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Research Article

STOCHASTIC MODEL FOR MANPOWER PLANNING WITH MULTIPLE DEPLETIONS

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ABSTRACT

In any organization, it is a routine fact that some employees leave the organization deliberately due to retirement, physical or psychological illness and demise. This is common whenever there is policy decision for ennoblement in salary, marketing and the objectives of the organization. This leads to wastage or attrition. To overcome this manpower wastage, new recruitment is indispensable. Based on the requirement of the organization, quality employees are selected and recruited at the right time in right quantity. Manpower planning helps in achieving this effectively, also recognize excess and inadequacy manpower areas and thereby maintains equilibrium. Continual recruitment affects the reputation and financial status of the organization. Hence recruitment can be adjourning till a point called breakdown point. Due to shortfall of manpower, beyond the breakdown point, the daily routine of the organization will be affected. This degree of allowable manpower reduction is called threshold. In this paper a stochastic model to estimate the expected time to recruitment with four sources of reduction of manpower wastage using correlated interarrival time has been determined. This gives maximum solution, as it deals with the expenses of continual recruitment, reduction of manpower and the expenditure of reduction of manpower using the result of Gurland (1955) and Esary *et. al.,* (1973), a Stochastic model for Manpower planning with Multiple Depletions has been discussed. Numerical illustrations are also provided.

INTRODUCTION

Manpower wastage is an element of employee turnover wastage is detachment from the organization, which includes, deliberate retirement, normal retirement, physical or psychological ailment, termination and death. This happens whenever the policy decision involving revision of pay and benefits and target in marketing to be accomplished. This leads to exit of employees from the organization called Wastage. An elevated rate of wastage in an organization leads to optimal recruitment, hiring and training cost. Recruitment of qualified and competent replacement is difficult to find when a productive employee resigns causing a loss of talent and skills within the organization. Unfair compensation and benefits, inability to cope with the demands of the job, lack of career growth opportunities, health problems are the major issues for manpower wastage. Manpower planning plays a very crucial role in addressing and preventing employee wastage. Replacement of exist employee is the biggest task for an organization as it involves high time, energy and money. But a appropriate solution has to be established to subside the reduction of manpower. Manpower wastage is allowed till the reduction crosses a level called the threshold. A stochastic model to determine the expected time to recruit with four sources of reduction of manpower wastage using correlated interarrival time is discussed in this paper. This gives an expression for expected time to recruit and gives a maximum solution for manpower wastage. The area of application of this Stochastic model is not restricted to particular industries but has immense scope in various field of interest. Using the result of Gurland (1955) and Esary *et. al.,* (1973), a Stochastic model for Manpower planning with Multiple Depletions has been discussed. Numerical illustrations are also provided.

Assumption

1. The diminution of manpower results due to proclamation of policy decisions.
2. The interarrival times between decision epochs are random variables are constantly correlated and exchangeable but not independent.

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3. The diminution of manpower also results owing to transmit of employee and the interarrival times between successive epochs of transmit are i.i.d. random variables having exponential distribution with parameter λ.
4. The total diminution due to the above three assumptions crosses the threshold which itself is a random variable, recruitment becomes necessary.
5. The four source of diminution are linear and additive.

Notations

1. X_i- a continuous random variable representing the amount of diminution of man hours due to the i^{th} epoch of policy decisions having exponential distribution with parameter α.
2. Y_j– a continuous random variable representing the amount of diminution of man hours due to the j^{th} event of transfer of personnel having exponential distribution with parameter μ.
3. Z_n – a continuous random variable representing the amount of diminution of man hours due to the n^{th} event of transfer of personnel having with exponential distribution parameter is denoted as η.
4. h – Probability density function of X_i, k – Probability density function of Y_i and m – Probability density function of Z_n
5. u_i – Random variable denoting the interarrival times between the successive decision epochs. The u_i's are constantly correlated with exponential distribution with parameter α.
6. v_j – Random variable representing the interarrival times between the successive epochs of transfer. v_j's are i.i.d. random variable with exponential distribution with parameter λ.
7. W_n – Random variable representing the interarrival times between the successive epochs of transfer of n^{th} event with parameter β.
8. $\widetilde{X} = X_1 + X_2 + X_3 + ... + X_m$, $\widetilde{Y} = Y_1 + Y_2 + Y_3 + ... + Y_m$ and $\widetilde{Z} = Z_1 + Z_2 + Z_3 + ... + Z_m$
9. L(t) = P(T < t) = Cumulative distribution function of the time to recruitment of the system.
10. S(t) = Survivor function P(T > t)
11. L*(S) = Laplace transform of L(t), h*(.) = Laplace transform of h(.),
12. k*(.) = Laplace transform of k(.) and p*(.) = Laplace transform of p(.)
13. f(.) = Probability density function of u_i, g(.) = Probability density function of v_i and s(.) = Probability density function of w_n.

