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STUDY OF PROPERTIES OF CERTAIN FAMILY OF UNIVALENT FUNCTIONS ASSOCIATED WITH SUBORDINATION

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ABSTRACT

There are many subclasses of univalent functions. The objectives of this paper is to introduce new classes and we have attempted to obtain Partial sums, Weighted mean, Arithmetic mean and Linear combination for the classes (A, B, α) and K (A, B, α)

INTRODUCTION

Let Tdenote the class of functions $f(z)$ of the form

$$f(z) = z \quad \sum_{k=2}^{\infty} a_k z^k, \qquad a_k \geq 0 \qquad \text{.........(1)}$$

which are univalent in the unit disc $U = \{z : z \in \quad and \mid z \mid < 1\}$

Definition: A function $f(z) \in T$ is said to be close to convex of order μ $(0 \leq \mu < 1)$ if $Re\,\{f'(z)\} > \mu$ for all $z \in U$

A function $f(z) \in T$ is said to be in the subclass $H(\mu)$ of starlike function if

$$Re\left(\frac{zf'(z)}{f(z)}\right) > \mu, \; z \in U \;\; 0 \leq \mu < 1$$

Definition: A function $f(z) \in T$ is said to be in the subclass $G(\mu)$ of convex function if

$$Re\left(1 + \frac{zf'(z)}{f(z)}\right) > \mu, \qquad z \in U$$

Definition: Let$f(z) = z \quad \sum_{k=2}^{\infty} a_k z^k$, $\quad g(z) = z \quad \sum_{k=2}^{\infty} a_k z^k$, $\; a_k \geq 0, b_k \geq 0$ then the convolution is defined as

$$f(z) \quad g(z) = z \quad \sum_{k=2}^{\infty} a_k b_k z^k \qquad \text{... ... (1.3.1)}$$

Definition: If f and g are regular in U, we say that f is subordinate to g, denoted by $f \quad g$ or $f(z) \quad g(z)$, if there exist a Schwarz function w, which is regular in U with $w(0) = 0$ and $|w(z)| < 1$
$z \in U$such that $f(z) = g(w(z))$, $z \in U$. In particular if g is univalent in U , we have the equivalence
$f(z) \quad g(z)$if and only if $f(0) = g(0)$ and $f(U) \subset g(U)$

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Definition: We say that a function $f(z) \in T$is in the class (A, B, α)if it satisfy

$$\frac{zf'(z) + \alpha z^2 f''(z)}{\alpha zf'(z) + (1 \quad \alpha) f(z)} \quad \frac{1 + Az}{1 + Bz} \qquad \ldots \ldots .. (1.5.1)$$

for $0 < \alpha \leq 1, \quad 1 \leq B < A \leq 1$

Furthermore a function $f(z) \in T$is said to belong to the class $K \quad (A, B, \alpha)$if and only if $zf'(z) \in \quad (A, B, \alpha)$.

***Theorem* A** function $f(z) = z \quad \sum_{k=2}^{\infty} a_k z^k, \quad a_k \geq 0$ is in (A, B, α) if and only if

$$\sum_{k=2}^{\infty} \{k + \alpha k^2 \quad 2\alpha k \quad (1 \quad \alpha) \quad [B(k + \alpha k(k \quad 1)) \quad A(\alpha k + 1 \quad \alpha)]\} a_k \leq (A \quad B)$$

Corollary If$f(z) \in \quad (A, B, \alpha)$then

$$a_k \leq \frac{(A \quad B)}{k + \alpha k^2 \quad 2\alpha k \quad (1 \quad \alpha) \quad [B(k + \alpha k(k \quad 1)) \quad A(\alpha k + 1 \quad \alpha)]}$$

and the equality holds for

$$f(z) = z \quad \frac{(A \quad B)}{k + \alpha k^2 \quad 2\alpha k \quad (1 \quad \alpha) \quad [B(k + \alpha k(k \quad 1)) \quad A(\alpha k + 1 \quad \alpha)]} z^k$$

Theorem: **A** function $f(z) = z \quad \sum_{k=2}^{\infty} a_k z^k, \quad a_k \geq 0$ is in$K \quad (A, B, \alpha)$ if and only if

$$\sum_{k=2}^{\infty} \{[k^2(1 \quad 2\alpha) + \alpha k^3 \quad k(1 \quad \alpha)] \quad A[\ \alpha k^2 \quad k(1 \quad \alpha)] \quad B[k^2 + \alpha k^2(k \quad 1)]\} a_k \leq (A \quad B)$$

