THE STATIONARY ANALYSIS OF A RETRIAL QUEUE WITH MULTISERVER IN n-LIMITED CAPACITY

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DOI: http://dx.doi.org/10.24327/IJRSR.2017.0804.0150

ABSTRACT

In this paper, we study the stationary analysis of the model M/M/3/n+1 with linear retrial rates and with state dependent parameters by introducing the bivariate process \{(C(t), Q(t)), t \geq 0\}. Some numerical results are also presented.

INTRODUCTION

A retrial queueing system is described by an arriving customer, finds the server busy, joins the retrial group to try again for service after a random amount of time. Retrial queueing systems have been widely used to model many problems in modern telephone switching systems, computer and communication systems. For detailed survey one can see Yang and Templeton (1987). Most papers assume that each orbiting customer seeks service independently of each other after a random time exponentially distributed with a fixed rate. Nevertheless, there are other queueing situations in which the retrial rate does not depend on the number of customers in the orbit. Some notable works in this directions are Fayolle (1986) and Martin and Artalejo (1995). Artalejo and Gomez-Corral (1997), in their paper incorporate both possibilities by assuming that time intervals between successive repeated attempts are exponentially distributed with parameter \(\alpha(1-\delta_0)+j\mu\), when the orbit size is \(j\).

The purpose of this paper is to analyse the retrial queueing model M/M/3/n+1 using the technique of Gomez-Corral and Ramalhoto (1999). The rest of the article is organized as follows: We describe the Mathematical model in section 2. In section 3, we carry out the stationary analysis M/M/3/n+1 retrial queueing model. Section 4 contains some numerical results corresponding to the model in section 3.

MATHEMATICAL MODEL

We consider a retrial queueing system with \(c\) servers and \(d\) waiting positions. When the \(c\) servers are busy, an arriving customer (called primary customer) occupies a waiting position and, when one server becomes free, one of the waiting customers immediately enters the server. Otherwise, when the \(c\) servers are busy and the \(d\) waiting positions are occupied, the customer immediately enters the orbit (called orbit customer). The state of the system at time \(t\) is described by the bivariate process \{(\(C(t), Q(t)), t \geq 0\)}, where \(C(t)\) is the total number of servers and waiting position occupied and \(Q(t)\) denotes the number of orbiting customers. The model is denoted by M/M/c/r+d. The arrival rates of the primary customer is \(\lambda_i\), if \(C(t) = i\) and the rate of orbit customer equals \(\beta_j\), when \(C(t) = i\) and \(Q(t) = j\). The service rate equals \(\nu_i\), when \(C(t) = i\). The state space \(S = \{0, 1, 2, \ldots, c\} \times Z_+\) noted in Asmussen (1987), of the Markov process \{\(X(t), t \geq 0\)\} is ergodic if and only if there exists a probability solution \(P = \{P_{0j}, P_{1j}, \ldots, P_{pj}\}, j \geq 0\) to equality \(Q = 0\), where \(Q\) is the infinitesimal matrix of the process \{\(X(t), t \geq 0\)\}. In this case the vector \(P\) is the stationary distribution of \{\(X(t), t \geq 0\)\}. In section 3, we take \(C(t) = i, i \in \{0, 1, 2, 3, 4\}\) called model 1.

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The Analysis

Let \( \{X(t), t \geq 0\} \) be a time homogeneous Markov process, where \( X(t) \) the bivariate process \((C(t), Q(t))\), \( C(t) \) is the number of customers in the system and \( Q(t) \) is the number in the orbit. Here the bivariate limit process \( X \) takes values on the lattice semi-strip \( S = \{0, 1, 2, 3, 4\} \times \mathbb{Z} \). The infinitesimal matrix is \( Q = (q_{ij}) \), where

\[
Q = \begin{pmatrix}
A_0 & C & 0 & 0 & 0 & 0 & \ldots \\
B_1 & A_1 & C & 0 & 0 & \ldots \\
0 & B_2 & A_2 & C & 0 & \ldots \\
0 & 0 & B_3 & A_3 & C & \ldots \\
0 & 0 & 0 & B_4 & A_4 & \ldots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\]

