0) \qquad \ldots(4.2)$$

where

$$A=\left[\frac{\left[(ª_{01}-\in)p\right]^{n-r-1}}{(n+1-c)P_{n-r-1}}+\frac{\left[(ª_{01}-\in)p\right]^{n-r-2}}{(n+1-c)P_{n-r-2}}\left[c(«-\in)+(r+1-c)¶p\right]+\ldots\right.$$
$$\left.+\left[c(«-\in)+(r+1-c)¶p\right]\ldots\left[c(«-\in)+((n-1)-c)¶p\right]\right]$$

$$B=\left[\frac{\left[(ª_{01}-\in)p\right]^{R-r-1}}{(R+1-c)P_{R-r-1}}+\frac{\left[(ª_{01}-\in)p\right]^{R-r-2}}{(R+1-c)P_{R-r-2}}\left[c(«-\in)+(r+1-c)¶p\right]\right.$$
$$\left.+\left[c(«-\in)+(r+1-c)¶p\right]\ldots\left[c(«-\in)+((R-1)-c)¶p\right]\right]$$

The Probability that the system is in slower rate of arrival is

$$P(1)=\sum_{n=r+1}^{R}P_n(1)+\sum_{n=R+1}^{K}P_n(1)$$

From (3.16) ,(3.17) ,(3.18) and (4.3), we get

$$P(1)=\sum_{n=r+1}^{R}\left(\frac{C}{D}\right)\frac{(ª_0-2\in)^c}{(«-\in)^c}\frac{p^c}{c!}\frac{\left[(ª_{01}-\in)p\right]^{R-c}}{(R-c+1)!}\frac{1}{\prod_{l=c+1}^{R}\left[c(«-\in)+(l-c)¶p\right]}P_0(0)$$
$$+\sum_{n=R+1}^{K}\left(\frac{D}{B}\right)\frac{(ª_0-2\in)^c}{(«-\in)^c}\frac{p^c}{c!}\frac{\left[(ª_{01}-\in)p\right]^{R-c}}{(R-c+1)!}\frac{1}{\prod_{l=c+1}^{K}\left[c(«-\in)+(l-c)¶p\right]}P_0(0) \qquad (4.4)$$

where

$$C=\left[\frac{\left[(ª_{11}-\in)p\right]^{n-r-1}}{(n+1-c)P_{n-r-1}}+\frac{\left[(ª_{11}-\in)p\right]^{n-r-2}}{(n+1-c)P_{n-r-2}}\left[c(«-\in)+(r+1-c)¶p\right]\right.$$
$$\left.+\left[c(«-\in)+(r+1-c)¶p\right]\ldots\left[c(«-\in)+((n-1)-c)¶p\right]\right]$$

$$D=\left[\frac{[(ª_{11}-\in)p]^{n-r-1}}{(n+1-c)P_{n-r-1}}+\frac{[(ª_{11}-\in)p]^{n-r-2}}{(n+1-c)P_{n-r-2}}[c(«-\in)+(r+1-c)¶p]\right.$$
$$\left.+\,..+\frac{[(ª_{11}-\in)p]^{n-r}}{(n+1-c)P_{n-r}}[c(«-\in)+(r+1-c)¶p]..[c(«-\in)+((R-1)-c)¶p]\right]$$

The Probability $P_0(0)$ that the system is empty can be calculated from the normalizing condition