Corollary If$f(z) \in K \quad (A, B, \alpha)$ then

$$a_k \leq \frac{(A \quad B)}{[k^2(1 \quad 2\alpha) + \alpha k^3 \quad k(1 \quad \alpha)] \quad A[\ \alpha k^2 \quad k(1 \quad \alpha)] \quad B[k^2 + \alpha k^2(k \quad 1)]}$$

and the equality holds for

$$f(z) = z \quad \frac{(A \quad B)}{[k^2(1 \quad 2\alpha) + \alpha k^3 \quad k(1 \quad \alpha)] \quad A[\ \alpha k^2 \quad k(1 \quad \alpha)] \quad B[k^2 + \alpha k^2(k \quad 1)]} z^k$$

Partial Sums

Following the earlier works by G. Murugusundaramoorthy, T. Rosy and K. Muthunagai [1] on partial sums of regular function . In this section we obtain lower bounds for the ratios of $Ref(z)$ to $f_k(z)$ and $f'(z)$ to $f'_k(z)$

Theorem Let$f(z) = z + \sum_{k=2}^{\infty} a_k z^k$, $f(z) \in \quad (A, B, \alpha)$and define the partial sums $f_1(z)$and $f_n(z)$, by

$f_1(z) = z$and$f_n(z) = z + \sum_{k=2}^{n} a_k z^k, n \in \quad /\{1\}$ ……(2.1.1)

Suppose also that

$\sum_{k=2}^{\infty} b_k |a_k| \leq 1$ ……………..(2.1.2)

Where

$$b_k = \frac{\{k + \alpha k^2 \quad 2\alpha k \quad (1 \quad \alpha) \quad [B(k + \alpha k(k \quad 1)) \quad A(\alpha k + 1 \quad \alpha)]\}}{A \quad B}$$

Then $\in \quad (A, B, \alpha)$. Furthermore,

$Re\left\{\frac{f(z)}{f_n(z)}\right\} > 1 \quad \frac{1}{b_{n+1}}, z \in U, n \in$ …….. (2.1.3)

And

$Re\left\{\frac{f_n(z)}{f(z)}\right\} > \frac{b_{n+1}}{1+b_{n+1}}$ …….. (2.1.4)

Proof- It is easily seen that $f_1(z) \in \quad (A, B, \alpha)$and $f \in \quad (A, B, \alpha)$

Next, for coefficient b_n we can verify that

$b_{k+1} > b_k > 1 \ldots \ldots (2.1.5)$

By setting

$$_1(z) = \ b_{n+1}\left\{\frac{f(z)}{f_n(z)} \quad \left(1 \quad \frac{1}{b_{n+1}}\right)\right\}$$

$$= 1 + \frac{b_{n+1}\sum_{k=n+1}^{\infty} a_k z^{k-1}}{1 + \sum_{k=2}^{n} a_k\, z^{k-1}}$$

We write

$$\left|\frac{{}_1(z) \quad 1}{{}_1(z)+1}\right| \leq \frac{b_{n+1}\sum_{k=n+1}^{\infty}|a_k|}{2 \quad 2\sum_{k=2}^{n}|a_k| \quad b_{n+1}\sum_{k=n+1}^{\infty}|a_k|} \qquad \ldots \ldots (2.1.6)$$

$$\left|\frac{{}_1(z) \quad 1}{{}_1(z)+1}\right| \leq 1$$

if

$$\frac{b_{n+1}\sum_{k=n+1}^{\infty}|a_k|}{2 \quad 2\sum_{k=2}^{n}|a_k| \quad b_{n+1}\sum_{k=n+1}^{\infty}|a_k|} \leq 1$$

Which is equivalent to

$$\sum_{k=2}^{n}|a_k| + b_{n+1}\sum_{k=n+1}^{\infty}|a_k| \leq 1 \qquad \ldots \ldots (2.1.7)$$

It is sufficient to show that the left hand side of (2.1.7) is bounded by $\sum_{k=2}^{\infty} b_k a_k$

Which is equivalent to

$$\sum_{k=2}^{\infty}(b_k \quad 1)\, a_k + \sum_{k=n+1}^{\infty}(b_k \quad b_{k+1})\, a_k \geq 0$$

This complete the proof of (2.1.3). In order to see that

$$f(z) = z + \frac{z^{n+1}}{b_{n+1}}$$

Gives the sharp result, we observe that for $z = re^{i\pi/n}$

$$\lim_{z\to 1_-}\frac{f(z)}{f_n(z)} = \lim_{z\to 1_-}\left(1 + \frac{z^n}{b_{n+1}}\right) = 1 \quad \frac{1}{b_{n+1}}$$