with \((A_0 + C) \mathbf{e} = 0, (B_1 + A_1 + C) \mathbf{e} = 0, \mathbf{c} = (1,1,1, \ldots),\)\

\[
A_i = \begin{pmatrix}
-(\lambda_0 + \beta_{0i}) & \lambda_0 & 0 & 0 & 0 \\
\beta_1 & -(\lambda_1 + \nu_1 + \beta_{ii}) & \lambda_1 & 0 & 0 \\
0 & \nu_2 & -(\lambda_2 + \nu_2 + \beta_{2i}) & \lambda_2 & 0 \\
0 & 0 & \nu_3 & -(\lambda_3 + \nu_3 + \beta_{3i}) & \lambda_3 \\
0 & 0 & 0 & \nu_4 & -(\lambda_4 + \nu_4)
\end{pmatrix}
\]

\[
B_i = \begin{pmatrix}
0 & \beta_{0i} & 0 & 0 & 0 \\
0 & 0 & \beta_{ii} & 0 & 0 \\
0 & 0 & 0 & \beta_{2i} & 0 \\
0 & 0 & 0 & 0 & \beta_{3i} \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

\[
C = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \lambda_4
\end{pmatrix}
\]

\( i = 0, 1, 2, 3, \ldots, \beta_{10} = 0, \beta_{20} = 0, \) and \( \beta_{30} = 0, \) where \( \beta_{ij} = \alpha_i(1-\delta_{ij})+j\mu_i.\)

The Markov process \( \{X(t), t \geq 0\} \) is Ergodic if and only if there exist a solution \( P = (P_0, P_1, P_2, \ldots, P_0, \ldots) \), where \( P_i = (P_{i0}, P_{i1}, P_{i2}, P_{i3}, P_{i4}) \), the matrix equation.

\[
PQ = 0
\]

This is equation to

\[
(\lambda_0 + \beta_{0j}) P_{0j} = \nu_1 P_{1j}, j \geq 0 \quad \ldots \quad (3.1)
\]

\[
(\lambda_1 + \nu_1 + \beta_{1j}) P_{1j} = \lambda_0 P_{0j} + \beta_{0j} P_{0j+1} + \nu_0 P_{2j}, j \geq 0 \quad \ldots \quad (3.2)
\]

\[
(\lambda_2 + \nu_2 + \beta_{2j}) P_{2j} = \lambda_1 P_{1j} + \beta_{1j} P_{1j+1} + \nu_1 P_{3j}, j \geq 0 \quad \ldots \quad (3.3)
\]

\[
(\lambda_3 + \nu_3 + \beta_{3j}) P_{3j} = \lambda_2 P_{2j} + \beta_{2j} P_{2j+1} + \nu_2 P_{4j}, j \geq 0 \quad \ldots \quad (3.4)
\]

\[
(\lambda_4 + \nu_4) P_{4j} = \lambda_3 P_{3j} + \beta_{3j} P_{3j+1} + \lambda_0 P_{0j+1}, j \geq 0 \quad \ldots \quad (3.5)
\]

We define generating functions

\[
P(z) = \sum_{j=0}^{\infty} P_{i0} z^j, i = 0, 1, 2, 3, 4 \quad \ldots \quad (3.6)
\]

