ISSN: 0976-3031

Available Online at http://www.recentscientific.com

CODEN: IJRSFP (USA)

International Journal of Recent Scientific Research
Vol. 8, Issue, 6, pp. 17488-17493, June, 2017

International Journal of Recent Scientific Research

DOI: 10.24327/IJRSR

Research Article

AN ITERATIVE ALGORITHM FOR SOLVING FRACTIONAL LINEAR PROGRAMMING PROBLEM

Srikant Gupta[1]., Ather Aziz Raina*[2] and Irfan Ali[1]

[1]Department of Statistics & Operations Research, Aligarh Muslim University, Aligarh (India)
[2]Department of Mathematical Sciences, BGSB University, Rajouri, Jammu & Kashmir (India)

DOI: http://dx.doi.org/10.24327/ijrsr.2017.0806.0364

ARTICLE INFO

Article History:

Received 15th March, 2017
Received in revised form 25th April, 2017
Accepted 28th May, 2017
Published online 28th June, 2017

Key Words:

Fractional Linear Programming (FLP) Problem; Single Objective FLP Problem; Bi-objective FLP Problem; Multi-objective FLP Problem; Beale's Method.

ABSTRACT

In this paper, an iterative procedure integrated with Beal's method is given to solve the fractional linear programming problem (FLPP). The standard procedures for solving the FLPP are quite complex and monotonous to obtaining the optimum solution. To avoid this complexity of standard procedures, the proposed iterative procedure can be used to solving any FLPP easily in a minimum number of iterations. The solution step of proposed iterative procedure to solve FLPP is illustrated with the numerical examples.

INTRODUCTION

Fractional linear programming (FLP) problem is a special kind of non-linear programming problem in which the objective function is a ratio of linear functions and the constraints are linear functions. Because of its applications in real life situations, it can be found in several important fields such as finance, industrial, corporate, health care, hospital and production planning. It has got a substantial research and interest of many researchers. In the literature, several methods (Cooper and Charnes, 1962; Bitran and Novaes, 1973; Grandforoush, 1983) were recommended for solving this problem. Tantawy, (2008) proposed an iterative method based on the conjugate gradient projection method for solving FLP problem. Jeflea, (2003) presented several algorithms to solve non-linear fractional programming problem and suggest some new exact algorithms or heuristic procedures for problems formulated by means of fractional programming. Proposed a new method namely, denominator objective restriction method for finding an optimal solution to linear fractional programming problems Pandian and Jayalakshmi, (2013) construct two linear programming problems from the given linear fractional programming problem such that one is of maximization type and the other is of minimization type. Tantawy, (2013) present a new feasible direction method to find all efficient extreme points for bi-criterion linear fractional programming problems and the method is based on the conjugate gradient projection method. An initial feasible point is used to generate all efficient extreme points for this problem through a sequence of feasible directions of movement. Recently, Ahmed and Mishra, (2015) present a transformation method for solving linear fractional programming problem when the objective function is ratio function and the set of constraints is in the form of linear inequality. A new homotopy perturbation method is used by Das and Mandal, (2015) to find the exact solutions for the system of linear fractional programming problem with equality constraints. Saha *et al*., (2015) develop a new technique for solving linear fractional programming problem by converting it into a single linear programming problem, which can be solved by using any type of linear fractional programming technique.

In this paper, we have given an iterative algorithm for solving different types of single objective FLPP, bi-objective FLPP, and multi-objective FLPP. Also, the proposed iterative algorithm integrated with Beale's method has been used to solving the Multi-objective FLPP. The Beale's method is generally used to solving the non-linear programming problems with linear constraints. Numerical examples are given to illustrate the advantages of the proposed work over the other existing algorithm.

*Corresponding author: **Ather Aziz Raina**
Department of Mathematical Sciences, BGSB University, Rajouri, Jammu & Kashmir (India)

The remaining contents of this article are presented in different sections. Section 2, contains the statement or proposed technique related to the defined problem has been given. In, Section 3, a multi-iterative algorithm for solving the problem has been given. The numerical example has been illustrated in Section 4 which includes different types of FLP problem. Finally, analysis of the solution and concluding remarks has been presented in Section 5.