$$P(0)+P(1)=1$$

From (4.2) ,(4.4) and (4.5), we get

$$P_0(0)=\frac{1}{1+\sum_{n=1}^{c}\left\{\frac{1}{n!}\left(\frac{(ª_0-2\in)p}{«-\in}\right)^n\right\}+\sum_{n=c+1}^{R-1}\frac{(ª_0-2\in)^c}{(«-\in)^c}\frac{p^c}{c!}\frac{[(ª_{01}-\in)p]^{n-c}}{(n-c+1)!}\frac{1}{\prod_{l=c+1}^{n}[c(«-\in)+(l-c)¶p]}}$$
$$-\frac{\sum_{n=r+1}^{R-1}}{\prod_{l=c+1}^{R-1}[c(«-\in)+(l-c)¶p]}\left[\frac{A}{B}\frac{[(ª_{01}-\in)p]^{R-c}}{(«-\in)^c}\frac{p^c}{c!}\right]$$
$$+\sum_{n=r+1}^{R}\left(\frac{C}{D}\right)\frac{(ª_0-2\in)^c}{(«-\in)^c}\frac{p^c}{c!}\frac{[(ª_{01}-\in)p]^{R-c}}{(R-c+1)!}\frac{1}{\prod_{l=c+1}^{R}[c(«-\in)+(l-c)¶p]} \quad (4.5)$$
$$+\sum_{n=R+1}^{K}\left(\frac{D}{B}\right)\frac{(ª_0-2\in)^c}{(«-\in)^c}\frac{p^c}{c!}\frac{[(ª_{01}-\in)p]^{R-c}}{(R-c+1)!}\frac{1}{\prod_{l=c+1}^{K}[c(«-\in)+(l-c)¶p]}$$

The expected number of customers in the system is give by

$$L_s=L_{s0}+L_{s1} \quad (4.6)$$

where

$$L_{s0}=\sum_{n=0}^{r}\mathrm{n}P_n(0)+\sum_{n=r+1}^{R-1}\mathrm{n}P_n(0) \quad (4.7)$$

and

$$L_{s1}=\sum_{n=r+1}^{R-1}\mathrm{n}P_n(1)+\sum_{n=R}^{K}\mathrm{n}P_n(1) \quad (4.8)$$

From (3.15), (3.18), (4.7) and (4.8),we get

$$L_s = \sum_{n=0}^{c}\left\{\frac{1}{(n-1)!}\left(\frac{(a_0-2\in)p}{\text{«}-\in}\right)^n\right\}P_0(0)$$

$$+\sum_{n=c+1}^{R-1}\frac{(a_0-2\in)^c}{(\text{«}-\in)^c}\frac{p^c n}{c!}\frac{\left[(a_{01}-\in)p\right]^{n-c}}{(n-c+1)!}\frac{1}{\prod\limits_{l=c+1}^{n}\left[c(\text{«}-\in)+(l-c)\P p\right]}P_0(0)$$

$$-\frac{n\sum\limits_{n=r+1}^{R-1}}{\prod\limits_{l=c+1}^{R-1}\left[c(\text{«}-\in)+(l-c)\P p\right]}\left[\frac{A}{B}\frac{\left[(a_{01}-\in)p\right]^{R-c}}{(\text{«}-\in)^c}\frac{p^c}{c!}\right]P_0(0)$$

$$+\sum_{n=r+1}^{R}n\left(\frac{C}{D}\right)\frac{(a_0-2\in)^c}{(\text{«}-\in)^c}\frac{p^c}{c!}\frac{\left[(a_{01}-\in)p\right]^{R-c}}{(R-c+1)!}\frac{1}{\prod\limits_{l=c+1}^{R}\left[c(\text{«}-\in)+(l-c)\P p\right]}P_0(0)$$

$$+\sum_{n=R+1}^{K}n\left(\frac{D}{B}\right)\frac{(a_0-2\in)^c}{(\text{«}-\in)^c}\frac{p^c}{c!}\frac{\left[(a_{01}-\in)p\right]^{R-c}}{(R-c+1)!}\frac{1}{\prod\limits_{l=c+1}^{K}\left[c(\text{«}-\in)+(l-c)\P p\right]}P_0(0)$$

Using Little's formula, the expected waiting time of the customers in the system is given by

$$W_s = \frac{L_s}{\bar{a}} \qquad \text{Where } \bar{a} = a_0 P(0) + a_1 P(1) \tag{4.9}$$

Numerical Illustrations

For various values of $a_0, a_1, \text{«}, r, R, \in, K$ the values of $P_0(0)$, P(0), P(1) L_S and W_S are computed and tabulated in the following Table.