The probability that the total diminution of manpower on m occasions of decision making and n, p and q occasions of transfer of personnel does not exceed the threshold level is given as,

$$P\left[\widetilde{A}+\widetilde{B}+\widetilde{C}+\widetilde{D}<Z\right]=\int_0^\infty Q_{\widetilde{A}+\widetilde{B}+\widetilde{C}+\widetilde{D}}(x)c\,e^{-cx}dx$$

$$=\int_0^\infty Q_{\widetilde{A}+\widetilde{B}+\widetilde{C}+\widetilde{D}}(x)c\,e^{-cx}dx$$

$$= C\,Q^*_{\widetilde{A}+\widetilde{B}+\widetilde{C}+\widetilde{D}}(x)$$

But we know that

$$F_m^*(s)=\frac{1}{s}f_m^*(s)$$

$$G_n^*(s)=\frac{1}{s}g_n^*(s)$$

$$R_p^*(s)=\frac{1}{s}r_p^*(s)$$

$$S_q^*(s)=\frac{1}{s}s_q^*(s)$$

$$P\left[\widetilde{A}+\widetilde{B}+\widetilde{C}+\widetilde{D}<Z\right]=c\frac{q^*_{\widetilde{A}+\widetilde{B}+\widetilde{C}+\widetilde{D}}(c)}{c}$$

$$=q^*_{\widetilde{A}}(c)\,q^*_{\widetilde{B}}(c)\,q^*_{\widetilde{C}}(c)q^*_{\widetilde{D}}(c)$$

$$=\left[q^*(c)\right]^m\left[q^*(c)\right]^n\left[q^*(c)\right]^p\left[q^*(c)\right]^q$$

$$=\left[h^*(c)\right]^m\left[k^*(c)\right]^n\left[p^*(c)\right]^p\left[p^*(c)\right]^q \qquad \text{…(1)}$$

Since A, B,C and D are independent and x_i, y_j and z_n

RESULTS

$s(t)=P(T<t)$ = Probability that there are exactly m occasions of policy making and n, p and q occasions of transfer and the total depletion does not cross the threshold Z in (0, t).

$$s(t)=\sum_{m=0}^{\infty}[F_m(t)-F_{m+1}(t)]h^*(c)^m\sum_{n=0}^{\infty}[G_n(t)-G_{n+1}(t)]\,k^*(c)^n$$
$$\sum_{p=0}^{\infty}[R_p(t)-R_{p+1}(t)]u^*(c)^p\sum_{q=0}^{\infty}[S_q(t)-S_{q+1}(t)]\,v^*(c)^q \qquad \ldots(2)$$

$$s(t)=\left\{1-\left(1-h^*(c)\right)\left(\sum_{m=1}^{\infty}F_m(t)\,h^*(c)^{m-1}\right)\right\}\left\{1-\left(1-k^*(c)\right)\left(\sum_{n=1}^{\infty}G_n(t)\,k^*(c)^{n-1}\right)\right\}$$
$$\left\{1-\left(1-u^*(c)\right)\left(\sum_{p=1}^{\infty}R_p(t)\,u^*(c)^{p-1}\right)\right\}\left\{1-\left(1-v^*(c)\right)\left(\sum_{q=1}^{\infty}S_q(t)\,v^*(c)^{q-1}\right)\right\} \qquad \ldots(3)$$

