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Research Article

COMPARISON BETWEEN TWO-SAMPLE ADAPTIVE TESTS AND TRADITIONAL TESTS UNDER SYMMETRIC AND SKEWED DISTRIBUTIONS FOR LOCATION PROBLEM

Chikhla Jun Gogoi and Bipin Gogoi

Department of Statistics, Dibrugarh University, Dibrugarh Assam, India, PIN-786004

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ABSTRACT

The most familiar test for two sample location problem is the parametric t test and non parametric wilcoxon test provided the parent distribution is normal. But if the data set follows a skewed distribution then these two tests may give an inaccurate result. In this paper it is tried to give a solution to this problem by the use of adaptive tests. Power comparisons are made here between t test, wilcoxon test and proposed adaptive tests by monte carlo simulation method.

INTRODUCTION

One of the fundamental problems of statistics, often encountered in applications, is the two-sample location problem. In the two-sample location problem the application of the t-test depends on very restrictive assumptions such as normality and equal variances of the two random variables X_1 and X_2. If the assumptions of the t-test are not satisfied it is more appropriate to apply a robust version of the t-test, like the Welch test or the trimmed t-test, or a nonparametric test, like the Wilcoxon. But usually we have no information about the underlying distribution of the data. Therefore, an adaptive test should be applied. It would be desirable, therefore to use data itself to determine the nature of F(.), and on the basis of that information, we could choose an appropriate set of scores. We would then use that same data to perform the test. Such two-stage analyses are termed as adaptive test. In the past seven decades many important distribution free tests for differences in location between samples had been developed. In the mid 1940s the Wilcoxon -Mann- Whitney test was introduced for testing differences in location between two samples and it was developed by Wilcoxon and extended by Mann and Whitney. But further, it turns out that there exist simple adaptive rank tests that can discover differences between distributions more easily than WMW tests. These adaptive non parametric procedures display significant improvements in power over the parametric t- test in samples of large and moderate sizes. The purpose of this chapter is two folds, first to introduce the selector statistics, secondly compare the t-test with adaptive distribution-free test like Wilcoxon test, test based on scores under normality and under different models of nonnormality, like heavy tailed or asymmetric distributions Adaptive tests are important in applications because the practicing statistician usually has no information about the underlying distribution. The adaptive testing procedures that are truly nonparametric distribution-free. That is, the two stages of the inference process are constructed in such a way that it control the overall α -level. Monte-Carlo simulations are used for comparison of the tests with respect to level α and powerβ.

Selector statistics for selection of test

We apply the concept of Hogg (1974) that is based on following lemma:

1. Let F denote the class of distributions under consideration. Suppose that each of k tests $T_1, T_2, \ldots, T_k$ is distribution –free over F , that is $Pr_{Ho}(T_i \in C_i) = \alpha$ for each $F \in F$, $h = 1, \ldots, k$.
2. Let S be some statistic (called a selector statistic) that is, under Ho, independent of $T_1, \ldots, T_k$ for each $F \in F$. Suppose we use S to decide which test T_h to conduct. Specially, let M_s denote the set of all values of S with the following decomposition: $M_s = D_1 \cup D_2 \cup \ldots \cup D_h$, $D_i \cap D_j = \emptyset \; for \; i \neq j$. So that $S \in D_h$ corresponds to the

Corresponding author:* **Chikhla Jun Gogoi
Department of Statistics, Dibrugarh University, Dibrugarh Assam, India, PIN-786004

decision to use test T_h.

The overall testing procedure is then defined by: If $S \in D_h$ then reject Ho if $T_h \in C_h$. This two-staged adaptive test is distribution-free under Ho over the class F ,i.e. it maintains the level α for each $F \in F$.

