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Research Article

REAL OPTION ANALYSIS CASE WITH DYNAMIC RISK NEUTRAL PROBABILITY

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ABSTRACT

This paper constructs a model with dynamic risk neutral probabilities of double stochastic variables and multistage constructions for the cellulosic ethanol project in China. Based on real option analysis, the investors can estimate the unit market value of the cellulosic ethanol project. Because of the great reduction of the gasoline price and the huge increment of the corn cob price, there are some negative decision values. Specially, action “invest” is still the optimal decision if only the stage-1 construction has been completed. Due to the regulation of the Chinese government, the dynamic risk neutral probabilities of the gasoline price and corn cob price are around 0.5, that are obviously different with the fixed risk neutral probabilities.

INTRODUCTION

As one kind of renewable energy, the cellulosic ethanol project was gradually developed in China in recent years. The first large-scale cellulosic ethanol producer with annual capacity of 50,000 tons had been put into operation in Shandong province. However, the Chinese cellulosic ethanol industry developed slowly by the complex technology, high cost and uncertainties.

Real option is a typical approach used in renewable energy field. Some researchers (Lee *et al*., 2010; Sharma *et al*., 2013; Zhang *et al*., 2014) had investigated the benefits of investing renewable energy policy using binomial tree model. Sharma *et al*. (2013) constructed a real option model for a hypothetical, vertically integrated lignocellulosic enterprise that produces cellulosic ethanol and biosuccinic acid. In their model, binomial tree was used to present the uncertainties in bioproduct demands and prices. Different with the previous research, this paper establishes a quadrinomial lattice tree model based on dynamic risk-neutral probability with two construction stages and double stochastic variables.

Parameters Used In Real Option Model

The parameters considered in the real option model conclude the prices of the main raw materials and the products, the subsidy level and the carbon emission cost. We divide them to two types, stochastic variables and non-stochastic parameters. Since the fuel ethanol priceis set at 0.9111 times the price of No.93 gasoline from 1 May, 2011 in China, the gasoline price is one key stochastic variable in cellulosic ethanol investment project. As the main raw material, corn cob price is another stochastic variable. Furthermore, we suppose that these two stochastic variables follow Ornstein-Uhlenbeck processes and they are independent. Let $p^g(i,t)$ denote the gasoline price with t periods elapsed and i downward moves, $p^c(j,t)$ denote the corn cob price with t periods elapsed and j downward moves, where $0 \le t \le T$, $0 \le i, j \le t$. T is the total number of time periods. Specially, we only considers two move cases - up and down - about the stochastic variables, u_g and d_g are the upward and downward move size of gasoline price, u_c and d_c are the upward and downward move size of corn cob price. The binomial tree about the stochastic variable can be presented as Exhibit1, which ignores the subscript g and c. Then the binomial trees of gasoline price and corn cob price can be constructed starting with the related average prices in 2015.

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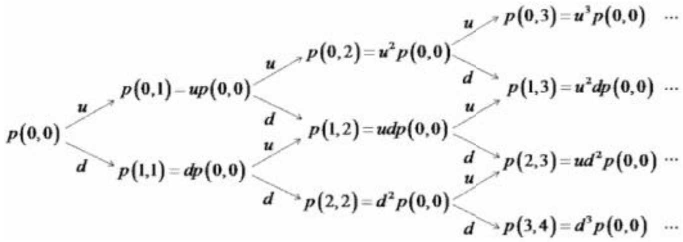
Exhibit 1

The binomial tree of stochastic variable (without subscript g and c)



Other by-products and raw materials in the cellulosic ethanol producing process are considered as non-stochastic parameters. According to the report in the conference of Proceeding of the 6th Stakeholder Plenary Meeting of EBTP by Kang (2014), xylitol, pure lignin are the main by-products, zymin is another important raw material in the cellulosic ethanol project. Let p^x, p^l denote the prices of xylitol and pure lignin, p^z present the expense of zymin for every ton cellulosic ethanol. Meanwhile, we assume that the investors should pay the carbon emission cost, which is denoted by p^{cb}. Actually, multistage investment gives investors more time to consider, so we consider the case with two construction stages before the project completed. Let C_{other} denote the total construction costs (such as land, equipment and so on), then the stage-1 construction cost $J_1 = aC_{other}$, the stage-2 construction cost $J_2 = (1-a)C_{other}$. Here, a is the stage-1 construction cost proportion. Meanwhile, Q presents the capacity of the cellulosic ethanol project, S indicates the subsidy for every ton cellulosic ethanol, r_f denotes the risk-free interest rate. Furthermore, the cellulosic ethanol investment right will be lost if the construction program cannot be completed on or before the expiration date T. All these non-stochastic parameters are shown in Exhibit2.