$$Now, L(t)=p[T<t]=1-s(t)$$
$$=1-\left\{1-\left(1-h^*(c)\right)\left(\sum_{m=1}^{\infty}F_m(t)\,h^*(c)^{m-1}\right)\right\}\left\{1-\left(1-k^*(c)\right)\left(\sum_{n=1}^{\infty}G_n(t)\,k^*(c)^{n-1}\right)\right\}$$
$$\left\{1-\left(1-u^*(c)\right)\left(\sum_{q=1}^{\infty}R_p(t)\,u^*(c)^{p-1}\right)\right\}\left\{1-\left(1-v^*(c)\right)\left(\sum_{q=1}^{\infty}S_q(t)\,v^*(c)^{q-1}\right)\right\} \qquad \ldots(4)$$

$$=\left(1-h^*(c)\right)\left(\sum_{m=1}^{\infty}F_m(t)\,h^*(c)^{m-1}\right)+\left(1-k^*(c)\right)\left(\sum_{n=1}^{\infty}G_n(t)\,k^*(c)^{n-1}\right)$$
$$-\left(1-h^*(c)\right)\left(1-k^*(c)\right)\left(\sum_{m=1}^{\infty}F_m(t)\,h^*(c)^{m-1}\right)\left(\sum_{n=1}^{\infty}G_n(t)\,k^*(c)^{n-1}\right)$$
$$+\left(1-u^*(c)\right)\left(\sum_{p=1}^{\infty}R_p(t)\,u^*(c)^{p-1}\right)$$
$$-\left(1-h^*(c)\right)\left(1-u^*(c)\right)\left(\sum_{m=1}^{\infty}F_m(t)\,h^*(c)^{m-1}\right)\left(\sum_{p=1}^{\infty}R_p(t)\,u^*(c)^{p-1}\right)$$
$$-\left(1-k^*(c)\right)\left(1-u^*(c)\right)\left(\sum_{n=1}^{\infty}G_n(t)\,k^*(c)^{n-1}\right)\left(\sum_{p=1}^{\infty}R_p(t)\,u^*(c)^{p-1}\right)$$
$$+\left(1-h^*(c)\right)\left(1-k^*(c)\right)\left(1-u^*(c)\right)\left(\sum_{m=1}^{\infty}F_m(t)\,h^*(c)^{m-1}\right)\left(\sum_{n=1}^{\infty}G_n(t)\,k^*(c)^{n-1}\right)\left(\sum_{p=1}^{\infty}R_p(t)\,u^*(c)^{p-1}\right)$$
$$+\left(1-v^*(c)\right)\left(\sum_{q=1}^{\infty}S_q(t)\,v^*(c)^{q-1}\right)-\left(1-h^*(c)\right)\left(1-v^*(c)\right)\left(\sum_{m=1}^{\infty}F_m(t)\,h^*(c)^{m-1}\right)\left(\sum_{q=1}^{\infty}S_q(t)\,v^*(c)^{q-1}\right)$$
$$-\left(1-k^*(c)\right)\left(1-v^*(c)\right)\left(\sum_{n=1}^{\infty}G_n(t)\,k^*(c)^{n-1}\right)\left(\sum_{q=1}^{\infty}S_q(t)\,v^*(c)^{q-1}\right)$$
$$+\left(1-h^*(c)\right)\left(1-k^*(c)\right)\left(1-v^*(c)\right)\left(\sum_{m=1}^{\infty}F_m(t)\,h^*(c)^{m-1}\right)\left(\sum_{n=1}^{\infty}G_n(t)\,k^*(c)^{n-1}\right)\left(\sum_{q=1}^{\infty}S_q(t)\,v^*(c)^{q-1}\right)$$
$$-\left(1-u^*(c)\right)\left(1-v^*(c)\right)\left(\sum_{p=1}^{\infty}R_p(t)\,u^*(c)^{p-1}\right)\left(\sum_{q=1}^{\infty}S_q(t)\,v^*(c)^{q-1}\right)$$
$$+\left(1-h^*(c)\right)\left(1-u^*(c)\right)\left(1-v^*(c)\right)\left(\sum_{m=1}^{\infty}F_m(t)\,h^*(c)^{m-1}\right)\left(\sum_{p=1}^{\infty}R_p(t)\,u^*(c)^{p-1}\right)\left(\sum_{q=1}^{\infty}S_q(t)\,v^*(c)^{q-1}\right)$$
$$+\left(1-k^*(c)\right)\left(1-u^*(c)\right)\left(1-v^*(c)\right)\left(\sum_{n=1}^{\infty}G_n(t)\,k^*(c)^{n-1}\right)\left(\sum_{p=1}^{\infty}R_p(t)\,u^*(c)^{p-1}\right)\left(\sum_{q=1}^{\infty}S_q(t)\,v^*(c)^{q-1}\right)$$
$$-\left(1-h^*(c)\right)\left(1-k^*(c)\right)\left(1-u^*(c)\right)\left(1-v^*(c)\right)\left(\sum_{m=1}^{\infty}F_m(t)\,h^*(c)^{m-1}\right)\left(\sum_{n=1}^{\infty}G_n(t)\,k^*(c)^{n-1}\right)$$
$$\left(\sum_{p=1}^{\infty}R_p(t)\,u^*(c)^{p-1}\right)\left(\sum_{q=1}^{\infty}S_q(t)\,v^*(c)^{q-1}\right) \qquad \ldots\ldots(5)$$