The proof of this lemma is given by Randle and Wolfe(1979).Using the lemma, as a selector statistic, we use a function of order statistics of combined sample. We choose the selector statistic as

$$S = (\widehat{Q_1}, \widehat{Q_2})$$

Table 1 Theoretical values of Q_1 and Q_2 for selected distributions

Distribution	Q_1	Q_2
Uniform(0,1)	1	1.9
Normal	1	2.585
Exponential(with λ=1)	4.569	2.864

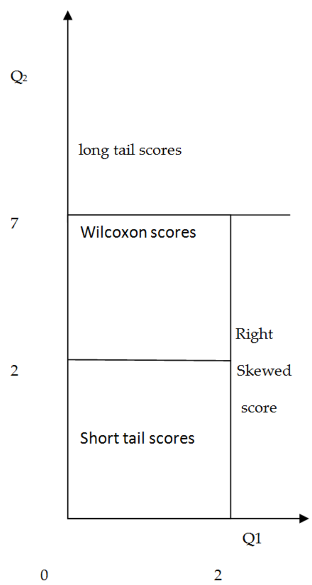
Where

$$\widehat{Q_1} = \frac{\widehat{U}_{0.05} - \widehat{M}_{0.5}}{\widehat{M}_{0.5} - \widehat{L}_{0.05}}$$

and

$$\widehat{Q}_2 = \frac{\widehat{\widehat{U}}_{0.05} - \widehat{L}_{0.05}}{\widehat{U}_{0.5} - \widehat{L}_{0.5}}$$

And Hogg's(1974) measures for skewness and tailweight, and $\widehat{L}_\gamma$, $\widehat{M}_\gamma$ and $\widehat{U}_\gamma$ denote the average of the smallest , middle and largest γN order statistics, respectively, in the combined sample; fractional items are used when γN is not an integer. Obviously $\widehat{Q}_1$= 1 if the data are symmetric and $\widehat{Q}_1$<1 (>1) if the data are skewed to the left(right). The longer the tails the greater is $\widehat{Q}_2$. Table 1 shows the theoretical measures of Q_1 and Q_2 for selected distributions. As in Buning (1996), we define the adaptive test as follows (Fig..1)



If $Q_1 \leq 2, Q_2 \leq 2$ perform the Gastwirth test,
If $\widehat{Q}_1 \leq 2$, $2 < \widehat{Q}_2 \leq 3$ perform Wilcoxon test,
If $\widehat{Q}_1 > 2$, $2 < \widehat{Q}_2 \leq 3$ perform HFR test, and
If $\widehat{Q}_2$>3 perform the LT test.

However, sometimes a larger critical value for $\widehat{Q}_2$ was used to differentiate between tests(Hogg1975). Therefore, we define a second adaptive test as follows:

If $\widehat{Q}_1 \leq 2, \widehat{Q}_2 \leq 2$ perform the Gastwirth test,
If $\widehat{Q}_1 \leq 2$, $2 < \widehat{Q}_2 \leq 5$ perform Wilcoxon test,
If $\widehat{Q}_1 > 2$, $2 < \widehat{Q}_2 \leq 5$ perform HFR test, and
If $\widehat{Q}_2$>5 perform the LT test.

Test Procedures

Let $x_{i1}, x_{i2}, \ldots, x_{in_i}$, i = 1,2 be independent random samples from parent populations with continuous distribution function F[(x-μ_i)] . Let μ_i represent the location parameters and σ_i the scale parameters of the populations assumed to be same. The problem is to test Ho: $\mu_1 = \mu_2$ against H_1: $\mu_1 \neq \mu_2$ or $\mu_1 > \mu_2$ or $\mu_1 < \mu_2$. In our case we have considered only the alternative $\mu_1 > \mu_2$. For testing this hypothesis the procedures are

Student's t- test

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \qquad (2.1)$$

where $s^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$; and $\bar{x}_i$'s and s_i^2 's are the means and variances of the two samples. The statistic t follows Student's t distribution with $n_1 + n_2$ -2 degrees of freedom.

Wilcoxon Test

$$\mathbf{W} = \sum_{i=1}^{N} iZ_i \;, \mathbf{N} = \mathbf{n_1} + \mathbf{n_2}$$

Where Z_i is a indicator variable. It take the value 1, if the ith observation from first sample and zero ,otherwise.

