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Research Article

SUM SQUARE PRIME LABELING OF SOME TREE GRAPHS

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ABSTRACT

Sum square prime labeling of a graph is the labeling of the vertices with {0,1,2---------,p-1} and the edges with square of the sum of the labels of the incident vertices. The greatest common incidence number of a vertex (***gcin***) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the gcin of each vertex of degree greater than one is one, then the graph admits sum square prime labeling. Here we identify some tree graphs for sum square prime labeling.

INTRODUCTION

All graphs in this paper are trees, connected, finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4]. Some basic concepts are taken from [1] and [2]. In this paper we investigated sum square prime labeling of some tree graphs.

Definition: Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

MAIN RESULTS

Definition: Let G = (V(G),E(G)) be a graph with p vertices and q edges . Define a bijection
f : V(G) → {0,1,2,3,----------------,p-1} by $f(v_i)$ = i 1 , for every i from 1 to p and define a 1-1 mapping f_{ssp} : E(G) → set of natural numbers N by $f_{ssp}(uv)$ = $\{f(u) + f(v)\}^2$. The induced function f_{ssp} is said to be a sum square prime labeling, if for each vertex of degree at least 2, the gcin is one labels of the incident edges is 1.

Definition: A graph which admits sum square prime labeling is called a sum square prime graph.

Definition: A graph G (V,E) obtained by a path by attaching exactly two pendent edges to each vertices of the path is called a centipede graph.

Definition: A graph G (V,E) obtained by a path by attaching exactly two pendent edges to each internal vertices of the path is called a Twig graph.

Definition: A coconut tree CT (m,n) is the graph obtained from the path P_n by appending m new pendant edges at an end vertex of P_n.

Definition: The Bistar graph B(m,n) is the graph obtained from path P_2 by joining *m* pendant edges to one end and *n* pendant edges to other end.

Definition: The graph $S_{m,n}$ is the graph obtained by joining the end vertices of m copies of path P_n to a single vertex .

Definition: The H- graph of a path P_n is the graph obtained from two copies of P_n with vertices v_1,v_2,-------------------,v_n

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and u_1,u_2,------------------,u_n by joining the vertices $v_{(\frac{n+1}{2})}$ and $u_{(\frac{n+1}{2})}$ if n is odd and the vertices $v_{(\frac{n+2}{2})}$ and $u_{\frac{n}{2}}$ if n is even.

Theorem: Centipede graph C(2,n) admits sum square prime labeling.

Proof: Let G = C(2,n) and let v_1,v_2,----------------,v_{3n} are the vertices of G

Here |V(G)| = 3n and |E(G)| = 3n-1
Define a function f : V → {0,1,2,3,----------------,3n-1 } by
$f(v_i)$ = i-1 , i = 1,2,------,3n

For the vertex labeling f, the induced edge labeling f_{ssp} is defined as follows

$f_{ssp}(v_{3i-2}\ v_{3i-1})$ = $(6i-5)^2$, i = 1,2,------------,n
$f_{ssp}(v_{3i-1}\ v_{3i})$ = $(6i-3)^2$, i = 1,2,------------,n
$f_{ssp}(v_{3i-1}\ v_{3i+2})$ = $(6i-1)^2$, i = 1,2,------------,n-1

Clearly f_{ssp} is an injection.
gcin of (v_{3i-1})= gcd of $\{f_{ssp}(v_{3i-2}v_{3i-1}), f_{ssp}(v_{3i}\ v_{3i-1})\}$
= gcd of {6i-5,6i-3}
= 1, 1 ≤ i ≤n

So, ***gcin*** of each vertex of degree greater than one is 1.
According to this pattern C(2,n), admits sum square prime labeling.

Theorem: Twig graph $T_w(2,n)$ admits sum square prime labeling. , when n is even.