Table 5 1

r =4, R = 6, K = 10										
S.No	**c**	a_0	a_{01}	a_{11}	«	Č	p	**P_0(0)**	**P(0)**	**P(1)**
1	1	4	2	2	5	1	1	0.643960856	0.999999999	$2.20875*10^{-8}$
2	2	4	2	2	5	1	1	0.612677051	0.999999899	$4.27358*10^{-8}$
3	1	5	2	2	5	1	1	0.54664699	0.99999993	$2.81245*10^{-8}$
4	2	5	2	2	5	1	1	0.488422475	0.999999818	$7.66547*10^{-8}$
5	1	3	2	2	5	1	1	0.783426021	0.999999966	$1.34355*10^{-8}$
6	2	3	2	2	5	1	1	0.779395699	0.999999968	$1.35912*10^{-8}$
7	1	4	2	2	5	1	1	0.643960856	0.999999945	$2.20875*10^{-8}$
8	2	4	2	2	5	1	1	0.612677051	0.999999899	$4.27358*10^{-8}$
9	1	4	2	2	4	1	1	0.569534395	0.999999854	$5.68011*10^{-8}$
10	2	4	2	2	4	1	1	0.524812632	0.999999639	$1.47348*10^{-7}$
11	1	4	2	2	6	1	1	0.696875881	0.999999976	$9.80964*10^{-9}$
12	2	4	2	2	6	1	1	0.673972829	0.999999965	$1.5327*10^{-8}$
13	1	6	2	2	5	1	1	0.474883678	0.999999918	$3.25764*10^{-8}$
14	2	6	2	2	5	1	1	0.395456203	0.999999739	$1.10336*10^{-7}$
15	1	4	2	2	5	1	1	0.643960856	0.999999945	$2.20875*10^{-8}$
16	2	4	2	2	5	1	1	0.612677051	0.999999899	$4.27358*10^{-8}$
17	1	4	2	2	5	1	1	0.643960856	0.999999945	$2.20875*10^{-8}$
18	2	4	2	2	5	1	1	0.612677051	0.999999899	$4.27358*10^{-8}$
19	1	4	2	2	5	0	1	0.513633235	0.999998873	$4.58104*10^{-7}$
20	2	4	2	2	5	0	1	0.464949605	0.999998422	$6.71096*10^{-7}$
21	1	4	2	2	5	0.5	1	0.566585141	0.999999656	$1.38514*10^{-7}$
22	2	4	2	2	5	0.5	1	0.524566152	0.999999946	$2.30573*10^{-7}$
23	1	4	2	2	5	1	0.5	0.659019550	0.999999964	$1.64036*10^{-8}$

Table 2

r =4, R = 6, K = 10									
S.No	**c**	a_0	a_{01}	a_{11}	«	Č	*p*	L_S	W_S
1	1	4	2	2	5	1	1	0.391982675	0.097995765
2	2	4	2	2	5	1	1	0.512288174	0.128072052
3	1	5	2	2	5	1	1	0.499121623	0.099824329
4	2	5	2	2	5	1	1	0.735725966	0.147145213
5	1	3	2	2	5	1	1	0.238437365	0.079479123
6	2	3	2	2	5	1	1	0.260346837	0.086782281
7	1	4	2	2	5	1	1	0.391982675	0.097995672
8	2	4	2	2	5	1	1	0.512288174	0.128072051
9	1	4	2	2	4	1	1	0.484612846	0.121153224
10	2	4	2	2	4	1	1	0.687333052	0.171833306
11	1	4	2	2	6	1	1	0.328653755	0.082163440
12	2	4	2	2	6	1	1	0.407658762	0.101914693
13	1	6	2	2	5	1	1	0.578130604	0.096355109
14	2	6	2	2	5	1	1	0.927182118	0.154530478
15	1	4	2	2	5	1	1	0.391982675	0.097995671
16	2	4	2	2	5	1	1	0.512288174	0.128072050
17	1	4	2	2	5	1	1	0.391982675	0.097995673
18	2	4	2	2	5	1	1	0.512288174	0.128072054
19	1	4	2	2	5	0	1	0.568412275	0.142103180
20	2	4	2	2	5	0	1	0.84033716	0.210084516
21	1	4	2	2	5	0.5	1	0.493264464	0.123316146
22	2	4	2	2	5	0.5	1	0.692575683	0.173143984
23	1	4	2	2	5	1	0.5	0.350501942	0.087625488

The observations made from the table 5.1 and 5.2 are

1. When the mean dependence rate increases and the other parameters are kept constant, L_S and W_S decrease
2. When the arrival rate increases and the other parameters are kept constant, Ls and Ws increase.
3. When the service rate increases and the other parameters are kept constant, Ls and Ws decrease.

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