Two sample tests based on some scores

$$T = \sum_{i=1}^{N} g(i) V_i$$

where g(i) are real valued scores, and V_i = 1 when the ith smallest of the $N=n_1+n_2$ observation s is from the first sample and V_i = 0 otherwise. Two-sample tests on T are distribution – free, under Ho, we have

$$E(T) = \frac{n_1}{N} \sum_{i=1}^{N} g(i)$$

$$Var(T) = \frac{n_1 n_2}{N^2(N-1)} [N \sum_{i=1}^{N} g^2(i) - (\sum_{i=1}^{N} g(i)^2]$$

And the standardized statistic

$$\frac{T - E(T)}{\sqrt{Var(T)}}$$

Follows asymptotically a standard normal distribution (Hajek *et al.* 1999).When some condition about the scores g(i) are fulfilled, T can be asymptotically normal under an alternative, too (Chernoff and Savage, 1958). However, in general, the rejection probability under the alternative depends on the distribution of the data. Therefore, different choices for scores $g(i)$ were proposed.

Now we will discuss some scores on which adaptive tests A(s) based such as short tailed test, medium tests, long tailed tests and right skewed tail tests.

Gastwirth test (short tails)

$$g(i) = \begin{cases} i - \dfrac{N+1}{4} & for \quad i \le \dfrac{N+1}{4} \\ 0 \quad for & \dfrac{N+1}{4} \le i \le \dfrac{3(N+1}{4} \\ i - \dfrac{3(N+1)}{4} & for \quad i \ge \dfrac{3(N+1)}{4} \end{cases}$$

Wilcoxon test (median tails): $g(i) = i$
Long tails Test (long tails):

$$g(i) = \begin{cases} -\left(\left[\dfrac{N}{4}\right] + 1\right) & for \quad i < \left[\dfrac{N}{4}\right] + 1 \\ i - \dfrac{N+1}{2} \quad for & \left[\dfrac{N}{4}\right] + 1 \le i \le \left[\dfrac{3(N+1)}{4}\right] \\ \left[\dfrac{N}{4}\right] + 1 \quad for & i > \left[\dfrac{3(N+1)}{4}\right] \end{cases}$$

Hogg-Fisher- Randles(HFR) test (right skewed)

$$g(i) = \begin{cases} i - \frac{N+1}{2} & for \quad i \le \frac{N+1}{2} \\ 0 & for \quad i > \frac{N+1}{2} \end{cases}$$

The Monte Carlo Study

For the simulation study of the t- test, Wicoxon test, Gastwirth test, Long-tail test and short-tail test (HFR) six families of distributions are selected. These are-the Normal, the Logistic and the Exponential. In studying the significant levels, we first considered distributions with location parameter equal to zero and with equal scale parameters. Specifically, we considered the distribution functions F(x $-\mu_i$), where μ_i were the location parameters. For each set of sample N = $\sum_i n_i$, i =1,2 , the experiment was repeated 5,000 times and proportion of rejection of the true null hypothesis was recorded and presented in table 2 to 4.

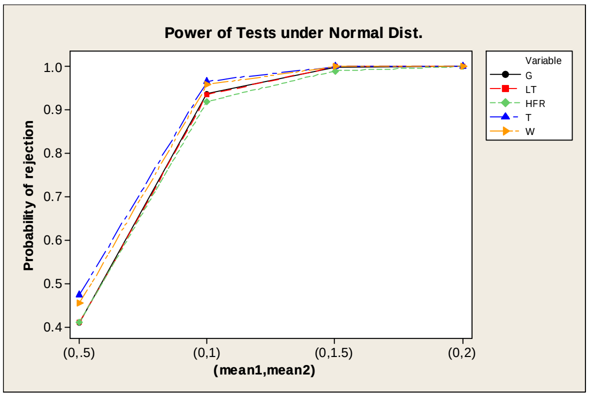


Fig 2 Empirical power of tests under Normal distribution for $n_1=n_2=30$ at 5% level

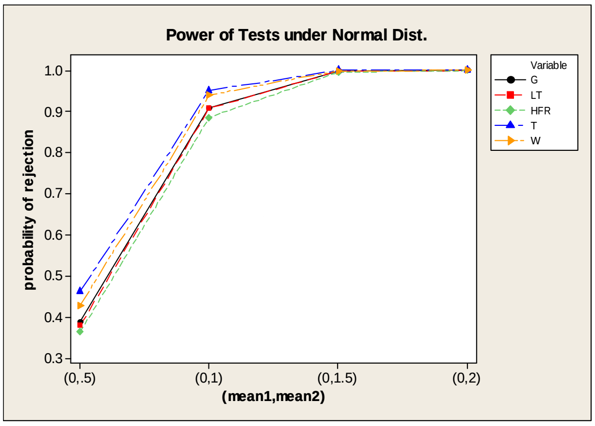


Fig 3 Empirical power of tests under Normal distribution for $n_1=25$, $n_2=30$ at 5% level