Applying (3.9) on both sides of (3.2), (3.3), (3.4), (3.5), (3.6) we get

\[
... (3.7)
\]
\[(\lambda_0+\alpha_0) P_0(z) + \mu z P'_0(z) = v_1 P_0(z) + \alpha_0 P_{00}\]  \hspace{1cm} \ldots (3.8)

\[\mu z^2 P'_0(z) + (\lambda_1+\nu_1+\alpha_1) z P_0(z) + \alpha_1 P_0 = \mu z P'_0(z) + (\lambda_0+\alpha_0) P_0(z) + v_2 z P_3(z) + \alpha_2 z P_{10}\]  \hspace{1cm} \ldots (3.9)

\[\mu z^2 P_{02}(z) + (\lambda_2+\nu_2+\alpha_2) z P_0(z) + \alpha_1 P_2 = \mu z P_{02}(z) + (\lambda_1+\alpha_1) P_0(z) + v_3 z P_3(z) + \alpha_2 z P_{20}\]  \hspace{1cm} \ldots (3.10)

\[\mu z^2 P'_1(z) + (\lambda_3+\nu_3+\alpha_3) z P_0(z) + \alpha_1 P'_0 = \mu z P'_1(z) + (\lambda_2+\alpha_2) P_0(z) + v_2 z P_3(z) + \alpha_2 z P_{30}\]  \hspace{1cm} \ldots (3.11)

\[\lambda_4 z P_3(z) = \sum_{i=0}^3 \mu_i z P'_i(z) + \alpha_i (P_0(z) - P_{00})\]  \hspace{1cm} \ldots (3.13)

Differentiating the equation (3.8) with respect to \(z\)

\[v_1 P'_0(z) = (\lambda_0+\alpha_0) P_0(z) + \mu z \cdot P'_0(z)\]  \hspace{1cm} \ldots (3.14)

From (3.2), (3.8) and (3.14) we can write (3.9) as

\[v_1 \cdot z P'_0(z) = \mu \mu z^2 P''_0(z) + ((\lambda_0 + \alpha_0 + \mu_0) \cdot \mu_1 + \mu_0(\lambda_1 + \alpha_1 + \mu_1)) z^2\]  \hspace{1cm} \ldots (3.15)

\[-\mu_0 v_1 z P'_0(z) + ((\lambda_0 + \alpha_0)(\lambda_1 + \alpha_1 + \mu_1)) z^2 \cdot \alpha_0 v_1 P_0(z) + (-\alpha_0(\lambda_1 + \lambda_0) \cdot z \cdot \alpha_0 v_1 P_{00}\]  \hspace{1cm} \ldots (3.15)

Differenting (3.15) with respect to \(z\) and multiplying the resulting relation by \(z\) and after some algebraic manipulation we get

\[v_1 \cdot v_2 z P''_0(z) = \mu \mu z^2 P''_m(z) + ((\lambda_0 + \alpha_0 + \mu_0) \cdot \mu_1 + \mu_0(\lambda_1 + \alpha_1 + \mu_1)) z^2\]  \hspace{1cm} \ldots (3.16)

\[\mu \mu z^2 P''_0(z) + ((\lambda_0 + \alpha_0)(\lambda_1 + \alpha_1 + \mu_1)) z^2 \cdot \alpha_0 v_1 P_0(z) + (-\alpha_0(\lambda_1 + \lambda_0) \cdot z \cdot \alpha_0 v_1 P_{00}\]  \hspace{1cm} \ldots (3.16)

Substituting (3.2), (3.3), (3.8) and (3.14) into (3.10) and rearranging leads to the following equation

\[v_1 v_2 v_3 z P''_0(z) = A z P''_0(z) + (B z^2 + C z) P''_0(z) + (D z^2 + E z) P''_0(z) + (F z + G) P''_0(z) + (H z + I) P''_0(z) + (J z + K) P''_0(z) + \ldots (3.17)

Where

\[A = \mu_0 \cdot \mu_1 \cdot \mu_2\]

\[B = \mu_0 \cdot \mu_1 (\lambda_0+\nu_1+\alpha_0) + (\lambda_0 + \alpha_0 + 3 \mu_0) \cdot \mu_1 + \mu_0(\lambda_1 + \alpha_1 + \mu_1) \mu_2\]

\[C = \mu_0 (v_1 \cdot v_2 \cdot \beta_1)\]

\[D = \mu_0(v_1 \cdot v_2 \cdot \beta_2)\]

\[E = \mu_0(v_1 \cdot v_2 \cdot \beta_3)\]

\[F = \mu_0(v_1 \cdot v_2 \cdot \beta_4)\]

\[G = \mu_0(v_1 \cdot v_2 \cdot \beta_5)\]

\[H = \mu_0(v_1 \cdot v_2 \cdot \beta_6)\]

\[I = \mu_0(v_1 \cdot v_2 \cdot \beta_7)\]