pdf is given by

$$
\begin{aligned}
l(t) &= \left(1-h^*(c)\right)\sum_{m=1}^{\infty} f_m(t)\,h^*(c)^{m-1} + \left(1-k^*(c)\right)\sum_{n=1}^{\infty} g_n(t)\,k^*(c)^{n-1} \\
&-\left(1-h^*(c)\right)\left(1-k^*(c)\right)\left[\sum_{m=1}^{\infty} f_m(t)h^*(c)^{m-1}\sum_{n=1}^{\infty} G_n(t)k^*(c)^{n-1} + \sum_{m=1}^{\infty} F_m(t)h^*(c)^{m-1}\sum_{n=1}^{\infty} g_n(t)k^*(c)^{n-1}\right] \\
&+\left(1-u^*(c)\right)\sum_{p=1}^{\infty} r_p(t)\,u^*(c)^{p-1} \\
&-\left(1-h^*(c)\right)\left(1-u^*(c)\right)\left[\sum_{m=1}^{\infty} f_m(t)h^*(c)^{m-1}\sum_{p=1}^{\infty} R_p(t)u^*(c)^{p-1} + \sum_{m=1}^{\infty} F_m(t)h^*(c)^{m-1}\sum_{p=1}^{\infty} r_p(t)u^*(c)^{p-1}\right] \\
&-\left(1-k^*(c)\right)\left(1-u^*(c)\right)\left[\sum_{n=1}^{\infty} g_n(t)k^*(c)^{n-1}\sum_{p=1}^{\infty} R_p(t)u^*(c)^{p-1} + \sum_{n=1}^{\infty} G_n(t)k^*(c)^{n-1}\sum_{p=1}^{\infty} r_p(t)u^*(c)^{p-1}\right] \\
&+\left(1-h^*(c)\right)\left(1-k^*(c)\right)\left(1-u^*(c)\right)\left[\begin{array}{l}
\sum_{m=1}^{\infty} f_m(t)h^*(c)^{m-1}\sum_{n=1}^{\infty} G_n(t)k^*(c)^{n-1}\sum_{p=1}^{\infty} R_p(t)u^*(c)^{p-1} \\
+\sum_{m=1}^{\infty} F_m(t)h^*(c)^{m-1}\sum_{n=1}^{\infty} g_n(t)k^*(c)^{n-1}\sum_{p=1}^{\infty} R_p(t)u^*(c)^{p-1} \\
+\sum_{m=1}^{\infty} F_m(t)h^*(c)^{m-1}\sum_{n=1}^{\infty} G_n(t)k^*(c)^{n-1}\sum_{p=1}^{\infty} r_p(t)u^*(c)^{p-1}
\end{array}\right] \\
&+\left(1-v^*(c)\right)\sum_{q=1}^{\infty} s_q(t)\,v^*(c)^{q-1} \\