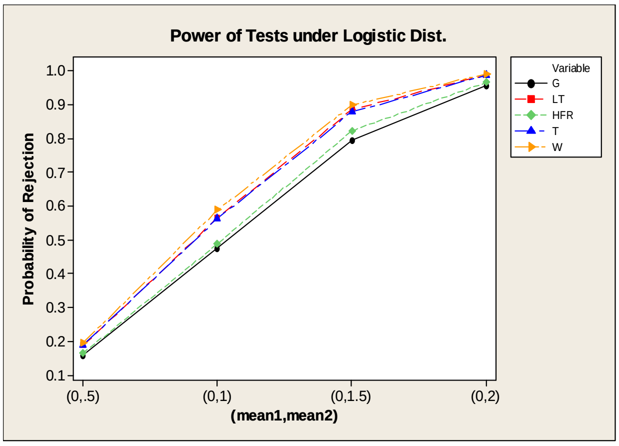


Fig 4 Empirical power of tests under Logistic distribution for $n_1=n_2=30$ at 5% Level

Table 2 Empirical Level and power of tests under Normal distribution for equal sample sizes

Sample sizes n_i	Location parameter μ_i	G 5%	G 1%	LT 5%	LT 1%	HFR 5%	HFR 1%	T 5%	T 1%	W 5%	W 1%
	0 0	.0547	.0032	.0542	.0096	.0551	.0082	.0540	.0106	.0559	.0097
30 30	0 0.5	.1609	.0190	.1738	.0534	.1847	.2952	.1951	.0631	.1947	.0558
	0 1.0	.4588	.1152	.5110	.2544	.5206	.3802	.5675	.3059	.5523	.2732
	0 1.5	,7625	.3595	.8388	.6126	.8122	.5089	.8874	.6789	.8758	.6380
	0 2.0	.9301	.6579	.9695	.8838	.9742	.7007	.9874	.9288	.9828	.9056
	0 0	.0534	.0086	.0532	.0100	.0526	.0098	.0612	.0122	.0560	.0088
25 30	0 0.5	.3870	.1556	.3794	.1738	.3634	.1596	.4608	.2446	.4282	.1974
	0 1.0	.9098	.7202	.9076	.7540	.8858	.7138	.9516	.8598	.9398	.8032
	0 1.5	.9990	.9842	.9984	.9902	.9960	.9794	1.000	.9986	.9996	.9954
	0 2.0	1.000	.9998	1.000	1.000	1.000	.9980	1.000	1.000	1.000	1.000

Table 3 Empirical level and power of tests under Logistic distribution for equal sample sizes

Sample sizes n_i	Location parameter μ_i	G 5%	G 1%	LT 5%	LT 1%	HFR 5%	HFR 1%	T 5%	T 1%	W 5%	W 1%
30 30	0 0	.0481	.0085	.0483	.0082	.0483	.0073	.0499	.0083	.0498	.0079
	0 0.5	.1592	.0427	.1863	.0637	.1640	.0503	.1873	.0610	.1934	.0646
	0 1.0	.4739	.2233	.5646	.3192	.4879	.2531	.5622	.3169	.5880	.3354
	0 1.5	.7939	.5470	.8849	.7141	.8223	.6012	.8777	.7034	.8978	.7373
	0 2.0	.9572	.8327	.9880	.9424	.9658	.8753	.9867	.9370	.9912	.9521
25 30	0 0	.0496	.0064	.0508	.0104	.0468	.0112	.0558	.0124	.0500	.0096
	0 0.5	.1516	.0372	.1820	.0578	.1646	.0526	.1946	.0722	.1894	.0598
	0 1.0	.4504	.1978	.5190	.2872	.4732	.2496	.5506	.3270	.5496	.3020
	0 1.5	.7614	.4954	.8534	.6454	.7982	.5774	.8566	.6888	.8736	.6744
	0 2.0	.9334	.7784	.9804	.9132	.9576	.8558	.9804	.9256	.9864	.9242

