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A NEW SUBCLASS OF MEROMORPHIC p-VALENT FUNCTIONS WITH POSITIVE COEFFICIENTS

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ABSTRACT

this paper we have introduced a family of meromorphic p- valent functions, by making use of the generalized hypergeometric function and study some properties as coefficients inequalities, distortion theorems, closure theorems and radii of starlikeness and convexity.

INTRODUCTION

Let A^*_p denote the class of functions f(z) of the form

$$f(z) = z^{-p} + \sum_{n=p}^{\infty} a_n z^n \ (a_n \geq 0, n \geq p, p \in N),(1.1) \quad (1.1)$$

which are analytic and p-valent in the punctured unit disk $U^* = \{z \in C: 0 < |z| < 1\}$.

Define the Hadamard product of the function $f(z) \in A_p^*$ given by (1.1) and $g(z) \in A_p^*$ given

$$g(z) = z^{-p} + \sum_{n=p}^{\infty} a_n z^n \ (a_n \geq 0, n \geq p, p \in N) \quad (1.2)$$

as

$$(f * g)(z) = z^{-p} + \sum_{n=p}^{\infty} a_n b_n z^n = (g * f)(z).$$

For complex parameters $\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m (\beta_i \neq 0, -1, \dots; i = 1,2, \dots m)$the generalized

Hypergeometricfunction${}_lF_m(z)$ is defined by

$${}_lF_m(z) \equiv {}_l F_m(\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m; z) = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n \dots\dots (\alpha_l)_n}{(\beta_1)_n \dots\dots (\beta_m)_n} \frac{z^n}{n!} \quad (1.3)$$

$(l \leq m + 1; l, m = 0,1,2, \dots, z \in U^*)$,

Where $(x)_n$ denotes the Pochhammer symbol defined by

$$(x)_n = \begin{cases} x(x+1)(x+2)\dots(x+n-1), & n = 1,2,\dots \\ 1, & n = 0 \end{cases} \quad (1.4)$$

For positive real values of $\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m (l \leq m + 1; l, m = 0,1,2, \dots,)$,let$H(\alpha_1, \dots, \alpha_l; \beta_1, \dots, \beta_m) : A_p^* \to A_p^*$ be a linear operator defined by

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$H_m^l[\alpha,\beta]f(z) = H(\alpha_1,\ldots,\alpha_l;\beta_1,\ldots,\beta_m)f(z)$
$= [z^{-p}{}_lF_m(\alpha_1,\ldots,\alpha_l;\beta_1,\ldots,\beta_m;z)] * f(z)$
$= z^{-p} + \sum_{n=p}^{\infty}\omega_n(\alpha,\beta;l;m)a_n z^n$ (1.5)

where

$$\omega_n(\alpha,\beta;l;m) = \frac{(\alpha_1)_{n+p}\ldots\ldots(\alpha_l)_{n+p}}{(\beta_1)_{n+p}\ldots\ldots(\beta_m)_{n+p}}\frac{1}{(n+p)!} \quad (1.6)$$

is a positive number for all n=0,1,2,...... Here $H_m^l[\alpha,\beta]$ is used for shorter notation of $H(\alpha_1,\ldots,\alpha_l;\beta_1,\ldots,\beta_m)$.
Let f and g be analytic in unit disk U, then g is said to be subordinate to f, written as $g \prec f$ or $g(z) \prec f(z)$, if there exists a Schwartz function ω, which is analytic in U with ω(0)=0 and $|\omega(z)| < 1 (z\epsilon U)$ such that g(z) = f(ω(z)). In particular, if the function f is univalent in U, we have the following equivalence ([4][5])

$$g(z) \prec f(z)(z \in U) \Leftrightarrow g(0) = f(0) and g(U) \subseteq f(U).$$

Motivated with the work of Aqlam,Kulkarni [1] and Dziok, Murugusundaramoorthy and Sokol [3] we define $H_m^l(p,A,B,b,\mu)$ as a class of functions of the form (1.1) which satisfies the condition

$$p - \frac{1}{b}\left\{\frac{z\left(H_m^l[\alpha,\beta]f(z)\right)'}{H_m^l[\alpha,\beta]f(z)} + p\right\} \prec p - \mu p + \mu p\frac{1+Az}{1+Bz}, \quad (1.7)$$

Where $0 < \mu \le 1 - 1 \le B < A \le 1, p \in N, b$ non zero complex number.
We can re-write the condition (1.7) as

$$\left|\frac{z(H_m^l[\alpha,\beta]f(z))' + pH_m^l[\alpha,\beta]f(z)}{Bz(H_m^l[\alpha,\beta]f(z))' + [Bp(1-\mu b) + Ab\mu p\}H_m^l[\alpha,\beta]f(z)}\right| < 1. \quad (1.8)$$

We note that the concept of generalized hypergeometric function $H_m^l[\alpha,\beta]$ was motivated by Carlson – Shaffer [2].