Statement of the Problem

Consider the modelling of a complex system where several fractional objectives are to be optimized at a time to get the optimal result. The general form of multi-objective fractional linear programming (MOFLP) problem function

$$Z_i^p(x) = \frac{p_i(x)}{d_i(x)} = \frac{\sum_{j=1}^{n} p_{ij}x_j + p_0}{\sum_{j=1}^{n} d_{ij}x_j + d_0}, \quad i = 1,2,...,k; \quad j = 1,2,.....,n; \quad p = 1,2,.....,l.$$

which must be maximized (or minimized) subject to

$$\begin{aligned} &\sum_{j=1}^{n} a_{ij}x_j \le b_i, \, i = 1,2,...,m_1, \\ &\sum_{j=1}^{n} a_{ij}x_j \ge b_i, \, i = m_1+1, \, m_1+2,...,m_2, \\ &\sum_{j=1}^{n} a_{ij}x_j = b_i, \, i = m_2+1, \, m_2+2,...,m, \\ &x_j \ge 0, \, j = 1,2,...,n, \end{aligned} \tag{1}$$

where $Z_i^p(x)$ is a vector of objective functions (criteria vector) and we suppose that $d(x) \ne 0, \; \forall \; x = (x_1, x_2, ..., x_n) \in S$ where S denotes a feasible set or set of the feasible solution defined by constraints. Because denominator $d(x) \ne 0, \; \forall \; x \in S$ without loss of generality we can assume that $d(x) > 0, \; \forall \; x \in S$. In this case, $d(x) < 0$, we can multiply numerator $p(x)$ and denominator $d(x)$ of objective function $Q(x)$ with (-1). We refer to a fractional programming problem in the most general form. Consider maximization FLP problem may be stated as:

where x, p, d are $n \times 1$ vectors, b is a $m \times 1$ vector and α, β are the scalar constants. It is assumed that the constraints $S = \{x : Ax \le \underline{b}, \; \underline{x} \ge 0\}$ is non-empty and bounded. Assume that the objective function $Z_i^{(p)}(x), \; i = 1,2,...,k$ is concave. Let $\underline{x}_B = \begin{pmatrix} x_{B1} \\ \vdots \\ x_{Bm} \end{pmatrix}$ be a basic feasible solution to FLP problem (1). It is also assumed that all the basic feasible solutions are non-degenerate. Let $\underset{m\times m}{B}$ denote the basis matrix corresponding to $\underline{x}_B$. Let the matrix $\underset{m\times n}{A}$ and the vector $\underset{n\times 1}{\underline{x}}$ is partitioned as $\underset{m\times n}{A} = \left(\underset{m\times m}{B} \Big/ \underset{m\times(n-m)}{R} \right)$ and $\underline{x} = \begin{pmatrix} \underset{m\times 1}{x_B} \\ \underset{(n-m)\times 1}{x_R} \end{pmatrix}$, where R consists of those columns of A which are not in B and $\underline{x}_R$ consists of $(n-m)$ non-basic variables.

The constraint equations $A\underline{x} = \underline{b}$ can be expressed as

$$(B/R)\begin{pmatrix} \underline{x}_B \\ \underline{x}_R \end{pmatrix} = \underline{b}$$

or $B\underline{x}_B + R\underline{x}_R = \underline{b}$

or $B\underline{x}_B = \underline{b} - R\underline{x}_R$

Pre-multiplication B^{-1} gives

$B^{-1}B\underline{x}_B = B^{-1}\underline{b} - B^{-1}R\underline{x}_R$

or $\underline{x}_B = B^{-1}\underline{b} - B^{-1}R\underline{x}_R$

$$\text{or } \underline{x}_B = \underline{y}_0 - \left(\underline{y}_1, \underline{y}_2, \dots, \underline{y}_j, \dots, \underline{y}_{n-m}\right)\underline{x}_R \tag{2}$$

where $B^{-1}\underline{b} = \underline{y}_0 = \begin{pmatrix} y_{10} \\ y_{20} \\ \vdots \\ y_{m0} \end{pmatrix}$ and $\underline{y}_j = \begin{pmatrix} y_{1j} \\ y_{2j} \\ \vdots \\ y_{mj} \end{pmatrix}$;

$j = 1,2,...,n-m$

is the j^{th} column of the $(m \times (n-m))$ matrix $B^{-1}R$. From equation (2) the i^{th} component x_{Bi} of $\underline{x}_B$ is given as

$$x_{Bi} = y_{i0} - \sum_{i=1}^{n-m} y_{ij}x_{Rj}, \quad j = 1,2,...,m \tag{3}$$

Using (3), Objective function can be expressed as a function of non-basic variables as

$$Z_i^{(p)}(x) = \frac{\alpha + p^T \underline{x}_R}{\beta + d^T \underline{x}_R}, \; i = 1,2,...,k \tag{4}$$

where α, β are the constant terms and $p^T\underline{x}_R$, $d^T\underline{x}_R$ are the linear part.