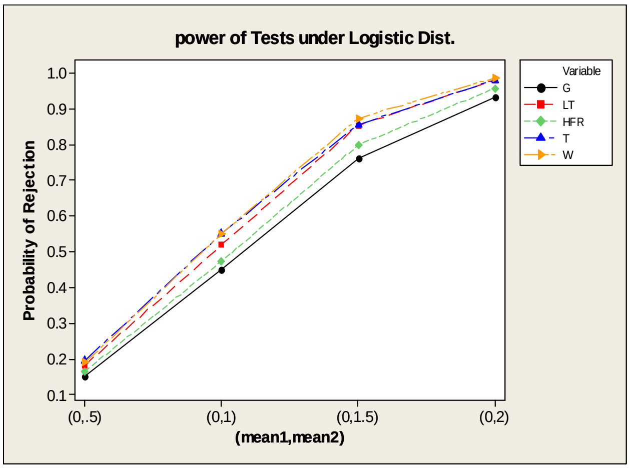


Fig 5 Empirical power of tests under Logistic distribution for n_1=25 n_2=30 at 5% level

DISCUSSION

For comparison purposes we have considered various combinations of sample sizes with equal and unequal sample sizes. We have also considered different sets of μ_i's for the study.

Table 2 shows the power of tests under normal distribution for equal and unequal sample sizes respectively. We have seen that power of t- test is higher than the other tests in this distribution in presence of various combinations of location parameters and sample sizes. Power of Wilcoxon test is found to be slightly less than the t-test but more than other score base tests in both the two cases.

Table 3 displays the power of tests under logistic distribution. We have seen that Wilcoxon, t- test and LT test are more powerful than other two tests with both equal and unequal sample sizes and at 5% and 1% level. However, power of W test is more than all tests.

Table 4 Empirical level and power of tests under Exponential distribution for equal sample sizes

Sample sizes n_i	Location parameter μ_i	G 5%	G 1%	LT 5%	LT 1%	HFR 5%	HFR 1%	T 5%	T 1%	W 5%	W 1%
30 30	0 0	.0481	.0085	.0483	.0082	.0483	.0073	.0483	.0087	.0498	.0079
	0 0.5	.8464	.5982	.6474	.4202	.9235	.7844	.5051	.2771	.7633	.5373
	0 1.0	.9907	.9181	.9903	.9610	.9997	.9988	.9540	.8680	.9942	.9752
	0 1.5	.9993	.9873	1.000	.9991	1.000	1.000	.9991	.9951	1.000	.9996
	0 2.0	1.000	.9981	1.000	1.000	1.000	1.000	1.000	.9999	1.000	1.000
25 30	0 0	.0496	.0064	.0508	.0104	.0468	.0112	.0488	.0104	0500	.0096
	0 0.5	.8370	.5912	.5908	.3628	.8910	.7330	.5038	.2860	.7252	.4796
	0 1.0	.9914	.9246	.9764	.9204	.9998	.9966	.9466	.8582	.9912	.9546
	0 1.5	.9998	.9860	.9996	.9982	1.000	1.000	.9988	.9924	.9998	.9994
	0 2.0	1.000	.9980	1.000	1.000	1.000	1.00	1.000	1.000	1.000	1.000

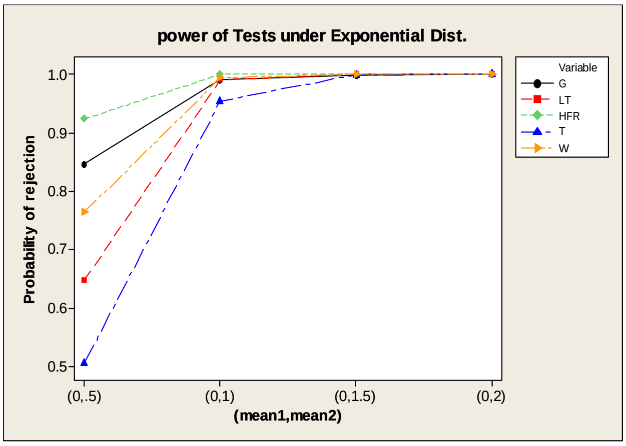


Fig 6 Empirical power of tests under Exponential distribution for n_1=n_2=30 at 5% level

Table 4 shows the empirical power of tests under exponential distribution. Here we obtained similar results as like the lognormal distribution. That is, power of HFR is the highest of all followed by Wicoxon, G and others.