Proof: Let G = $T_w(2,n)$ and let v_1,v_2,----------------,v_{3n-4} are the vertices of G
Here |V(G)| = 3n-4 and |E(G)| = 3n-5

Define a function f : V → {0,1,2,3,----------------,3n-5 } by

$f(v_i)$ = i-1 , i = 1,2,------,3n-4

For the vertex labeling f, the induced edge labeling f_{ssp} is defined as follows

$f_{ssp}(v_{3i-2}\ v_{3i-1})$ = $(6i-5)^2$, i = 1,2,------------,n-2
$f_{ssp}(v_{3i-1}\ v_{3i})$ = $(6i-3)^2$, i = 1,2,------------,n-2
$f_{ssp}(v_{3i-1}\ v_{3i+2})$ = $(6i-1)^2$, i = 1,2,------------,n-3
$f_{ssp}(v_{3n-7}\ v_{3n-5})$ = $(6n-14)^2$.
$f_{ssp}(v_2\ v_{3n-4})$ = $(3n-4)^2$.

Clearly f_{ssp} is an injection.

gcin of (v_{3i-1}) =gcd of $\{f_{ssp}(v_{3i-2}v_{3i-1}), f_{ssp}(v_{3i}\ v_{3i-1})\}$
= gcd of {6i-5,6i-3}
= 1, 1 ≤ i ≤n-2
So, ***gcin*** of each vertex of degree greater than one is 1.

According to this pattern $T_w(2,n)$, admits sum square prime labeling.

Theorem: Twig graph $T_w(2,n)$ admits sum square prime labeling, when n is odd.

Proof: Let G = $T_w(2,n)$ and let v_1,v_2,----------------,v_{3n-4} are the vertices of G

Here |V(G)| = 3n-4 and |E(G)| = 3n-5
Define a function f : V → {0,1,2,3,----------------,3n-5 } by

$f(v_i)$ = i-1 , i = 1,2,------,3n-4

For the vertex labeling f, the induced edge labeling f_{ssp} is defined as follows

$f_{ssp}(v_{3i-2}\ v_{3i-1})$ = $(6i-5)^2$, i = 1,2,------------,n-2
$f_{ssp}(v_{3i-1}\ v_{3i})$ = $(6i-3)^2$, i = 1,2,------------,n-2
$f_{ssp}(v_{3i-1}\ v_{3i+2})$ = $(6i-1)^2$, i = 1,2,------------,n-3
$f_{ssp}(v_2\ v_{3n-5})$ = $(3n-5)^2$.
$f_{ssp}(v_{3n-7}\ v_{3n-4})$ = $(6n-13)^2$.

Clearly f_{ssp} is an injection.

gcin of (v_{3i-1})=gcd of $\{f_{ssp}(v_{3i-2}v_{3i-1}), f_{ssp}(v_{3i}\ v_{3i-1})\}$
= gcd of {6i-5,6i-3}
= 1,
1 ≤ i ≤n-2
So, ***gcin*** of each vertex of degree greater than one is 1.
According to this pattern $T_w(2,n)$, admits sum square prime labeling.

Theorem: Coconut Tree graph CT (m,n) admits sum square prime labeling.

Proof: Let G = CT (m,n) and let v_1,v_2,----------------,v_{m+n} are the vertices of G

Here |V(G)| = m+n and |E(G)| = m+n-1

Define a function f : V → {0,1,2,3,----------------,m+n-1 } by
$f(v_i)$ = i-1 , i = 1,2,------,m+n

For the vertex labeling f, the induced edge labeling f_{vnsp} is defined as follows

$f_{vnsp}(v_i\ v_{i+1})$ = $(2i-1)^2$, i = 1,2,------------,n-1
$f_{ssp}(v_n\ v_{n+i})$ = $(2n+i-2)^2$, i = 1,2,------------,m
Clearly f_{ssp} is an injection.

gcin of (v_{i+1}) = gcd of $\{f_{ssp}(v_i v_{i+1}), f_{ssp}(v_{i+1}\ v_{i+2})\}$
= gcd of $\{(2i-1)^2,(2i+1)^2\}$
= gcd of {(2i-1),(2i+1)}
= 1,
1 ≤ i ≤n-1

So, ***gcin*** of each vertex of degree greater than one is 1.

According to this pattern CT(m,n), admits sum square prime labeling.

Theorem: Bistar graph B(m,n) admits sum square prime labeling.