In this paper, coefficient inequalities, distortion theorem as well as closure theorem for the class $H_m^l(p,A,B,b,\mu)$ are obtained.

2. Coefficients Inequality

Theorem2.1: A necessary and sufficient for f(z) of the form (1.1) to be in the class $H_m^l(p,A,B,b,\mu)$ is that

$$\sum_{n=p}^{\infty}[n(1-B) + p\{1 - B - \mu|b|(A-B)\}]\omega_n(\alpha,\beta;l;m)a_n \le p\mu|b|(A-B). \quad (2.1)$$

The result is sharp for the function f(z) given by

$$f(z) = z^{-p} + \left(\frac{p\mu|b|(A-B)}{[n(1-B)+p\{1-B-\mu|b|(A-B)\}]}\right)\frac{1}{\omega_n(\alpha,\beta;l;m)}z^k, (k \ge p, n \in N). \quad (2.2)$$

Proof: Let $f(z) \in H_m^l(p,A,B,b,\mu)$, then we have

$$\left|\frac{z(H_m^l[\alpha,\beta]f(z))' + pH_m^l[\alpha,\beta]f(z)}{Bz(H_m^l[\alpha,\beta]f(z))' + [Bp(1-\mu b) + Ab\mu p\}H_m^l[\alpha,\beta]f(z)}\right|$$

$$= \left|\frac{\sum_{n=p}^{\infty}(n+p)\omega_n(\alpha,\beta;l;m)a_n z^n}{p\mu|b|(A-B) + \sum_{n=p}^{\infty}[B(n+p) + p\mu|b|(A-B)]\omega_n(\alpha,\beta;l;m)a_n z^n}\right| < 1$$

If we choose z to be real and $z \to 1^-$, we get

$$\sum_{n=p}^{\infty}[n(1-B) + p\{1 - B - \mu|b|(A-B)\}]\omega_n(\alpha,\beta;l;m)a_n \le p\mu|b|(A-B).$$

Assuming that the inequality (2.1) holds true then from (1.8),we find that

$$\left|\frac{z(H_m^l[\alpha,\beta]f(z))' + pH_m^l[\alpha,\beta]f(z)}{Bz(H_m^l[\alpha,\beta]f(z))' + [Bp(1-\mu b) + Ab\mu p\}H_m^l[\alpha,\beta]f(z)}\right| \le \frac{\sum_{n=p}^{\infty}(n+p)\omega_n(\alpha,\beta;l;m)a_n}{p\mu|b|(A-B) + \sum_{n=p}^{\infty}[B(n+p) + p\mu|b|(A-B)]\omega_n(\alpha,\beta;l;m)a_n} < 1.$$

$(z \in U^*, z \in C, |z| < 1)$.

Hence, by the Maximum Modulus Theorem we have $f(z) \in H_m^l(p,A,B,b,\mu)$.

3. DISTORTION THEOREM

Theorem 3.1: Let $f(z) \in H_m^l(p,A,B,b,\mu)$, then for $0 < |z| = r < 1$, we have

$$r^{-p} - \left(\frac{\mu|b|(A-B)(2p)!}{[2(1-B) - \mu|b|(A-B)]}\frac{(\beta_1)_{2p}\ldots\ldots(\beta_m)_{2p}}{(\alpha_1)_{2p}\ldots\ldots(\alpha_l)_{2p}}\right)r^p \le |f(z)|$$

$$\le r^{-p} + \left(\frac{\mu|b|(A-B)(2p)!}{[2(1-B)-\mu|b|(A-B)]}\frac{(\beta_1)_{2p}\ldots\ldots(\beta_m)_{2p}}{(\alpha_1)_{2p}\ldots\ldots(\alpha_l)_{2p}}\right)r^p \quad (3.1)$$

where equality holds true for the function

$$f(z) = z^{-p} + \left(\frac{\mu|b|(A-B)(2p)!}{[2(1-B)-\mu|b|(A-B)]}\frac{(\beta_1)_{2p}\ldots\ldots(\beta_m)_{2p}}{(\alpha_1)_{2p}\ldots\ldots(\alpha_l)_{2p}}\right)z^p. \quad (3.2)$$