Exhibit2

The non-stochastic parameters used in the real option model

Parameter	Value	Provenance
Q	50,000 tons	the report from the first cellulosic ethanol producer
p^x	23,000 yuan/ton	the average value from the data (Jan.to May in 2015) based on China Beijing Environmental Exchange.
p^l	4,500 yuan/ton	the 6th stakeholder Plenary Meeting of EBTP
p^z	2,600 yuan/ton cellulosic ethanol	the 6th stakeholder Plenary Meeting of EBTP
p^{cb}	50 yuan/ton	the average value from the data in 2015 based on China Beijing Environmental Exchange.
S	800 yuan/ton cellulosic ethanol	the report from the first cellulosic ethanol producer
r_f	0.032	the average interest rate of treasury bondsin 2015 in China
C_{other}	166 millions yuan	the report from the first cellulosic ethanol producer
a	0.5	initial hypothesis
T	5	the investment right is valid from 2015 to 2020 (2015 is the start year denoted as number 0)

Parameters Estimation

By assumption, the gasoline price and the corn cob price follow OU processes as $d\ln p_t^g = \beta_g\left(\theta_g - \ln p_t^g\right)dt + \sigma_g dB_{1t}$, $d\ln p_t^c = \beta_c\left(\theta_c - \ln p_t^c\right)dt + \sigma_c dB_{2t}$, p_t^g and p_t^c are the prices of gasoline and corn cob at any time t, β_1 and β_2 are the rates of mean reversions, θ_1 and θ_2 are the related long-run levels, σ_1 and σ_2 are the related volatilities, B_{1t} and B_{2t} are different Brownian motions.

Actually, the OU process $d\ln p_t = \beta\left(\theta - \ln p_t\right)dt + \sigma dB_t$ can be estimated following the AR(1) process $\Delta g_t = \varphi_0 + \varphi_1 g_t + \varepsilon_{t+1}$, $\varepsilon_{t+1} \sim N\left(0, \delta^2\right)$, $g_t = \ln p_t$. Hence, the estimated values $\hat{\beta}$, $\hat{\theta}$, $\hat{\sigma}$ can be obtained as

$$\hat{\beta} = -\frac{1}{\Delta t}\ln\left(1+\hat{\varphi}_1\right),\ \hat{\theta} = -\frac{\hat{\varphi}_0}{\hat{\varphi}_1},\ \hat{\sigma} = \hat{\delta}\sqrt{\frac{2\ln\left(1+\hat{\varphi}_1\right)}{\hat{\varphi}_1\left(2+\hat{\varphi}_1\right)\Delta t}},$$

where $\hat{\varphi}_0$, $\hat{\varphi}_1$, $\hat{\delta}_1$ are estimated values of φ_0, φ_1, δ_1. According to the daily history data of No.93 gasoline price and corn cob price of Shandong province from 2011 to 2015, the volatilities of the logarithm of gasoline price with the year as unit are $\sigma_g = 1.2467$, $\sigma_c = 3.4299$, the upward move sizes $u_g = e^{\sigma_g} = 3.4789$, $u_c = e^{\sigma_c} = 30.8738$, the downward move sizes $d_g = e^{-\sigma_g} = 0.2874$, $d_c = e^{-\sigma_c} = 0.0323$.

Following the idea of binomial tree, it is not hard to obtain the value of each node in the binomial tree of gasoline price or corn cob price. Exhibit 3 and Exhibit4 show all these values in the binomial tree of the gasoline price with initial data $p^g(0,0) = 8372$, $p^c(0,0) = 450$ yuan per ton, which is the average price of the data in Shandong province in 2015.

Exhibit3

The binomial tree of the gasoline price (yuan/ton)

2015	2016	2017	2018	2019	2020
8372	29133	101382	352811	1227781	4272679
	2428	8449	29401	102315	356057
		704	2450	8526	29671
			204	711	2473
				59	206
					17

Exhibit4

The binomial tree of the corn cob price (yuan/ton)

2015	2016	2017	2018	2019	2020
450	13895	428928	13241003	408749750	12618104781
	14	417	12868	397230	12262492
		0	13	386	11917
			0	0	12
				0	0
					0