The basic feasible solution $\underline{x}_B$ and the corresponding value of the objective function are obtained by putting $x_{Rj} = 0$, $j = 1,2,...,n-m$ in equation (3) & equation (4) respectively.

Use Dinkelbach (1967) transformation method that reduces the solution of a problem to the solution of a sequence of linear programming problems.

$$F_p(x)=\sum_{i=1}^{k} p_i(x)-Z_i^{(p)}d_i(x),\ i=1,2,\ldots,k \tag{5}$$

where $Z_i^{(1)}$ is the initial value of the objective function at $x^{(i)}=\underline{x}=\begin{pmatrix} x_B \\ x_R \end{pmatrix}$

Using (5), $F_i^{(1)}$ can be differentiated w.r.t. x_{Rj} and we get

$$\frac{\partial F_p(x)}{\partial x_{Rj}}=\frac{\partial}{dx_{Rj}}\left(F_p(x)=\sum_{i=1}^{k} p_i(x)-Z_i^{(p)}d_i(x)\right) \tag{6}$$

$\dfrac{\partial F_i^{(1)}}{\partial x_{Rj}}$ gives the rate of change in objective function w.r.t. x_{Rj}. Thus, the maximum rate of change in objective function is to be considered which depends upon the value of x_{Rj} given by (6). Suppose the desired value of x_{Rj} after differentiation is $x_{Rj}^{(1)}$, $x_{Rj}^{(2)}$.

If max of $\left\{x_{Rj}^{(1)},x_{Rj}^{(2)}\right\}=x_{Rj}^{(1)}$, that is $x_{Rj}^{(1)}$ becomes a new basic variable. By making x_{Rj} a basic variable, we have to find minimum value at which one of the previous basic variables say $\underline{x}_B$ becomes a new non-basic variable by putting the value of x_{Rj} in (3).

Now, we have a new non-basic feasible solution with $m+1$ positive variables, m old basic variables and x_{Rj} as the new basic variables. The present solution is now a basic feasible solution to the set of constraints. When x_{Rj} becomes a basic variable equation (2) changes to

$$x_{Rj}=\frac{y_{i0}}{y_{ij}}-\sum_{i=1}^{n-m}\frac{x_{Bi}}{y_{ij}} \tag{7}$$

Using (7) x_{Rj} can be removed from (4) and we have basic variable and the objective function in terms of non-basic variable alone. The procedure is then repeated with the new basic feasible solution.

The process will terminate when $\dfrac{\partial F_i^{(1)}}{\partial x_{Rj}}\le 0$ for all non-basic variables.

Theorem 1: Transformation of FLP problem to LP problem

Vector $\underline{x}$ is an initial optimal solution of the FLP problem if and only if

$$F_p(\underline{x})=\max_{x\in s}\left\{\sum_{i=1}^{k} p_i(x)-Z_i^{(p)}d_i(x)\right\}=0$$

where $Z_i^{(p)}=\dfrac{p_i(\underline{x})}{d_i(\underline{x})}$ is the value of i^{th} the objective function at p^{th} iteration.

$F_p(x)$ is the linearization form of FLP problem at p^{th} iteration.

Proof: If vector $\underline{x}$ is initial optimal solution of the problem then

$$Z_i^{(p)}=\frac{p_i(\underline{x})}{d_i(\underline{x})}=\frac{p(x)}{d(x)},\quad \forall x\in s$$

The latter means that

$$p_i(x)-Z_i^{(p)}d_i(x)\le 0,\quad \forall x\in s$$

Taking into account equality, we obtain

$$\max_{x\in s}\left\{\sum_{i=1}^{k} p_i(x)-Z_i^{(p)}d_i(x)\right\}=0$$

Conversely, if vector $\underline{x}$ is an initial optimal solution of the problem then,

$$\max_{x\in s}\left\{\sum_{i=1}^{k}p_i(x)-Z_i^{(p)}d_i(x)\right\}\le\max_{x\in s}\left\{\sum_{i=1}^{k}p_i(\underline{x})-Z_i^{(p)}d_i(\underline{x})\right\}=0,\ x\in s$$

This means that vector $\underline{x}$ is an initial optimal solution of the FLP problem

Algorithm

Step1: Optimise fractional objectives and marked with $\left(Z_1^{(1)},Z_2^{(1)},\ldots,Z_k^{(1)}\right)$, subject to linear constraints.