Differenting (3.17) with respect to \(z\) and multiplying by \(z\), we get

\[v_1 v_2 v_3 z^2 P''_0(z) = \mu \mu z^2 P''_0(z) + ((3 \mu_0 \mu_1 \mu_2 + A) z^2 + B z^3) P''_0(z) + ((2 A + C) z^2 + (B + D) z^3) P''_0(z) + \ldots (3.18)

Substituting (3.2), (3.3), (3.4), (3.9), and (3.15) into (3.18) into (3.11), we get

\[v_1 v_2 v_3 z^2 P''_0(z) = G_2 z^2 P''_0(z) + (G_2 z^2 + G_2 z) P''_0(z) + (G_2 z^2 + G_2 z) P''_0(z) + \ldots (3.19)

where

\[G_2 = (3 \mu_0 \mu_1 \mu_2 + A) \mu_1 + (\lambda_1 + \nu_1 + \alpha_1) \mu_0 \mu_1 \mu_2\]

\[G_3 = (B v_1 + v_2 + \mu_1 \mu_2\]

\[G_4 = ((A + C) \mu_1 + A \lambda_0 + \nu_1 + \alpha_0 \mu_1 - \lambda_3 \mu_0 \mu_1 v_1\]

\[G_5 = (B + D) \mu_1 + B \lambda_0 + \nu_1 + \alpha_0 \mu_1 - \lambda_3 \mu_0 + \mu_1 (\lambda_1 + \nu_1 + \alpha_1)\]

\[G_6 = \mu_0 \mu_1 \mu_2 v_1\]

\[G_7 = \mu_0 \mu_1 \mu_2 v_1\]

\[G_8 = \mu_0 \mu_1 \mu_2 v_1\]

\[G_9 = \mu_0 \mu_1 \mu_2 v_1\]

\[G_{10} = \mu_0 \mu_1 \mu_2 v_1\]

\[G_{11} = \mu_0 \mu_1 \mu_2 v_1\]

\[G_{12} = \mu_0 \mu_1 \mu_2 v_1\]
From (3.16) we deduce that

\[ v_1 v_2 v_3 P_{10}(z) = a_1 z P_0(z) + a_2 P_0(z) + a_3 P_{00}(z) \]  

(3.20)

where

\[ a_1 = \mu_0 v_1 v_2 v_3; a_2 = (\lambda_0 + \alpha_0) v_1 v_2 v_3 \text{ and } a_3 = -\alpha_0 v_2 v_3. \]

From (3.14), we have that

\[ v_1 v_2 v_3 P_{10}(z) = b_1 z P_{00}(z) + b_2 P_0(z) \]  

(3.21)

where

\[ b_1 = \mu_0 v_1 v_2 v_3; b_2 = (\lambda_0 + \alpha_0 + \mu_0) v_2 v_3. \]

We can write the equations (3.15) as

\[ v_1 v_2 v_3 P_2(z) = c_1 z^2 P_0(z) + (cz + d) P_0(z) + (cz + d) P_{00}(z) \]  

(3.22)

where

\[ c_1 = \mu_0 v_1 v_2 v_3; c_2 = ((\lambda_0 + \alpha_0 + \mu_0) + (\lambda_0 + \alpha_0 + \mu_0) + (\lambda_0 + \alpha_0 + \mu_0) ) v_2 v_3 \]

\[ c_3 = -\alpha_0 v_1 v_2 v_3; c_4 = (s_0 + s_0 + s_0 + s_0) v_2 v_3. \]