&-\left(1-h^*(c)\right)\left(1-v^*(c)\right)\left[\sum_{m=1}^{\infty} f_m(t)h^*(c)^{m-1}\sum_{q=1}^{\infty} S_q(t)v^*(c)^{q-1} + \sum_{m=1}^{\infty} F_m(t)h^*(c)^{m-1}\sum_{q=1}^{\infty} s_q(t)v^*(c)^{q-1}\right] \\
&-\left(1-k^*(c)\right)\left(1-v^*(c)\right)\left[\sum_{n=1}^{\infty} g_n(t)k^*(c)^{n-1}\sum_{q=1}^{\infty} S_q(t)v^*(c)^{q-1} + \sum_{n=1}^{\infty} G_n(t)k^*(c)^{n-1}\sum_{q=1}^{\infty} s_q(t)v^*(c)^{q-1}\right] \\
&+\left(1-h^*(c)\right)\left(1-k^*(c)\right)\left(1-v^*(c)\right)\left[\begin{array}{l}
\sum_{m=1}^{\infty} f_m(t)h^*(c)^{m-1}\sum_{n=1}^{\infty} G_n(t)k^*(c)^{n-1}\sum_{q=1}^{\infty} S_q(t)v^*(c)^{q-1} \\
+\sum_{m=1}^{\infty} F_m(t)h^*(c)^{m-1}\sum_{n=1}^{\infty} g_n(t)k^*(c)^{n-1}\sum_{q=1}^{\infty} S_q(t)v^*(c)^{q-1} \\
+\sum_{m=1}^{\infty} F_m(t)h^*(c)^{m-1}\sum_{n=1}^{\infty} G_n(t)k^*(c)^{n-1}\sum_{q=1}^{\infty} s_q(t)v^*(c)^{q-1}
\end{array}\right] \\
&-\left(1-u^*(c)\right)\left(1-v^*(c)\right)\left[\sum_{p=1}^{\infty} r_p(t)u^*(c)^{p-1}\sum_{q=1}^{\infty} S_q(t)v^*(c)^{q-1} + \sum_{p=1}^{\infty} R_p(t)u^*(c)^{p-1}\sum_{q=1}^{\infty} s_q(t)v^*(c)^{q-1}\right] \\
&+\left(1-h^*(c)\right)\left(1-u^*(c)\right)\left(1-v^*(c)\right)\left[\begin{array}{l}
\sum_{m=1}^{\infty} f_m(t)h^*(c)^{m-1}\sum_{p=1}^{\infty} R_p(t)u^*(c)^{p-1}\sum_{q=1}^{\infty} S_q(t)v^*(c)^{q-1} \\
+\sum_{m=1}^{\infty} F_m(t)h^*(c)^{m-1}\sum_{p=1}^{\infty} r_p(t)u^*(c)^{p-1}\sum_{q=1}^{\infty} S_q(t)v^*(c)^{q-1} \\
+\sum_{m=1}^{\infty} F_m(t)h^*(c)^{m-1}\sum_{p=1}^{\infty} R_p(t)u^*(c)^{p-1}\sum_{q=1}^{\infty} s_q(t)v^*(c)^{q-1}
\end{array}\right]
\end{aligned}
$$