Proof: Since $f(z) \in H_m^l(p, A, B, b, \mu)$ then from (2.1)

$$\sum_{n=p}^{\infty} [n(1-B) + p\{1 - B - \mu|b|(A-B)\}] \frac{(\alpha_1)_{n+p} \dots \dots (\alpha_l)_{n+p}}{(\beta_1)_{n+p} \dots \dots (\beta_m)_{n+p}} \frac{1}{(n+p)!} a_n \le p\mu|b|(A-B)$$

or

$$[p(1-B) + p\{1 - B - \mu|b|(A-B)\}] \frac{(\alpha_1)_{2p} \dots \dots (\alpha_l)_{2p}}{(\beta_1)_{2p} \dots \dots (\beta_m)_{2p}} \frac{1}{(2p)!} \sum_{n=p}^{\infty} a_n \le p\mu|b|(A-B)$$

or

$$\sum_{n=p}^{\infty} |a_n| \le \left(\frac{\mu|b|(A-B)(2p)!}{[2(1-B) - \mu|b|(A-B)]} \frac{(\beta_1)_{2p} \dots \dots (\beta_m)_{2p}}{(\alpha_1)_{2p} \dots \dots (\alpha_l)_{2p}} \right). \quad (3.3)$$

Thus for $0 < |z| = r < 1$,

$$|f(z)| \le |z|^{-p} + \sum_{n=p}^{\infty} |a_n| z^n \le r^{-p} + r^p \sum_{n=p}^{\infty} |a_n|$$

or

$$|f(z)| \le r^{-p} + \left(\frac{\mu|b|(A-B)(2p)!}{[2(1-B) - \mu|b|(A-B)]} \frac{(\beta_1)_{2p} \dots \dots (\beta_m)_{2p}}{(\alpha_1)_{2p} \dots \dots (\alpha_l)_{2p}} \right) r^p \quad (3.4)$$

and

$$|f(z)| \ge |z|^{-p} - \sum_{n=p}^{\infty} |a_n| z^n \ge r^{-p} - r^p \sum_{n=p}^{\infty} |a_n|$$

or

$$|f(z)| \ge r^{-p} - \left(\frac{\mu|b|(A-B)(2p)!}{[2(1-B) - \mu|b|(A-B)]} \frac{(\beta_1)_{2p} \dots \dots (\beta_m)_{2p}}{(\alpha_1)_{2p} \dots \dots (\alpha_l)_{2p}} \right) r^p \quad (3.5)$$

On using (3.4) and (3.5) inequality (3.1) follows.

4. Closure Theorem

Theorem 4.1: Let

$$f_{p-1}(z) = z^{-p} \text{ and } f_n(z) = z^{-p} + \left(\frac{p\mu|b|(A-B)}{[n(1-B) + p\{1 - B - \mu|b|(A-B)\}]} \right) \frac{1}{\omega_n(\alpha, \beta; l; m)} z^n \quad (4.1)$$

for $n \ge p$, then $f(z) \in H_m^l(p, A, B, b, \mu)$ if and only if it can be expressed in the form

$$f(z) = \sum_{n=p-1}^{\infty} \lambda_n f_n(z), \text{ where } \lambda_n \ge 0 \text{ and } \sum_{n=p-1}^{\infty} \lambda_n = 1. \quad (4.2)$$

Proof: Let f (z) can be expressed in the form (4.2), then

$$f(z) = \sum_{n=p-1}^{\infty} \lambda_n f_n(z) = z^{-p} + \sum_{n=p}^{\infty} \left(\frac{p\mu|b|(A-B)}{[n(1-B) + p\{1 - B - \mu|b|(A-B)\}]} \right) \frac{\lambda_n}{\omega_n(\alpha, \beta; l; m)} z^n .$$

Then

$$\sum_{n=p}^{\infty} [n(1-B) + p\{1 - B - \mu|b|(A-B)\}] \omega_n(\alpha, \beta; l; m) \times \left(\frac{p\mu|b|(A-B)}{[n(1-B) + p\{1 - B - \mu|b|(A-B)\}]} \right) \frac{\lambda_n}{\omega_n(\alpha, \beta; l; m)}$$

$$= \sum_{n=p}^{\infty} p\mu|b|(A-B)\lambda_n = p\mu|b|(A-B) \sum_{n=p}^{\infty} \lambda_n \le p\mu|b|(A-B).$$