From (3.16) we obtain

\[ v_1 v_2 v_3 z P_{10}(z) = f_1 z P_0(z) + (f_2 z + f_3 z) P_{00}(z) \]  

(3.25)

where

\[ f_1 = \mu_0 v_1 v_2 v_3; f_2 = (s_0 + s_0 + s_0 + s_0) v_2 v_3; f_3 = (2A + C) v_2 v_3; f_4 = (B + D) v_2 v_3 \]

\[ f_5 = (C + E) v_2 v_3; f_6 = F v_2 v_3; f_7 = -G v_2 v_3; f_8 = H v_2 v_3. \]

From (3.19) we obtain

\[ \lambda_0 v_1 v_2 v_3 z P_2(z) = g_{10} z^2 P_{00}(z) + (g_{01} z + g_{00} z) P_0(z) + (g_{01} z + g_{00} z) P_{00}(z) \]  

(3.26)

where

\[ g_1 = \lambda_0 G_1; g_2 = \lambda_0 G_2; g_3 = \lambda_0 G_3; g_4 = \lambda_0 G_4; g_5 = \lambda_0 G_5; g_6 = \lambda_0 G_6 \]

Now using the set of equations (3.20) to (3.25) we have that, after some tedious algebra the equality (3.13) can be expressed as follows

\[ v_1 v_2 v_3 P_{10}(z) = (l_1 z + l_2 z^2) P_0(z) + (l_1 z + l_2 z^2) P_{00}(z) \]  

(3.27)

where

\[ l_1 = \mu_0 v_1 v_2 v_3; l_2 = (s_0 + s_0 + s_0 + s_0) v_2 v_3 \]

Then we deduce from (3.26) and (3.27) that the generating function \( P_d(z) \) satisfies the following fourth-order differential equations.

\[ (A_1 z^2 + A_2 z^3) P_{10}(z) + (A_1 z^2 + A_2 z^3) P_{00}(z) \]  

\[ + (A_1 z^2 + A_2 z^3) P_0(z) + (A_1 z^2 + A_2 z^3) P_{00}(z) = 0 \]  

(3.28)

where

\[ A_1 = g_1; A_2 = l_1; A_3 = g_2; A_4 = g_3; A_5 = l_2; A_6 = g_4; A_7 = g_5; A_8 = g_6 - l_5; A_9 = g_7; A_{10} = l_6; A_{11} = g_{10}; A_{12} = g_{11}; A_{13} = g_{12}; A_{14} = g_{13}; A_{15} = g_{14}. \]
Numerical calculations were performed to obtain the values of the probabilities, for fixed values of parameters \( \{X(t), t \geq 0 \} \). Notice, that the stationary probabilities \( P_{ij} \) where

\[
P_{ij} = \lim_{t \to \infty} P_{ij}(t),
\]

and its derivatives in the above differential equation and rearranging its terms, we conclude that the sequence \( \{P_{ij}, j \geq 0 \} \) satisfies

\[
P_0 = x_{j=1} P_{0j} \chi_j \chi_{j=2} P_{02j} \geq 3 \quad ... (3.29)
\]

where

\[
\chi_{j=1} = - (A_{14} + j - 1)A_{14} + (j - 1)(j - 2)A_{14} + (j - 1)(j - 2)(j - 3)A_{14} + (j - 1)(j - 2)(j - 3)(j - 4)A_{14} \quad ... j \geq 4
\]

\[
\chi_{j=2} = - (A_{15} + j - 1)A_{15} + (j - 1)(j - 2)A_{15} + (j - 1)(j - 2)(j - 3)A_{15} + (j - 1)(j - 2)(j - 3)(j - 4)(j - 5)A_{15} \quad ... j \geq 4
\]

Replacing the generating function \( P(z) \) and its derivatives in the above differential equation and rearranging its terms, we conclude that

\[
\chi_{j=1} = -(A_{15} + A_{14} + A_{10}) \quad \chi_{j=2} = -(A_{15} + 2A_{10} + 2A_{7}) - A_{14} + 2A_{11} + 2A_{6} \quad ... A_{14} + 3A_{11} + 6A_{6} + A_{5}
\]