Proof: Let G = B(m,n) and let v_1,v_2,----------------,v_{m+n+2} are the vertices of G

Here |V(G)| = m+n+2 and |E(G)| = m+n+1
Define a function f : V → {0,1,2,3,----------------,m+n+1 } by

$f(v_i)$ = i-1 , i = 1,2,------,m+n+2

For the vertex labeling f, the induced edge labeling f_{ssp} is defined as follows

$f_{ssp}(v_1\ v_{i+1})$ = i^2, i = 1,2,------------,m
$f_{ssp}(v_1\ v_{m+2})$ = $(m+1)^2$
$f_{ssp}(v_{m+2}\ v_{m+2+i})$ = $(2m+i+2)^2$, i = 1,2,------------,n
Clearly f_{ssp} is an injection.
gcin of (v_1) = 1.
gcin of (v_{m+2}) = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.
According to this pattern B(m,n), admits sum square prime labeling.

Theorem: The graph $S_{m,n}$ admits sum square prime labeling, when n is odd.

Proof: Let G = $S_{m,n}$ and let $v_1,v_2,$----------------$,v_{mn+1}$ are the vertices of G

Here |V(G)| = mn+1 and |E(G)| = mn

Define a function f : V → {0,1,2,3,----------------,mn } by
$f(v_i) = i$, i = 1,2,------, mn+1
f(u) = 0

For the vertex labeling f, the induced edge labeling f_{ssp} is defined as follows

$f_{ssp}(u\, v_{(i-1)n+1})$ = $\{(i-1)\, n+1\}^2$, i = 1,2,------------,m
$f_{ssp}(v_{(j-1)n+i}\, v_{(j-1)n+i+1})$ = $(2(j-1)\, n+2i+1)^2$, j = 1,2,-----------,m
i = 1,2,-------------,n-1

Clearly f_{ssp} is an injection.

gcin of (u) = 1.
gcin of (v_{5i-4}) = 1, $1 \le i \le m$
gcin of ($v_{(j-1)n+i+1}$) = 1,
$1 \le j \le m$, $1 \le i \le n-2$

So, ***gcin*** of each vertex of degree greater than one is 1.
According to this pattern $S_{m,n}$, admits sum square prime labeling.

Theorem: H graph of path P_n admits sum square prime labeling.
Proof: Let G = H(P_n) and let $v_1,v_2,$----------------$,v_{2n}$ are the vertices of G

Here |V(G)| = 2n and |E(G)| = 2n-1
Define a function f : V → {1,2,3,----------------,2n-1 } by

$f(v_i) = i-1$, i = 1,2,------,2n

Case(i) n is even

For the vertex labeling f, the induced edge labeling f_{ssp} is defined as follows

$f_{ssp}(v_i\, v_{i+1})$ = 2i-1, i = 1,2,------------,n-1
$f_{ssp}(v_{n+i}\, v_{n+i+1})$ = 2n+2i-1, i = 1,2,------------,n-1
$f_{ssp}(v_{(\frac{n+2}{2})}\, v_{(\frac{3n}{2})})$ = 2n-1

Clearly f_{ssp} is an injection.

gcin of (v_{i+1}) = 1, $1 \le i \le n-2$
gcin of (v_{n+i+1}) = 1, $1 \le i \le n-2$

So, ***gcin*** of each vertex of degree greater than one is 1.
According to this pattern H(P_n), admits sum square prime labeling.

Case (ii) n is odd

For the vertex labeling f, the induced edge labeling f_{ssp} is defined as follows

$f_{ssp}(v_i\, v_{i+1})$ = 2i-1, i = 1,2,------------,n-1
$f_{ssp}(v_{n+i}\, v_{n+i+1})$ = 2n+2i-1, i = 1,2,------------,n-1
$f_{ssp}(v_{(\frac{n+1}{2})}\, v_{(\frac{3n+1}{2})})$ = 2n-1

Clearly f_{ssp} is an injection.

gcin of (v_{i+1}) = 1, $1 \le i \le n-2$
gcin of (v_{n+i+1}) = 1, $1 \le i \le n-2$
So, ***gcin*** of each vertex of degree greater than one is 1.
According to this pattern H graph of path P_n, admits sum square prime labeling.

CONCLUSION

In this paper we proved that some tree graphs admits sum square prime labeling. We developed sum square prime labeling based on the definition 'greatest common incidence number'. We think that other tree graphs must also satisfy sum square prime labeling. So this work is open for all researchers to do extension work.

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