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GENERALIZATION OF PAIRWISE QUASI-H-CLOSED SPACES

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ABSTRACT

In this paper, we prove some important theorems on pair wise quasi-H-closed modulo an ideal spaces and study some of its properties.

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INTRODUCTION

In 1963, the concept of bitopological space was introduced by Kelly [6] when he was studying properties of asymmetric distance function on a non empty set *X*. Since bitopological space is a generalization of topological space, it is therefore worth-while to investigate that how the existing notions of topological spaces can be extended to bitopological setting. H-closed and Quasi H-closed (QHC) spaces are considered to be interesting and important topics of study for as long as the last seventy year or so. The intensive study of such spaces by eminent topologists during this long period has motivated many others to generalize the existing results to bitopological spaces. The pairwise QHC-spaces were first introduced by Mukherjee [7]. Subsequently such spaces have further been studied in detail by Kariofillis [2] and Sen *et al.* [10]. In 2002, the concept of pairwise compactness modulo an ideal in bitopological space was introduced by Lal *et al.* [8] inspired by the concept of compactness modulo an ideal introduced by Newcomb [1] in general topology.

In this paper, we proved some important theorems on pair wise quasi-H-closed modulo an ideal spaces and study some of its properties. In the process, we get some interesting characterizations of such spaces.

Preliminaries

Definition: A point *x* in space (X, τ_1, τ_2) is said to be ij-θ-contact [2] point of a subset *A* of *X* if for any τ_i-open neighborhood U of *x*, $\tau_j\text{-cl}(U) \cap A \neq \phi$. The set of all ij-$\theta$-contact points of *A* is said to be ij-θ-closure of *A* and denoted by ij-θ-cl(*A*).

Definition: A point *x* in a space (X, τ_1, τ_2) is said to be ij-θ-adherent point [2] of filter base B on *X* [3] if it is ij-θ-contact point of every member of B or point *x* in a space (X, τ_1, τ_2) is said to be ij-θ-adherent point of filter base B on *X* if $x \in \bigcap_{B \in \mathcal{B}} \text{ij-}\theta\text{-cl}(B)$. The set of all ij-$\theta$-adherent points of B is called ij-θ-adherence of B and denoted by ij-θ-adhB.

Definition: A subset *A* in a space (X, τ_1, τ_2) is said to be ij-regularly open [4] if $A = \tau_i\text{-int}(\tau_j\text{-cl}(A))$. The complements of ij-regularly open sets are called ij-regularly closed sets.

Definition: A bitopological space (X, τ_1, τ_2) is said to be pairwise quasi-H-closed [7] if for every τ_i-open cover U of *X*,

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there exists a finite subfamily $\{U_1, U_2, U_3, ...U_n\}$ of U such that $X = \bigcup_{k=1}^{n} \tau_j\text{-cl}(U_k)$.

Definition: A bitopological space $(X, \tau_1, \tau_2, \mathscr{I})$ is said to be pairwise compact modulo an ideal or just (I)FHP-compact [8] if every pairwise open cover U of X has a finite subfamily $\{U_1, U_2, U_3, ..., U_n\}$ of U such that $X - \bigcup_{k=1}^{n} U_k \in I$.

Lemma: [2]. Let (X, τ_1, τ_2) be a bitopological space, then we have:

1. If $A \in \tau_j$, then $\tau_i\text{-cl}(A) = \text{ij-}\theta\text{-cl}(A)$.
2. If $\{A_k : k \in K\}$ is a collection of subsets of X, Then $\text{ij-}\theta\text{-cl}(\bigcap\{A_k : k \in K\}) \subset \bigcap\{\text{ij-}\theta\text{-cl}(A_k) : k \in K\}$.

Pair wise quasi-H-closed modulo an ideal

Theorem: Let (X, τ_1, τ_2) be bitopological space and let I be an ideal on X. Then following conditions are equivalent:

1. (X, τ_1, τ_2) is pairwise (I)QHC,
2. any filter base in $\wp(X) - I$ has non-empty ij-θ-adherence,
3. for each family U of subsets of X such that U $\subset$ $\wp(X) - I$, having (I)FIP, one has $\bigcap\{\text{ij-}\theta\text{-cl}(G) : G \in \mathscr{U}\} \neq \phi$.