Dynamic Risk-Neutral Probability

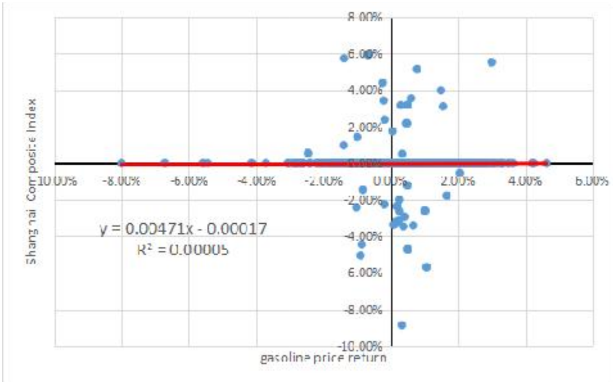
The dynamic risk-neutral probabilities of the upward move size of gasoline price $®_{g,u}(i,n)$ and corn cob price $®_{c,u}(j,n)$ satisfy $®_{g,u}(i,n) = ¡_{g,u}(i,n) - \frac{E[R_M]-(1+r_f)}{u_g - d_g}\ _g$, $®_{c,u}(j,n) = ¡_{c,u}(j,n) - \frac{E[R_M]-(1+r_f)}{u_c - d_c}\ _c$. Here, $¡_{g,u}(i,n)$ and $¡_{c,u}(j,n)$ are the actual probabilities of the upward move size,

$$
¡_u(\ ,n) = \begin{cases} 0, & \frac{1}{2} + \frac{\left(1-e^{-\ \Delta t_m}\right)\left(« - \ln p(\ ,n)\right)}{2± \sqrt{\Delta t_m}} \le 0 \\ \frac{1}{2} + \frac{\left(1-e^{-\ \Delta t_m}\right)\left(« - \ln p(\ ,n)\right)}{2± \sqrt{\Delta t_m}}, & 0 < \frac{1}{2} + \frac{\left(1-e^{-\ \Delta t_m}\right)\left(« - \ln p(\ ,n)\right)}{2± \sqrt{\Delta t_m}} < 1 \\ 1, & \frac{1}{2} + \frac{\left(1-e^{-\ \Delta t_m}\right)\left(« - \ln p(\ ,n)\right)}{2± \sqrt{\Delta t_m}} \ge 1 \end{cases}, \quad (1)
$$

which ignores the subscript g and c, and the symbol denote i or j. Here, R_M is the one period return on the market portfolio. Based on the history data of Shanghai composite index from 2012 to 2015, The expected continuously compound return of the market portfolio $E[R_M] = 0.08\%$. Since the CAPM betas of gasoline price and corn cob price satisfy $_g = \frac{Cov\left[R_{g,t+1}/R_{g,t}, R_M\right]}{Var[R_M]}$, $_c = \frac{Cov\left[R_{c,t+1}/R_{c,t}, R_M\right]}{Var[R_M]}$, $R_{g,t}$ and $R_{c,t}$ are the prices of the gasoline and corn cob at time t. Using EXCEL, the estimated values of CAPM beta can usually be calculated by its trend line and its equation as shown in Exhibit5 and Exhibit6. Thus, $_g = 0.00471$,. $_c = 0.0531$

Exhibit5

Scatter diagram of Shanghai composite index and gasoline price return



Hence, the upward dynamic risk-neutral probability at each node of each stochastic variable can be shown in Exhibit 7.

Real Option Model

According to the report given by Kang (2014), producing one ton cellulosic ethanol needs $\frac{20}{3}$ tons corn cob, $\frac{4}{5}$ tons xylitol, one ton pure lignin.

Exhibit 6

Scatter diagram of Shanghai composite index and corn cob price return

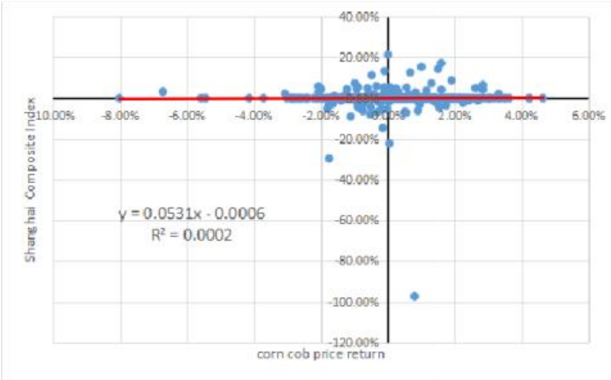


Exhibit 7

The dynamic upward risk-neutral probabilities (only retain four decimal) of gasoline price (G) and corn cob price (C)

2016		2017		2018		2019		2020	
G	C	G	C	G	C	G	C	G	C
0.50152	0.50000	0.50151	0.49996	0.50151	0.49991	0.50150	0.49987	0.50149	0.49982
		0.50153	0.50005	0.50152	0.50000	0.50151	0.49996	0.50151	0.49991
				0.50154	0.50009	0.50153	0.50005	0.50152	0.50000
						0.50155	0.50014	0.50154	0.50009
								0.50156	0.50018