$$+\left(1-k^*(c)\right)\left(1-u^*(c)\right)\left(1-v^*(c)\right)\begin{bmatrix}\sum_{n=1}^{\infty} g_n(t)k^*(c)^{n-1}\sum_{p=1}^{\infty} R_p(t)u^*(c)^{p-1}\sum_{q=1}^{\infty} S_q(t)v^*(c)^{q-1}\\ +\sum_{n=1}^{\infty} G_n(t)k^*(c)^{n-1}\sum_{p=1}^{\infty} r_p(t)u^*(c)^{p-1}\sum_{q=1}^{\infty} S_q(t)v^*(c)^{q-1}\\ +\sum_{n=1}^{\infty} G_n(t)k^*(c)^{n-1}\sum_{p=1}^{\infty} R_p(t)u^*(c)^{p-1}\sum_{q=1}^{\infty} s_q(t)v^*(c)^{q-1}\end{bmatrix}$$

$$-\left(1-h^*(c)\right)\left(1-k^*(c)\right)\left(1-u^*(c)\right)\left(1-v^*(c)\right)$$

$$\begin{bmatrix}\sum_{m=1}^{\infty} f_m(t)h^*(c)^{m-1}\sum_{n=1}^{\infty} G_n(t)k^*(c)^{n-1}\sum_{p=1}^{\infty} R_p(t)u^*(c)^{p-1}\sum_{q=1}^{\infty} S_q(t)v^*(c)^{q-1}\\ +\sum_{m=1}^{\infty} F_m(t)h^*(c)^{m-1}\sum_{n=1}^{\infty} g_n(t)k^*(c)^{n-1}\sum_{p=1}^{\infty} R_p(t)u^*(c)^{p-1}\sum_{q=1}^{\infty} S_q(t)v^*(c)^{q-1}\\ +\sum_{m=1}^{\infty} F_m(t)h^*(c)^{m-1}\sum_{n=1}^{\infty} G_n(t)k^*(c)^{n-1}\sum_{p=1}^{\infty} r_p(t)u^*(c)^{p-1}\sum_{q=1}^{\infty} S_q(t)v^*(c)^{q-1}\\ +\sum_{m=1}^{\infty} F_m(t)h^*(c)^{m-1}\sum_{n=1}^{\infty} G_n(t)k^*(c)^{n-1}\sum_{p=1}^{\infty} R_p(t)u^*(c)^{p-1}\sum_{q=1}^{\infty} s_q(t)v^*(c)^{q-1}\end{bmatrix}$$

……(6)

$$l(t)=\left(1-h^*(c)\right)\sum_{m=1}^{\infty} f_m(t)h^*(c)^{m-1}+\left(1-k^*(c)\right)\sum_{n=1}^{\infty}\lambda e^{-\lambda t}\frac{(\lambda t)^{n-1}}{(n-1)!}k^*(c)^{n-1}$$

$$-\left(1-h^*(c)\right)\left(1-k^*(c)\right)\begin{bmatrix}\sum_{m=1}^{\infty} f_m(t)h^*(c)^{m-1}\sum_{n=1}^{\infty}\int_0^t\left(\lambda e^{-\lambda t}\frac{(\lambda t)^{n-1}}{(n-1)!}k^*(c)^{n-1}\right)dt\\ +\sum_{m=1}^{\infty} F_m(t)h^*(c)^{m-1}\sum_{n=1}^{\infty}\lambda e^{-\lambda t}\frac{(\lambda t)^{n-1}}{(n-1)!}k^*(c)^{n-1}\end{bmatrix}$$

$$+\left(1-u^*(c)\right)\sum_{p=1}^{\infty}\mu e^{-\mu t}\frac{(\mu t)^{p-1}}{(p-1)!}u^*(c)^{p-1}$$

$$-\left(1-h^*(c)\right)\left(1-u^*(c)\right)\begin{bmatrix}\sum_{n=1}^{\infty} f_m(t)h^*(c)^{m-1}\sum_{p=1}^{\infty}\int_0^t\left(\mu e^{-\mu t}\frac{(\mu t)^{p-1}}{(p-1)!}u^*(c)^{p-1}\right)dt\\ +\sum_{m=1}^{\infty} F_m(t)h^*(c)^{m-1}\sum_{p=1}^{\infty}\mu e^{-\mu t}\frac{(\mu t)^{p-1}}{(p-1)!}u^*(c)^{p-1}\end{bmatrix}$$

$$-\left(1-k^*(c)\right)\left(1-u^*(c)\right)\begin{bmatrix}\sum_{n=1}^{\infty}\lambda e^{-\lambda t}\frac{(\lambda t)^{n-1}}{(n-1)!}k^*(c)^{n-1}\sum_{p=1}^{\infty}\int_0^t\left(\mu e^{-\mu t}\frac{(\mu t)^{p-1}}{(p-1)!}u^*(c)^{p-1}\right)dt\\ +\sum_{n=1}^{\infty}\int_0^t\left(\lambda e^{-\lambda t}\frac{(\lambda t)^{n-1}}{(n-1)!}k^*(c)^{n-1}\right)dt\sum_{p=1}^{\infty}\mu e^{-\mu t}\frac{(\mu t)^{p-1}}{(p-1)!}u^*(c)^{p-1}\end{bmatrix}$$