Proof (1) $\Rightarrow$ (2). Let B be any filter base in $\wp(X) - I$ without ij-θ-adherent point. Then for each $x \in X$ there exists τ_i-neighbourhood V_x of X and $F_x \in \mathscr{B}$ such that $\tau_j\text{-cl}(V_x) \cap F_x = \phi$. Let $\mathcal{G} = \{X - \tau_j\text{-cl}(V_x) : x \in X\}$, then G is τ_j-open filter base in $\wp(X) - I$ without τ_i-adherent point. So $(X, \tau_1, \tau_2, \mathscr{I})$ is not pair wise (I)QHC. Hence, we have contradiction.

(2) $\Rightarrow$ (1). Let B be any τ_i-open filter base in $\wp(X) - I$. By Lemma 2.6, we have $\{\text{ij-}\theta\text{-cl}(B) = \tau_i\text{-cl}(B)$ for each $B \in \mathscr{B}\}$. By hypothesis, there exists $x \in X$ such that

$$x \in \bigcap\{\text{ij-}\theta\text{-}(B) : B \in \mathscr{B}\}.$$

We have, $x \in \bigcap\{\tau_j\text{-cl}(B) : B \in \mathscr{B}\}$ for each $B \in \mathscr{B}$

$\Rightarrow V_x \cap F \neq \phi$ for each $B \in \mathscr{B}$ and for each $V_x \in N_i(x)$,

$\Rightarrow$ x is τ_i-adherent point of B. Hence $(X, \tau_1, \tau_2, \mathscr{I})$ is pair wise (I) QHC.

(2) $\Rightarrow$ (3). Let $\mathscr{U} = \{G_k : k \in K\}$ be collection of subsets of X such that U $\subset$ $\wp(X) - I$ having (I)FIP. Therefore, we have $\bigcap_{k=1}^{n}\{G_k\} \notin \mathscr{I}$.

Then $\Im = \{\bigcap_{\lambda \in \Lambda} G_\lambda : \Lambda \subset K, \Lambda \text{ is finite}\}$ does not contain empty set and also for the intersection of any two members of $\Im$, there exists a member of $\Im$ that is contained in the intersection of that two members. Therefore $\Im = \{\bigcap_{\lambda \in \Lambda} G_\lambda : \Lambda \subset K, \Lambda \text{ is finite}\}$ is a filter base on X. Clearly $\Im \subset \wp(X) - \mathscr{I}$. Hence by hypothesis, $\Im$ has non-empty ij-θ-adherence i.e.

$$x \in \bigcap\left\{\text{ij-}\theta\text{-cl}\left(\bigcap_{\lambda \in \Lambda} G_\lambda : \Lambda \subset K, \Lambda \text{ is finite,}\right)\right\}.$$

Because we know that if $\{G_k : k \in K\}$ is a collection of subsets of X, then

$$\text{ij-}\theta\text{-cl}(\bigcap\{G_k : k \in K\}) \subset \bigcap\{\text{ij-}\theta\text{-cl}(G_k) : k \in K\},$$

It follows that,

$$x \in \bigcap\left\{\bigcap_{\lambda \in \Lambda}(\text{ij-}\theta\text{-cl}(G_\lambda) : \Lambda \subset K, \Lambda \text{ is finite})\right\}$$

$$\Rightarrow \bigcap\{\text{ij-}\theta\text{-cl}(G_k) : k \in K\} \neq \phi.$$

(3) $\Rightarrow$ (2). Let U be any filter base exists in $\wp(X) - I$. Clearly it has (I)FIP, if not, there exists a finite number of members in U such that $\bigcap_{k=1}^{n} U_k \in \mathscr{I}$. Now by the definition of filter base, there exists $H \in \mathscr{U}$ such that $H \subset \bigcap_{k=1}^{n} U_k$ which implies that $H \in \mathscr{I}$ but this contradicts the fact that U be any filter base in $\wp(X) - I$. By hypothesis, $\bigcap\{\text{ij-}\theta\text{-cl}(U) : U \in \mathscr{U}\} \neq \phi$,

$\Rightarrow$ Filter base U in $\wp(X) - I$ has non-empty ij-adherence.