Step2: Convert the FLP problems into standard form by introducing the slack variables.

Step3: Express the constraints in terms of non-basic variables.

Step4: Find the initial solution of $\left(Z_1^{(1)},Z_2^{(1)},\ldots,Z_k^{(1)}\right)$.

Step5: Convert the FLP problem into a linear form by using the Theorem (1).

Step6: After converting the FLP problem into a linear form (expressed by $F_p(x)$), check whether the initial solution is optimal or not.

Step7: Differentiate the linear form of FLP problem by non-basic variables. If the differentiating value of a linear form of FLP problem is negative then STOP. It will be our optimal solution, otherwise, goto next step.

Step8: Consider the maximum value of the differentiating function and the non-basic variable corresponds to that maximum value becomes basic.

Step9: While transforming the non-basic variable into the basic variable, we have to find the minimum value at which basic variable transformed into non-basic variable through the set of constraints defined earlier in terms of non-basic variables.

Step10: Now we have a new set of basic and non-basic variables put these variables in $\left(Z_i^p\right)$.

Step11: Find the next initial solution of FLP problem with new constraints and convert it into linear form and check whether it is optimal or not and go to step (5).

Numerical Illustration

Single Objective FLPP

$$\left.\begin{aligned} \max Z_1^{(1)} &= \frac{p(x)}{d(x)} = \frac{x_1+3x_2+2x_3}{2x_1+x_2+4x_3+1} \\ \textbf{s.t.}\quad & x_1+3x_2+6x_3 \le 8 \\ & 2x_1+x_2+4x_3 \le 5 \\ & x_1,x_2,x_3 \ge 0 \end{aligned}\right\} \tag{8}$$

Converting the FLP problem into standard form, we get

$$\left.\begin{aligned} \max Z_1^{(1)} &= \frac{p(x)}{d(x)} = \frac{x_1+3x_2+2x_3+0x_4+0x_5}{2x_1+x_2+4x_3+0x_4+0x_5+1} \\ \textbf{s.t.}\quad & x_1+3x_2+6x_3+x_4+0x_5 = 8 \\ & 2x_1+x_2+4x_3+0x_4+x_5 = 5 \\ & x_1,x_2,x_3,x_4,x_5 \ge 0 \end{aligned}\right\} \tag{9}$$

where $x_4, x_5 \ge 0$ are slack variables.

Now, express the basic variables in terms of non-basic variables

$$\left.\begin{aligned} x_4 &= 8-x_1-3x_2-6x_3 \\ x_5 &= 5-2x_1-x_2-4x_3 \end{aligned}\right\} \tag{10}$$

Let the first initial solution of the FLP problem (9) will be

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 8 \\ 5 \end{pmatrix} \text{ and } Z_1^{(1)} = 0$$

Now, by using the Theorem of Linearization, we have

$$F_1(x) = x_1+3x_2+2x_3 \tag{11}$$

Optimality check: Differentiating (11) w.r.t. x_1 and x_2, we get

$$\frac{\partial F_1(x)}{\partial x_1} = 1,\ \frac{\partial F_1(x)}{\partial x_2} = 3\ \&\ \frac{\partial F_1(x)}{\partial x_3} = 2$$

$$\max\left(\frac{\partial F_1(x)}{\partial x_1}, \frac{\partial F_1(x)}{\partial x_2}, \frac{\partial F_1(x)}{\partial x_3}\right) = \max(1,3,2) = 3$$

Thus x_2 will become basic with a value of $x_2 = 8/3$ and x_4 will become non-basic.