Similarly If we define

$${}_2(z) = \ (1 + b_{n+1})\left\{\frac{f_n(z)}{f(z)} \quad \left(1 \quad \frac{b_{n+1}}{1 + b_{n+1}}\right)\right\}$$

$$= 1 \quad \frac{(1 + b_{n+1})\sum_{k=n+1}^{\infty} a_k z^{k-1}}{1 + \sum_{k=2}^{\infty} a_k\, z^{k-1}}$$

$$\left|\frac{{}_2(z) \quad 1}{{}_2(z)+1}\right| \leq \frac{(1 + b_{n+1})\sum_{k=n+1}^{\infty}|a_k|}{2 \quad 2\sum_{k=2}^{n}|a_k| \quad (1 \quad b_{n+1})\sum_{k=n+1}^{\infty}|a_k|} \leq 1$$

This last inequality is equivalent to

$$\sum_{k=2}^{n} a_k + b_{n+1}\sum_{k=n+1}^{\infty} a_k \leq 1 \ \ldots \ldots (2.1.8)$$

It is sufficient to show that the left hand side of (2.1.8) is bounded by $\sum_{k=2}^{\infty} b_k\, a_k$

Which is equivalent to

$$\sum_{k=2}^{\infty}(b_k \quad 1)\, a_k + \sum_{k=n+1}^{\infty}(b_k \quad b_{k+1})\, a_k \geq 0$$

This complete the proof of (2.1.4).

Theorem If $f(z) = \ z + \sum_{k=2}^{\infty} a_k\, z^k$ such that $f(z) \in \quad (A, B, \alpha)$.Then

$$Re\left\{\frac{f'(z)}{f'_n(z)}\right\} \geq 1 \quad \frac{n+1}{b_{n+1}} \qquad \ldots \ldots \ldots \ldots \ldots (2.2.1)$$

Proof: By setting

$$H(z) = b_{n+1}\left\{\frac{f'(z)}{f'_n(z)} \quad \left(1 \quad \frac{n+1}{b_{n+1}}\right)\right\}$$

$$= 1 + \frac{\frac{b_{n+1}}{n+1}\sum_{k=n+1}^{\infty} k a_k z^{k-1}}{1 + \sum_{k=2}^{n} k a_k z^{k-1}}$$

$$\left|\frac{H(z) \quad 1}{H(z)+1}\right| \leq \frac{\frac{b_{n+1}}{n+1}\sum_{k=n+1}^{\infty} k|a_k|}{2 \quad 2\sum_{k=2}^{n} k|a_k| \quad \frac{b_{n+1}}{n+1}\sum_{k=n+1}^{\infty} k|a_k|}$$

Now

$$\left|\frac{H(z) \quad 1}{H(z)+1}\right| \leq 1$$

If

$$\sum_{k=2}^{n} k|a_k| + \frac{b_{n+1}}{n+1}\sum_{k=n+1}^{\infty} k|a_k| \leq 1 \qquad \ldots\ldots(2.2.2)$$

Since the left hand side of (2.2.2) is bounded above by $\sum_{k=2}^{\infty} b_k|a_k|$ if

$$\sum_{k=2}^{n}(b_k \quad k)|a_k| + \sum_{k=n+1}^{\infty}\left(b_k \quad \frac{b_{n+1}}{n+1}k|a_k|\right) \geq 0 \ldots\ldots(2.2.3)$$

This completes the proof. The result is sharp for

$$f(z) = z + \frac{z^{n+1}}{b_{n+1}}$$

Theorem If $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$ such that $f(z) \in \quad (A,B,\alpha)$.Then

$$Re\left\{\frac{f'_n(z)}{f'(z)}\right\} \geq \frac{b_{n+1}}{n+1+b_{n+1}} \ldots\ldots\ldots\ldots\ldots(2.3.1)$$

PROOF: By setting

$$H(z) = [(n+1) + b_{n+1}]\left\{\frac{f'_n(z)}{f'(z)} \quad \frac{b_{n+1}}{n+1+b_{n+1}}\right\}$$

$$= 1 \quad \frac{\left(1+\frac{b_{n+1}}{n+1}\right)\sum_{k=n+1}^{\infty} k a_k z^{k-1}}{1 + \sum_{k=2}^{n} k a_k z^{k-1}}$$

$$\left|\frac{H(z) \quad 1}{H(z)+1}\right| \leq \frac{\left(1+\frac{b_{n+1}}{n+1}\right)\sum_{k=n+1}^{\infty} k|a_k|}{2 \quad 2\sum_{k=2}^{n} k|a_k| \quad \left(1+\frac{b_{n+1}}{n+1}\right)\sum_{k=n+1}^{\infty} k|a_k|}$$