Theorem: Let (X, τ_1, τ_2) be a bitopological space and let I be an ideal on X. Then the following conditions are equivalent:

1. (X, τ_1, τ_2) is pair wise (I)QHC,
2. each ij-regularly open cover U of X, there exists a finite sub collection $\{U_1, U_2, U_3, \ldots, U_n\}$ of U such that $X - \bigcup_{k=1}^{n}\{\tau_j\text{-cl}(U_k)\} \in \mathscr{I}$,
3. for each family U of ij-regularly closed subsets having empty intersection, there exists a finite subfamily $\{U_1, U_2, U_3, \ldots, U_n\}$ of U such that $\bigcap_{k=1}^{n}\{\tau_j\text{-int}(U_k)\} \in \mathscr{I}$,
4. For each family U of ij-regularly-closed subsets such that $\{\tau_j\text{-int}(U) : U \in \mathscr{U}\}$ has (I) FIP, one has $\bigcap\{U : U \in \mathscr{U}\} \neq \phi$.

Proof (2) $\Rightarrow$ (1). Let U be any τ_i-open cover of X. Then $\{\tau_i\text{-}\mathrm{int}(\tau_j\text{-}\mathrm{cl}(U)) : U \in \mathscr{U}\}$ is ij-regularly open cover of X. By hypothesis, there exists a finite subfamily $\{\tau_j\text{-}\mathrm{int}(\tau_j\text{-}\mathrm{cl}(U_k)) : k = 1,2,3,\ldots,n\}$ such that

$$X - \bigcup_{k=1}^{n}\{\tau_j\text{-}\mathrm{cl}(\tau_i\text{-}\mathrm{int}(\tau_j\text{-}\mathrm{cl}(U_k)))\} \in \mathscr{I},$$

Since U_k is τ_i-open and for each τ_i-open set U of U, $\tau_j\text{-}\mathrm{cl}(\tau_i\text{-}\mathrm{int}(\tau_j\text{-}\mathrm{cl}(U))) = \tau_j\text{-}\mathrm{cl}(U)$.

Then we have $X - \bigcup_{k=1}^{n}\{\tau_j\text{-}\mathrm{cl}(U_k)\} \in \mathscr{I}$, which shows that (X, τ_1, τ_2) is pairwise (I)QHC.

1. $\Rightarrow$ (2). This is obvious, because every ij-regularly open set is τ_i-open,
2. $\Rightarrow$ (3). Let $(X, \tau_1, \tau_2, \mathscr{I})$ be pair iwise (I)QHC. Let U be any family of ij-regularly-closed subsets having empty intersection. Then $\{X - U : U \in \mathscr{U}\}$ is ij-regularly open cover of X and hence admits a subfamily $\{X - U_k : k = 1,2,3,\ldots,n\}$ such that
$$X - \bigcup_{k=1}^{n}\{\tau_j\text{-}\mathrm{cl}(X - U_k)\} \in \mathscr{I},$$
$\Rightarrow X - \bigcup_{k=1}^{n}\{X - \tau_j\text{-}\mathrm{int}(U_k)\} \in \mathscr{I}$, Hence we have
$$\bigcap_{k=1}^{n}\{\tau_j\text{-}\mathrm{int}(U_k)\} \in \mathscr{I},$$
3. $\Rightarrow$ (4). This is easy to be established.
4. $\Rightarrow$ (1). Suppose bitopological space $(X, \tau_1, \tau_2, \mathscr{I})$ is not pairwise quasi-H-closed modulo an ideal. Let U be any pairwise ij-regularly open cover of X, then there exists a finite subfamily $\{U_1, U_2, U_3, \ldots, U_n\}$ of U such that
$$X - \bigcup_{k=1}^{n}\{\tau_j\text{-}\mathrm{cl}(U_k)\} \in \mathscr{I},$$

Hence, $\{X - U : U \in \mathscr{U}\}$ is family of ij-regularly closed sets such that $\{\tau_j\text{-}\mathrm{int}(X - U) : U \in \mathscr{U}\}$ has (I) FIP. By hypothesis, $\bigcap\{X - U : U \in \mathscr{U}\} \neq \phi$. But

$X - \bigcup\{U \in \mathscr{U}\} \neq \phi$, that is, U is not ij-regularly open cover of X, a contradiction.

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