$\Rightarrow$ **(10)** becomes

$$\left.\begin{aligned} x_2 &= \frac{8}{3}-\frac{1}{3}x_1-2x_3-\frac{1}{3}x_4 \\ x_5 &= \frac{7}{3}-\frac{5}{3}x_1-2x_3+\frac{1}{3}x_4 \end{aligned}\right\}$$

Now, putting the value of the basic variable in (**9**), we get

$$Z_1^{(2)} = \frac{8-4x_3-x_4}{11/3+5/3x_1+2x_3-1/3x_4}$$

Let the second initial solution of the FLP problem, will be

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 8/3 \\ 0 \\ 0 \\ 7/3 \end{pmatrix} \text{ and } Z_1^{(2)} = \frac{24}{11}$$

Now again by using the Theorem of Linearization, we have

$$F_2(x) = -\frac{40}{11}x_1-\frac{92}{11}x_3-\frac{3}{11}x_4 \tag{12}$$

Optimality check: Differentiating (**12**) w.r.t. x_1 and x_2, we get

$$\frac{\partial F_2(x)}{\partial x_1} = -\frac{40}{11} < 0,\ \frac{\partial F_2(x)}{\partial x_3} = -\frac{92}{11} < 0\ \&$$

$$\frac{\partial F_2(x)}{\partial x_4} = -\frac{3}{11} < 0$$

$\Rightarrow$ It is an optimal solution with $x_1 = 0$, $x_2 = 8/3$ & $x_3 = 0$ and $Z^* = 24/11$

Bi- Objective FLP problem

$$\left.\begin{aligned} \max Z_1^{(1)} &= \frac{p_1(x)}{d_2(x)} = \frac{6x_1+3x_2+6}{5x_1+2x_2+5} \\ \max Z_2^{(1)} &= \frac{p_2(x)}{d_2(x)} = \frac{2x_1+3x_2+5}{x_1+x_2+7} \\ \textbf{s.t.}\quad & 4x_1-2x_2 \le 20;\ 3x_1+5x_2 \le 25; \\ & x_1,x_2 \ge 0 \end{aligned}\right\} \tag{13}$$

Converting the FLP problem into standard form, we get

$$\left.\begin{aligned} \max Z_1^{(1)} &= \frac{p_1(x)}{d_1(x)} = \frac{6x_1+3x_2+0x_3+0x_5+6}{5x_1+2x_2+0x_3+0x_4+5} \\ \max Z_2^{(1)} &= \frac{p_2(x)}{d_2(x)} = \frac{2x_1+3x_2+0x_3+0x_4+5}{x_1+x_2+0x_4+0x_5+07} \\ \textbf{s.t.}\ & 4x_1-2x_2+x_3+0x_4 = 20;\ 3x_1+5x_2+0x_3+x_4 = 25; \\ & x_1,x_2,x_3,x_4 \ge 0 \end{aligned}\right\} \tag{14}$$

where $x_3, x_4 \geq 0$ are slack variables.

Now, express the basic variables in terms of non-basic variables

$$\left.\begin{aligned} x_3 &= 20 - 4x_1 + 2x_2 \\ x_4 &= 25 - 3x_1 - 5x_2 \end{aligned}\right\} \tag{15}$$

Let the first initial solution of the BOFLP problem **(15)** will be

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 20 \\ 25 \end{pmatrix} \text{ with } Z_1^{(1)} = 6/5 \ \& \ Z_2^{(1)} = 5/7$$

Now by using the Theorem of Linearization, we have

$$F_1(x) = \frac{9}{7}x_1 + \frac{101}{35}x_2 \tag{16}$$

Optimality check: Differentiating (**16**) w.r.t. x_1 and x_2, we get

$$\frac{\partial F_1(x)}{\partial x_1} = 9/7 \ \& \ \frac{\partial F_1(x)}{\partial x_2} = 101/35$$

$$\max\left(\frac{\partial F_1(x)}{\partial x_1}, \frac{\partial F_1(x)}{\partial x_2}\right) = \max(9/7, 101/35) = 101/35$$

Thus x_2 will become basic with a value of $x_2 = 5$ and x_4 will become non-basic.