Now

$$\left|\frac{H(z) \quad 1}{H(z)+1}\right| \leq 1$$

If

$$\sum_{k=2}^{n} k|a_k| + \left(1+\frac{b_{n+1}}{n+1}\right)\sum_{k=n+1}^{\infty} k|a_k| \leq 1 \ldots\ldots(2.3.2)$$

Since the left hand side of (2.3.2) is bounded above by $\sum_{k=2}^{\infty} b_k|a_k|$ if

$$\sum_{k=2}^{n}(b_k \quad k)|a_k| + \sum_{k=n+1}^{\infty}\left(b_k \quad \left(1+\frac{b_{n+1}}{n+1}\right)k|a_k|\right) \geq 0 \ldots\ldots(2.3.3)$$

This completes the proof.

Weighted Mean, Arithmetic Mean and Linear Combination

Following the earlier works by W.G.Asthan, H.D. Mustafa and E.K.Mouajeeb[2] weighted mean, arithmetic mean and linear combination of regular function.

Definition: Let f, $g \in \quad (A,B,\alpha)$ then the weighted mean w_{fg} of f and g is defined as

$$w_{fg} = \frac{1}{2}[(1 \quad t)f(z) + (1+t)g(z)]\,, 0 < t < 1$$

Definition: Let $f_i(z) = z \quad \sum_{k=2}^{\infty} a_{i,k}\, z^k$, $i = 1,2,3,\dots,m$ be the functions in the class (A,B,α) then the arithmetic mean of f_i ($i = 1,2,3,\dots,m$) is defined by

$$g(z) = \frac{1}{m}\sum_{i=1}^{m} f_i(z)$$

Definition: Let $f_i(z) = z \quad \sum_{k=2}^{\infty} a_{i,k}\, z^k$, $i = 1,2,3,\dots,m$ be the functions in the class (A,B,α) then the linear combination of f_i ($i = 1,2,3,\dots,m$) is defined by

$G(z) = \sum_{i=1}^{m} k_i f_i(z)$, where $\sum_{i=1}^{m} k_i = 1$

Theorem Let $f, g \in \quad (A,B,\alpha)$. Then the weighted mean w_{fg} of f and g is also in the class (A,B,α).

Proof: By DEFINITION 3.1, we have

$$w_{fg} = \frac{1}{2}[(1 \quad t)f(z) + (1+t)g(z)]$$
$$= \frac{1}{2}\left[(1 \quad t)\left(z \quad \sum_{k=2}^{\infty} a_k\, z^k\right) + (1+t)\left(z \quad \sum_{k=2}^{\infty} b_k\, z^k\right)\right]$$
$$= z \quad \sum_{k=2}^{\infty} \frac{1}{2}[(1 \quad t)a_k + (1+t)b_k]\, z^k \dots\dots (3.1.1)$$

Since $,g \in \quad (A,B,\alpha)$ so by THEOREM 1 we have

$$\sum_{k=2}^{\infty}\{k + \alpha k^2 \quad 2\alpha k \quad (1 \quad \alpha) \quad [B(k + \alpha k(k \quad 1)) \quad A(\alpha k + 1 \quad \alpha)]\}a_k \le (A \quad B)$$

And

$$\sum_{k=2}^{\infty}\{k + \alpha k^2 \quad 2\alpha k \quad (1 \quad \alpha) \quad [B(k + \alpha k(k \quad 1)) \quad A(\alpha k + 1 \quad \alpha)]\}b_k \le (A \quad B)$$

Therefore

$$\sum_{k=2}^{\infty}\{k + \alpha k^2 \quad 2\alpha k \quad (1 \quad \alpha) \quad [B(k + \alpha k(k \quad 1)) \quad A(\alpha k + 1 \quad \alpha)]\}\left[\frac{1}{2}[(1 \quad t)a_k + (1+t)b_k]\right]$$
$$= \frac{1}{2}(1 \quad t)\sum_{k=2}^{\infty}\{k + \alpha k^2 \quad 2\alpha k \quad (1 \quad \alpha) \quad [B(k + \alpha k(k \quad 1)) \quad A(\alpha k + 1 \quad \alpha)]\}a_k$$
$$+\frac{1}{2}(1+t)\sum_{k=2}^{\infty}\{k + \alpha k^2 \quad 2\alpha k \quad (1 \quad \alpha) \quad [B(k + \alpha k(k \quad 1)) \quad A(\alpha k + 1 \quad \alpha)]\}b_k \le \frac{1}{2}(1 \quad t)(A \quad B) + \frac{1}{2}(1+t)(A \quad B)$$
$$= A \quad B$$

Therefore

$$w_{fg} \in \quad (A,B,\alpha)$$

Hence the proof of theorem is completed.