It follows by induction form (3.28) that

\[
P_{0j} = \frac{B_0}{C_0} P_{00} \quad P_{0j} = \eta_{j-1} P_{00} \quad ... (3.30)
\]

where

\[
\eta_{j-1} = \eta_j - 2\chi_{j=1} + \eta_j - 3\chi_{j=2} \quad ... j \geq 3
\]

Theorem 3.1. If \( \lim_{t \to \infty} \eta_j = + \infty \), then the stationary distribution of \( \{X(t), t \geq 0 \} \) is given by

\[
P_{00} = \left( \sum_{t=0}^{\infty} \sum_{j=0}^{\infty} M_{ij} \right)^{-1},
\]

\[
P_{ij} = M_{ij} P_{00}, \quad (i, j) \in \mathbb{E} - \{(0, 0)\}
\]

where

\[
M_{ij} = \left( \frac{\mu_i + \beta_{ij}}{\mu} \right) \eta_{j-1}, \quad j \geq 1
\]

\[
M_{ij} = \left( \frac{\mu_i + \beta_{ij}}{\mu} \right) \eta_{j-1}, \quad j \geq 0
\]

\[
M_{ij} = \left( \frac{\lambda + \beta_{ij}}{\mu} \right) \eta_{j-1}, \quad j \geq 0
\]

\[
M_{ij} = \left( \frac{\lambda + \beta_{ij}}{\mu} \right) \eta_{j-1}, \quad j \geq 0
\]

\[
M_{ij} = \left( \frac{\lambda + \beta_{ij}}{\mu} \right) \eta_{j-1}, \quad j \geq 0
\]

Notice, that the stationary probabilities \( P_{ij}(t) \) have been written in terms of \( P_{00} \). Hence, the computation of the stationary distribution of \( \{X(t), t \geq 0 \} \) is reduced to find \( P_{00} \) to any desired accuracy by using the equation (3.30).

**Numerical study**

Numerical calculations were performed to obtain the values of the probabilities, for fixed values of parameters \( \lambda_i = 1/i+1, \mu_i = 1/i+2 \) and \( \beta_{ij} = a_i(1-\delta_{ij}) + j\mu_i \) where \( \mu_i = 1/2i \) and \( a_i = 1/i+3, 0 \leq i \leq 4, j \geq 0 \). Some selective results are exhibited in table 4.1
The stationary analysis of a Retrial Queue With Multiserver In n-limited Capacity

Table 1 The steady state probabilities

<table>
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<tr>
<th>λn</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
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<td>p00</td>
<td>0.0052</td>
<td>0.0053</td>
<td>0.0055</td>
<td>0.0057</td>
<td>0.0059</td>
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<td>0.0001</td>
<td>0</td>
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<td>0.0004</td>
<td>0.0003</td>
<td>0.0002</td>
</tr>
<tr>
<td>p03</td>
<td>0.0003</td>
<td>0.0002</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>p04</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>p05</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>.</td>
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<td>.</td>
</tr>
<tr>
<td>p10</td>
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<td>0.0070</td>
<td>0.0109</td>
<td>0.0152</td>
<td>0.0198</td>
</tr>
<tr>
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<td>0.0006</td>
<td>0.0004</td>
<td>0.0002</td>
</tr>
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<td>0.0011</td>
<td>0.0010</td>
<td>0.0009</td>
</tr>
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<td>0.0004</td>
<td>0.0003</td>
<td>0.0003</td>
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<td>0</td>
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</tr>
<tr>
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<td>0.0137</td>
<td>0.0216</td>
<td>0.0302</td>
<td>0.0395</td>
</tr>
<tr>
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<td>0.0029</td>
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<td>0.0008</td>
</tr>
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We can extend it to n-limited capacity in similar manner.

CONCLUSION

If we use n-limited capacity model then we can get service in quickly.

References


How to cite this article:

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