$\Rightarrow$ **(15)** becomes

$$\left.\begin{aligned} x_2 &= 5 - \frac{3}{5}x_1 - \frac{1}{5}x_4 \\ x_3 &= 30 - \frac{26}{5}x_1 - \frac{2}{5}x_4 \end{aligned}\right\} \tag{17}$$

Now, putting the value of the basic variable in (**14**), we get

$$\left.\begin{aligned} Z_1^{(2)} &= \frac{21 + 21/5x_1 - 3/5x_4}{15 + 19/5x_1 - 2/5x_4} \\ Z_2^{(2)} &= \frac{20 + 1/5x_1 - 3/5x_4}{12 + 2/5x_1 - 1/5x_4} \end{aligned}\right\} \tag{18}$$

Then the second initial solution of the BOFLP problem, will be

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ 30 \\ 0 \end{pmatrix} \text{with } Z_1^{(2)} = \frac{21}{15} \ \& \ Z_2^{(2)} = \frac{20}{12}$$

Now again by using the Theorem of Linearization, we have

$$\left. F_2(x) = -\frac{119}{75}x_1 - \frac{23}{75}x_4 \right\} \tag{19}$$

Optimality check: Differentiating **(19)** w.r.t x_1 and x_4, we get

$$\frac{\partial F_2(x)}{\partial x_1} = -\frac{119}{75} < 0 \ \& \ \frac{\partial F_2(x)}{\partial x_4} = -\frac{23}{75} < 0$$

By linearizing $F_2(x)$, we get the optimal solution of the BOFLP problem.

The solution of the above BOFLP problem is $x_1 = 0, \ x_2 = 5$ with $Z_1^* = 7/5$ & $Z_2^* = 5/3$

Multi – Objective FLP problem

$$\left.\begin{aligned} \max Z_1^{(1)} &= \frac{p_1(x)}{d_1(x)} = \frac{2x_1 + 3x_2}{x_1 + x_2 + 7} \\ \max Z_2^{(1)} &= \frac{p_1(x)}{d_1(x)} = \frac{x_1 + 3x_2}{2x_1 + x_2 + 5} \\ \max Z_3^{(1)} &= \frac{p_1(x)}{d_1(x)} = \frac{2x_1 + x_2}{x_1 + 3x_2 + 4} \\ \textit{s.t.} \quad & 3x_1 + x_2 \leq 4; \ x_1 + x_2 \leq 1; \ x_1, x_2 \geq 0 \end{aligned}\right\} \tag{20}$$

Converting the FLP problem into standard form, we get

$$\left.\begin{aligned} \max Z_1^{(1)} &= \frac{p_1(x)}{d_1(x)} = \frac{2x_1 + 3x_2 + 0x_3 + 0x_4}{x_1 + x_2 + 0x_3 + 0x_4 + 7} \\ \max Z_2^{(1)} &= \frac{p_1(x)}{d_1(x)} = \frac{x_1 + 3x_2 + 0x_3 + 0x_4}{2x_1 + x_2 + 0x_3 + 0x_4 + 5} \\ \max Z_3^{(1)} &= \frac{p_1(x)}{d_1(x)} = \frac{2x_1 + x_2 + 0x_3 + 0x_4}{x_1 + 3x_2 + 0x_3 + 0x_4 + 4} \\ \textit{s.t.} \quad 3x_1 + x_2 + x_3 + 0x_4 &= 4; \\ x_1 + x_2 + 0x_3 + x_4 &= 1; \\ x_1, x_2, x, x &\geq 0 \end{aligned}\right\} \tag{21}$$

where $x_3, x_4 \geq 0$ are slack variables.

Now, express the basic variables in terms of non-basic variables

$$\left.\begin{aligned} x_3 &= 4 - 3x_1 - x_2 \\ x_4 &= 1 - x_1 - x_2 \end{aligned}\right\} \tag{22}$$

Let the first initial solution of the MOFL problem (21) will be

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \\ 1 \end{pmatrix} \text{with } Z_1^{(1)} = 0, \ Z_1^{(1)} = 0 \ \& \ Z_1^{(1)} = 0$$

Now by using the Theorem of Linearization, we have

$$F_1(x) = 5x_1 + 7x_2 \tag{23}$$

Optimality check: Differentiating **(23)** w.r.t. x_1 and x_2, we get

$$\frac{\partial F_1(x)}{\partial x_1} = 5 \ \& \ \frac{\partial F_1(x)}{\partial x_2} = 7$$

$$\max\left(\frac{\partial F_1(x)}{\partial x_1}, \frac{\partial F_1(x)}{\partial x_2}\right) = \max(5,7) = 7$$

Thus, x_2 will become basic with a value of $x_2 = 1$ and x_4 will become non-basic.