CONCLUSION

In case of symmetric distribution, under equal variances t-test is most the preferable test, as it maintain levels and shows more power than other tests. In case of skewed distribution rank test or score based test may be more preferable. The choice of a suitable rank test or score based test which is more efficient than t-test depends on the underlying distribution of data. Because the practicing statistician usually has no clear idea about the distribution, an adaptive test should be applied which takes into account the given data set.

References

1. Allingham, D., Rayner, J. (2011) Two-Sample Testing for Equality of Variances. *Fourth Annual ASEARC Conference, February 17-18, 2011, Paramatta, Australia.*
2. Bean, R.(1974). Asymptotically efficient adaptive rank estimates in location models., *Annals of Statistics,*2,63-74
3. Beran, R.(1978). An efficient and Robust Adaptive Estimator of Location. *the annals of statistics 1978, Vol. 6, No 2,292-313.*
4. Bickel,P.J.(1982).On Adaptive Estimation. *The Annals of Statistic, Vol. 1, No 3(SEP1982).pp 647-671*
5. Blair, R.C., and Thompson,G,L,(1990). Distribution Free Rank Like Test for Scale with Unequal Population Locations. *Research sponsored by NSF.*
6. Buning, H.(1991). *Robuste and Adaptive Tests*. De Gruyter, Berlin.
7. Buning, H.(1994). Robust and adaptive tests for the two –sample location problem. *OR Spektrum* 16,33-39.
8. Buning, H.(2000).Robustness and power of parametric ,nonparametric, robustified and adaptive tests-the multi-sample location problem., *Statistical Papers* 41,381-407.
9. Buning, H.(2002). An adaptive distribution-free test for the general two-sample Problem. *Computational Statistics* 17,297-313.
10. Bunning, H.(1996).,Adaptive tests for the c-sample location problem-the case of two-Sided alternatives', *Comm.Stat.-Theo. Meth.* 25,1569-1582.
11. Bunning, H. (1999)., Adaptive Jonckheere-Type Tests for Ordered Alternatives. *Journal of Applied Statistics*, 26(5),541-51
12. Buning, H. And Kossler, W.(1998). Adaptive tests for umbrella alternatives., *Biometrical journal*,40,573-587.
13. Buning, H. And Kossler,W.(1999). The asymptotic power of Jonckheere-Type tests for ordered alternatives. *Asutralian and New Zealand Journal of Stagbtistics*, 41(1), 67-77.
14. Buning, H. and Thadewald,T.(2000). An adaptive two-sample location-scale test of Lepage-type for symmetric distributions. *Journal of Staistical Computation and Simuation.*, 65,287-310.
15. Croux.C., Dehon, C., Robust estimation of location and scale. Dorota M. Dabrowska(1989). Rank Tests for Matched Pair Experiments with Censored Data. *Journal of Multivariate Analysis 28, 88-l 14* .
16. Faraway, J.J.(1992). Smoothing in Adaptive Estimation. *The Annals of Statistics.1992 vol.20, No 1,414-427*
17. Freidlin, B. And Gastwirth, J.(2000).Should the median test be retired from general use? *The American Statistician*,54(3),161-164.
18. Freidlin, B.;Miao,W. And Gastwirth, J.L.(2003a). On the use of the Shapiro-Wilk test In two- stage adaptive inference for paired data from moderate to very heavy tailed distributions. *Biometrical Journal* 45,887-900.
19. Gastwirth, J.(1965).Percentile Modification of Two-Sample Rank Tests, *Journal of theAmerican Statistical Association*, 60(312),1127-1141.,
20. Hajek, J.(1962). Asymptotically Most Powerful Rank-Order Tests *.Annuals of Mathematical Statistics*, 33,1124-1147

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