So, from (2.1), it follows that $f(z) \in H_m^l(p, A, B, b, \mu)$

Conversely, let $f(z) \in H_m^l(p, A, B, b, \mu)$. From theorem 2.1, we have

$$a_n \le \left(\frac{p\mu|b|(A-B)}{[n(1-B) + p\{1 - B - \mu|b|(A-B)\}]} \right) \frac{1}{\omega_n(\alpha, \beta; l; m)} \text{ for } n \ge p.$$

Setting

$$\lambda_n = \frac{n(1-B) + p\{1 - B - \mu|b|(A-B)\}}{p\mu|b|(A-B)} \omega_n(\alpha, \beta; l; m) \text{ for } n \ge p.$$

and $\lambda_{p-1} = \sum_{n=p}^{\infty} \lambda_n$

It follows that

$$f(z) = \sum_{n=p-1}^{\infty} \lambda_n f_n(z).$$

This completes the proof.

5. Radii of Starlikeness and Convexity

Theorem 5.1: Let the function f(z) defined by (1.1) be in the class $H_m^l(p, A, B, b, \mu)$ Then

(i)f is meromorphically p-valent starlike of order δ $(0 \le \delta \le p)$ in the disk $|z| < r_1$,where

$$r_1 = r_1(p, A, B, b, \mu, \delta) = \min_{n \ge p}[\frac{\{n(1-B)+p(1-B-\mu|b|(A-B))\}}{p\mu|b|(A-B)}\omega_n(\alpha, \beta; l; m)\left(\frac{p-\delta}{n+\delta}\right)]^{\frac{1}{n}}. \quad (5.1)$$

(ii) f is meromorphically p-valent convex of order δ $(0 \le \delta \le p)$ in the disk $|z| < r_2$,where

$$r_2 = r_2(p, A, B, b, \mu, \delta) = \min_{n \ge p}[\frac{\{n(1-B)+p(1-B-\mu|b|(A-B))\}}{p\mu|b|(A-B)}\omega_n(\alpha, \beta; l; m)\frac{p(p-\delta)}{n(n+\delta)}]^{\frac{1}{n}}. \quad (5.2)$$

Proof: (i) Using definition (1.1), we observe that

$$\left|\frac{zf'(z)+pf(z)}{zf'(z)+(2\delta-p)f(z)}\right| \le \frac{\sum_{n=p}^{\infty}(n+p)|a_n||z|^n}{2(p-\delta)-\sum_{n=p}^{\infty}(n-p+2\delta)|a_n||z|^n} \le 1, (|z| < r_1; 0 \le \delta < 1). \quad (5.3)$$

This last inequality (5.3) holds true if

$$\sum_{n=p}^{\infty}\left(\frac{n+\delta}{p-\delta}\right)|a_n||z|^n \le 1.$$

In view of (2.1), the last inequality is true if

$$\left(\frac{n+\delta}{p-\delta}\right)|z|^n \le \frac{\{n(1-B)+p(1-B-\mu|b|(A-B))\}}{p\mu|b|(A-B)}\omega_n(\alpha, \beta; l; m)(n \ge p, p \in N).$$

which on solving gives (5.1).

(ii) Using definition (1.1), we observe that

$$\left|\frac{zf''(z)+(1+p)f'(z)}{zf''(z)+(1+2\delta-p)f'(z)}\right| \le \frac{\sum_{n=p}^{\infty}n(n+p)|a_n||z|^n}{2p(p-\delta)-\sum_{n=p}^{\infty}n(n-p+2\delta)|a_n||z|^n} \le 1, (|z| < r_2; 0 \le \delta < 1). \quad (5.4)$$

This last inequality (5.4) holds true if

$$\sum_{n=p}^{\infty}\left(\frac{n(\ n+\delta)}{p\ (p-\delta)}\right)|a_n||z|^n \le 1.$$

In view of (2.1), the last inequality is true if

$$\left(\frac{n(\ n+\delta)}{p\ (p-\delta)}\right)|z|^n \le \frac{\{n(1-B)+p(1-B-\mu|b|(A-B))\}}{p\mu|b|(A-B)}\omega_n(\alpha, \beta; l; m)(n \ge p, p \in N).$$

which on solving gives (5.2).

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