$$+\left(1-h^*(c)\right)\left(1-k^*(c)\right)\left(1-u^*(c)\right)$$

$$\left[\begin{array}{l}\sum_{m=1}^{\infty} f_m(t)\,h^*(c)^{m-1}\sum_{n=1}^{\infty}\int_0^t\left(\lambda e^{-\lambda t}\frac{(\lambda t)^{n-1}}{(n-1)!}k^*(c)^{n-1}\right)dt\sum_{p=1}^{\infty}\int_0^t\left(\mu e^{-\mu t}\frac{(\mu t)^{p-1}}{(p-1)!}u^*(c)^{p-1}\right)dt\\ +\sum_{m=1}^{\infty} F_m(t)\,h^*(c)^{m-1}\sum_{n=1}^{\infty}\lambda e^{-\lambda t}\frac{(\lambda t)^{n-1}}{(n-1)!}k^*(c)^{n-1}\sum_{p=1}^{\infty}\int_0^t\left(\mu e^{-\mu t}\frac{(\mu t)^{p-1}}{(p-1)!}u^*(c)^{p-1}\right)dt\\ +\sum_{m=1}^{\infty} F_m(t)\,h^*(c)^{m-1}\sum_{n=1}^{\infty}\int_0^t\left(\lambda e^{-\lambda t}\frac{(\lambda t)^{n-1}}{(n-1)!}k^*(c)^{n-1}\right)dt\sum_{p=1}^{\infty}\mu e^{-\mu t}\frac{(\mu t)^{p-1}}{(p-1)!}u^*(c)^{p-1}\end{array}\right]$$

$$+\left(1-v^*(c)\right)\sum_{q=1}^{\infty}\theta e^{-\theta t}\frac{(\theta t)^{q-1}}{(q-1)!}v^*(c)^{q-1}$$

$$-\left(1-h^*(c)\right)\left(1-v^*(c)\right)\left[\begin{array}{l}\sum_{m=1}^{\infty} f_m(t)\,h^*(c)^{m-1}\sum_{q=1}^{\infty}\int_0^t\left(\theta e^{-\theta t}\frac{(\theta t)^{q-1}}{(q-1)!}v^*(c)^{q-1}\right)dt\\ +\sum_{m=1}^{\infty} F_m(t)\,h^*(c)^{m-1}\sum_{q=1}^{\infty}\theta e^{-\theta t}\frac{(\theta t)^{q-1}}{(q-1)!}v^*(c)^{q-1}\end{array}\right]$$

$$-\left(1-k^*(c)\right)\left(1-v^*(c)\right)\left[\begin{array}{l}\sum_{n=1}^{\infty}\lambda e^{-\lambda t}\frac{(\lambda t)^{n-1}}{(n-1)!}k^*(c)^{n-1}\sum_{q=1}^{\infty}\int_0^t\left(\theta e^{-\theta t}\frac{(\theta t)^{q-1}}{(q-1)!}v^*(c)^{q-1}\right)dt\\ +\sum_{n=1}^{\infty}\int_0^t\left(\lambda e^{-\lambda t}\frac{(\lambda t)^{n-1}}{(n-1)!}k^*(c)^{n-1}\right)dt\sum_{q=1}^{\infty}\theta e^{-\theta t}\frac{(\theta t)^{q-1}}{(q-1)!}v^*(c)^{q-1}\end{array}\right]$$

$$-\left(1-u^*(c)\right)\left(1-v^*(c)\right)\left[\begin{array}{l}\sum_{p=1}^{\infty}\mu e^{-\mu t}\frac{(\mu t)^{p-1}}{(p-1)!}u^*(c)^{p-1}\sum_{q=1}^{\infty}\int_0^t\left(\theta e^{-\theta t}\frac{(\theta t)^{q-1}}{(q-1)!}v^*(c)^{q-1}\right)dt\\ +\sum_{p=1}^{\infty}\int_0^t\left(\mu e^{-\mu t}\frac{(\mu t)^{p-1}}{(p-1)!}u^*(c)^{p-1}\right)dt\sum_{q=1}^{\infty}\theta e^{-\theta t}\frac{(\theta t)^{q-1}}{(q-1)!}v^*(c)^{q-1}\end{array}\right]$$