Theorem Let $f_i(z) = z \quad \sum_{k=2}^{\infty} a_{i,k}\, z^k$, $i = 1,2,3,\dots,m$ be the functions in the class (A,B,α) then the arithmetic mean of f_i ($i = 1,2,3,\dots,m$) is defined by

$g(z) = \frac{1}{m}\sum_{i=1}^{m} f_i(z)$ is also in the class (A,B,α)

Proof: Since $f_i(z) = z \quad \sum_{k=2}^{\infty} a_{i,k}\, z^k$, $i = 1,2,3,\dots,m$

Therefore

$$g(z) = \frac{1}{m}\sum_{i=1}^{m} f_i(z)$$
$$= \frac{1}{m}\sum_{i=1}^{m}\left(z \quad \sum_{k=2}^{\infty} a_{i,k}\, z^k\right)$$

$$= z \quad \sum_{k=2}^{\infty} \left(\frac{1}{m} \sum_{i=1}^{m} a_{i,k} \right) z^k$$

We have $f_i(z) = z \quad \sum_{k=2}^{\infty} a_{i,k} z^k$, $i = 1,2,3, \dots, m$ are in the class $\quad (A, B, \alpha)$
So by THEOREM 1 we prove that

$$\sum_{k=2}^{\infty} \{k + \alpha k^2 \quad 2\alpha k \quad (1 \quad \alpha) \quad [B(k + \alpha k(k \quad 1)) \quad A(\alpha k + 1 \quad \alpha)]\} \left(\frac{1}{m} \sum_{i=1}^{m} a_{i,k} \right)$$

$$= \frac{1}{m} \sum_{i=1}^{m} \left(\sum_{k=2}^{\infty} \{k + \alpha k^2 \quad 2\alpha k \quad (1 \quad \alpha) \quad [B(k + \alpha k(k \quad 1)) \quad A(\alpha k + 1 \quad \alpha)]\} a_{i,k} \right)$$

$$\leq \frac{1}{m} \sum_{i=1}^{m} (A \quad B) = A \quad B$$

Hence the proof of theorem is completed .

Theorem Let$f_i(z) = z \quad \sum_{k=2}^{\infty} a_{i,k} z^k$, $i = 1,2,3, \dots, m$ be the functions in the class $\quad (A, B, \alpha)$ then the linear combination of f_i ($i = 1,2,3, \dots, m$) is defined by

$G(z) = \sum_{i=1}^{m} k_i f_i(z)$, where $\sum_{i=1}^{m} k_i = 1$ is also in the class $\quad (A, B, \alpha)$

Proof Let $f_i(z) = z \quad \sum_{k=2}^{\infty} a_{i,k} z^k$, $i = 1,2,3, \dots, m$ be the functions in the class $\quad (A, B, \alpha)$
so by THEOREM 1 we have

$$\sum_{k=2}^{\infty} \{k + \alpha k^2 \quad 2\alpha k \quad (1 \quad \alpha) \quad [B(k + \alpha k(k \quad 1)) \quad A(\alpha k + 1 \quad \alpha)]\} a_{i,k} \leq (A \quad B)$$

$$G(z) = \sum_{i=1}^{m} k_i f_i(z)$$

$$G(z) = \sum_{i=1}^{m} k_i \left(z \quad \sum_{k=2}^{\infty} a_{i,k} z^k \right)$$

$$G(z) = z \quad \sum_{k=2}^{\infty} \left(\sum_{i=1}^{m} k_i a_{i,k} \right) z^k$$

So by THEOREM 1 we prove that

$$\sum_{k=2}^{\infty} \{k + \alpha k^2 \quad 2\alpha k \quad (1 \quad \alpha) \quad [B(k + \alpha k(k \quad 1)) \quad A(\alpha k + 1 \quad \alpha)]\} \left(\sum_{i=1}^{m} k_i a_{i,k} \right)$$

$$= \sum_{i=1}^{m} k_i \left(\sum_{k=2}^{\infty} \{k + \alpha k^2 \quad 2\alpha k \quad (1 \quad \alpha) \quad [B(k + \alpha k(k \quad 1)) \quad A(\alpha k + 1 \quad \alpha)]\} a_{i,k} \right)$$

$$\leq \sum_{i=1}^{m} k_i (A \quad B) = A \quad B$$

Hence the proof of theorem is completed.

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