It also needs to expense some zymin. Meanwhile, the producers obtain the subsidy from the government and they suffer the carbon emission cost. By BP carbon emission calculator, one ton fuel ethanol can be instead of one ton gasoline and release 3.15 tons carbon. Let $X(i,j,n)$ is the unit market value of the completed project at node (i,j,n), that is, the gasoline price has i downward movements and the corn cob price has j downward movements at date n .Let $V_m(i,j,n)$ denote the unit market value of the investment right at node (i,j,n), $m=1,2$ represents the number of construction stage to be invested. Then the unit market value of the investment right at each scenario can be obtained by the back induction method.

$$X(i,j,n) = 0.9111 p^g(i,n) + \frac{4}{5}p^x + \frac{2}{3}p^l + S - \frac{20}{3}p^c(j,n) - p^z - 3.15p^{cb}, \quad (2)$$

$$V_2(i,j,n) = \max\left\{-\frac{J_2}{Q} + X(i,j,n),\; \begin{array}{l} e^{-r_f}\left[\text{®}_{g,u}(i,n)\left[\text{®}_{c,u}(j,n)V_2(i,j,n+1) + (1-\text{®}_{c,u}(j,n))V_2(i,j+1,n+1)\right]\right. \\ \left. +(1-\text{®}_{g,u}(i,n))\left[\text{®}_{c,u}(j,n)V_2(i+1,j,n+1) + (1-\text{®}_{c,u}(j,n))V_2(i+1,j+1,n+1)\right]\right] \end{array}\right\}, \quad (3)$$

$$V_1(i,j,n) = \max\left\{\begin{array}{l} -\frac{J_1}{Q} + e^{-r_f}\left[\text{®}_{g,u}(i,n)\left[\text{®}_{c,u}(j,n)V_2(i,j,n+1) + (1-\text{®}_{c,u}(j,n))V_2(i,j+1,n+1)\right]\right. \\ \left. +(1-\text{®}_{g,u}(i,n))\left[\text{®}_{c,u}(j,n)V_2(i+1,j,n+1) + (1-\text{®}_{c,u}(j,n))V_2(i+1,j+1,n+1)\right]\right], \\ e^{-r_f}\left[\text{®}_{g,u}(i,n)\left[\text{®}_{c,u}(j,n)V_1(i,j,n+1) + (1-\text{®}_{c,u}(j,n))V_1(i,j+1,n+1)\right]\right. \\ \left. +(1-\text{®}_{g,u}(i,n))\left[\text{®}_{c,u}(j,n)V_1(i+1,j,n+1) + (1-\text{®}_{c,u}(j,n))V_1(i+1,j+1,n+1)\right]\right] \end{array}\right\}, \quad (4)$$

with the terminal conditions $V_2(i,j,T)=0$, $V_1(i,j,T)=0$. Here, $0 \le n \le T$, $0 \le i, j \le n$.

RESULTS AND CONCLSIONS

Based on the real option model, all the decision values at each scenario can be shown in Exhibit 8. Since there has 25 cases with the value 0 in 2020, it omits the following same parts as shown. Following this table, investors can make decisions and estimate the unit market value of the cellulosic ethanol project based on the information of the gasoline and corn cob prices. Obviously, the investors will not invest the cellulosic ethanol project intuitively at the expiration date, thus the decision values in the last column of the last tables must be zero.

The gray areas represent that the investors are better to wait for new information about the fuel market and invest the next construction stage latter. In the cellulosic ethanol investment, the fixed risk neutral probabilities of the upward moves of the gasoline price and corn cob price are $\text{®}_{g,u} = \frac{e^{r_f} - d_g}{u_g - d_g} = 0.2335$, $\text{®}_{c,u} = \frac{e^{r_f} - d_c}{u_c - d_c} = 0.0324$, which show that the gasoline price and corn cob price are more likely to reduce.

Exhibit 8

The decision values (million yuan, retain four decimal)

2016		2017		2018		2019		2020	
V2	V1	V2	V1	V2	V1	V2	V1	V2	V1
0.0224	0.0479	0.0578	0.0562	0.0613	0.0597	0	0	0	0
		0.1158	0.1141	0.1545	0.1529	0.2534	0	0	0
		0.0093	0.0076	0.2689	0.1846	0.3391	0	0	0
		0.0213	0.0197	0	0	0.3392	0	0	0
				0.0227	0.0139	0	0	0	0
				0.0313	0.0296	0	0	0	0
				0	0	0.0445	0	0	0
				0.0156	0.0075	0.0446	0	0	0
				0.0184	0.0167	0	0	0	0
						0	0	0	0
						0.0199	0	0	0
						0.0200	0	0	0
						0	0	0	0
						0	0	0	0
						0.0179	0	0	0
						0.0180	0	0	0
								0	0
								0	0
								⋮	⋮

However, the upward dynamic risk neutral probability at each node is around 0.5, which shows obvious difference. This comparison indicates that the dynamic risk neutral probability is more adapt to reflect the possible changes at each nodes, and more convenient for investors to make optimal decisions.

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