$\Rightarrow$ **(22)** becomes

$$\left.\begin{aligned} x_2 &= 1 - x_1 - x_4 \\ x_3 &= 3 - 2x_1 + x_4 \end{aligned}\right\} \quad (24)$$

Now, putting the value of the basic variable in (21), we get

$$\left.\begin{aligned} Z_1^{(2)} &= \frac{3 - x_1 - 3x_4}{8 + 0x_1 - x_4} \\ Z_2^{(2)} &= \frac{3 - 2x_1 - 3x_4}{6 + x_1 - x_4} \\ Z_3^{(2)} &= \frac{1 + x_1 - x_4}{7 - 2x_1 - 3x_4} \end{aligned}\right\} \quad (25)$$

Let the second initial solution of the MOFLP problem , will be

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 3 \\ 0 \end{pmatrix} \text{ with } Z_1^{(2)} = \frac{3}{8},\ Z_2^{(2)} = \frac{3}{6} \ \& \ Z_3^{(2)} = \frac{1}{7}$$

Now again by using the Theorem of Linearization, we have

$$F_2(x) = \frac{-31}{4}x_1 - \frac{367}{56}x_4 \quad (26)$$

Optimality check: Differentiating (26) w.r.t. x_1 and x_2, we get

$$\frac{\partial F_2(x)}{\partial x_1} = -\frac{31}{4} < 0 \ \& \ \frac{\partial F_2(x)}{\partial x_4} = -\frac{367}{56} < 0$$

By linearizing $F_2(x)$, we get the optimal solution of the MOFLP problem.

The solution of the above MOFLP problem is $x_1 = 1,\ x_2 = 0$ with $Z_1^* = 3/8,\ Z_2^* = 1/2,\ Z_3^* = 2/5$

CONCLUSION

In this paper, we have given an iterative procedure for solving FLP problem. This solution procedure is based on the concept of Beale's method which is frequently applied for solving nonlinear programming problem with linear constraints. Numerical results are been presented also to indicate that algorithm could solve reasonably large size problems in minimum numbers of iteration with minimum computational time. Thus, it has been concluded that Beale's method provides an optimal solution in the minimum number of iterations. Because of its simplicity, anyone can easily adopt this method to obtain the optimal solution for the fractional problem. The proposed iterative procedure can easily solve the single as well as multi-objective FLPP. The proposed iterative procedure is can easily solve the single as well as multi-objective FLPP. This unique feature of the proposed work is made it more useful over the other existing algorithms.

References

1. Ahmed, M.K.I., & Mishra, S.K. (2015). Transformation technique to solve multi-objective linear fractional programming problem. *International Journal of Applied Research*, 1(7), 813-815.
2. Bitran, G.R., & Novaes, A.G. (1973). Linear programming with a fractional function. *Operations Research*, 21(4), 22–29.
3. Charnes, A., & Cooper, W.W. (1962). Programs with linear fractional functions. *Naval Research Logistics Quarterly*, 9, 181–196.
4. Das, S.K., & Mandal, T. (2015). Solving linear fractional programming problems using a new Homotopy Perturbation method. *Operations Research and Applications : An International Journal*, 2(3), 1-15.
5. Grandforoush, A. (1983). An advance dual algorithm with constraint relaxation for all-integer programming. *Naval Research Logistics Quarterly*, 30, 133-143.
6. Jeflea, A. (2003). A parametric study for solving non-linear fractional problems. *An. St. Univ. Ovidius Constanta*, 11(2), 87–92.
7. Pandian, P., & Jayalakshmi, M. (2013). On solving linear fractional programming problems. *Modern Applied Science*, 7(6), 90-100.
8. Saha, S.K., Hossain, M.R., Uddin, M.K., & Mondal, R.N. (2015). A new approach of solving linear fractional programming problem (LFP) by using computer algorithm. *Open Journal of Optimization*, 4, 74-86.
9. Tantawy, S.F. (2008). A new procedure for solving linear fractional programming problems. *Mathematical and Computer Modelling*, 48, 969-973.
10. Tantawy, S.F. (2013). A new method for solving bi criterion linear fractional programming problems. *International Journal of Engineering and Innovative Technology*, 3(3), 128-133.

How to cite this article:

Srikant Gupta *et al.*2017, An Iterative Algorithm For Solving Fractional Linear Programming Problem. *Int J Recent Sci Res.* 8(6), pp. 17389-17393. DOI: http://dx.doi.org/10.24327/ijrsr.2017.0806.0364

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