$$+\left(1-h^*(c)\right)\left(1-u^*(c)\right)\left(1-v^*(c)\right)$$

$$\left[\begin{array}{l}\sum_{m=1}^{\infty} f_m(t)\,h^*(c)^{m-1}\sum_{p=1}^{\infty}\int_0^t\left(\mu e^{-\mu t}\frac{(\mu t)^{p-1}}{(p-1)!}u^*(c)^{p-1}\right)dt\sum_{q=1}^{\infty}\int_0^t\left(\theta e^{-\theta t}\frac{(\theta t)^{q-1}}{(q-1)!}v^*(c)^{q-1}\right)dt\\ +\sum_{m=1}^{\infty} F_m(t)\,h^*(c)^{m-1}\sum_{p=1}^{\infty}\mu e^{-\mu t}\frac{(\mu t)^{p-1}}{(p-1)!}u^*(c)^{p-1}\sum_{q=1}^{\infty}\int_0^t\left(\theta e^{-\theta t}\frac{(\theta t)^{q-1}}{(q-1)!}v^*(c)^{q-1}\right)dt\\ +\sum_{m=1}^{\infty} F_m(t)\,h^*(c)^{m-1}\sum_{p=1}^{\infty}\int_0^t\left(\mu e^{-\mu t}\frac{(\mu t)^{p-1}}{(p-1)!}u^*(c)^{p-1}\right)dt\sum_{q=1}^{\infty}\theta e^{-\theta t}\frac{(\theta t)^{q-1}}{(q-1)!}v^*(c)^{q-1}\end{array}\right]$$

$$+\left(1-k^*(c)\right)\left(1-u^*(c)\right)\left(1-v^*(c)\right)$$

$$\left[\begin{array}{l}\sum_{n=1}^{\infty}\lambda e^{-\lambda t}\frac{(\lambda t)^{n-1}}{(n-1)!}k^*(c)^{n-1}\sum_{p=1}^{\infty}\int_0^t\left(\mu e^{-\mu t}\frac{(\mu t)^{p-1}}{(p-1)!}u^*(c)^{p-1}\right)dt\sum_{q=1}^{\infty}\int_0^t\left(\theta e^{-\theta t}\frac{(\theta t)^{q-1}}{(q-1)!}v^*(c)^{q-1}\right)dt\\ +\sum_{n=1}^{\infty}\int_0^t\left(\lambda e^{-\lambda t}\frac{(\lambda t)^{n-1}}{(n-1)!}k^*(c)^{n-1}\right)dt\sum_{p=1}^{\infty}\mu e^{-\mu t}\frac{(\mu t)^{p-1}}{(p-1)!}u^*(c)^{p-1}\sum_{q=1}^{\infty}\int_0^t\left(\theta e^{-\theta t}\frac{(\theta t)^{q-1}}{(q-1)!}v^*(c)^{q-1}\right)dt\\ +\sum_{n=1}^{\infty}\int_0^t\left(\lambda e^{-\lambda t}\frac{(\lambda t)^{n-1}}{(n-1)!}k^*(c)^{n-1}\right)dt\sum_{p=1}^{\infty}\int_0^t\left(\mu e^{-\mu t}\frac{(\mu t)^{p-1}}{(p-1)!}u^*(c)^{p-1}\right)dt\sum_{q=1}^{\infty}\theta e^{-\theta t}\frac{(\theta t)^{q-1}}{(q-1)!}v^*(c)^{q-1}\end{array}\right]$$

$$
\begin{aligned}
&-\left(1-h^*(c)\right)\left(1-k^*(c)\right)\left(1-u^*(c)\right)\left(1-v^*(c)\right)\\
&\left[\begin{array}{l}
\sum_{m=1}^{\infty} f_m(t)\, h^*(c)^{m-1} \sum_{n=1}^{\infty}\int_0^t \left(\lambda e^{-\lambda t}\,\frac{(\lambda t)^{n-1}}{(n-1)!} k^*(c)^{n-1}\right)dt\\
\sum_{p=1}^{\infty}\int_0^t \left(\mu e^{-\mu t}\,\frac{(\mu t)^{p-1}}{(p-1)!} u^*(c)^{p-1}\right)dt \sum_{q=1}^{\infty}\int_0^t \left(\theta e^{-\theta t}\,\frac{(\theta t)^{q-1}}{(q-1)!} v^*(c)^{q-1}\right)dt\\
+\sum_{m=1}^{\infty} F_m(t)\, h^*(c)^{m-1} \sum_{n=1}^{\infty} \lambda e^{-\lambda t}\,\frac{(\lambda t)^{n-1}}{(n-1)!} k^*(c)^{n-1}\\
\sum_{p=1}^{\infty}\int_0^t \left(\mu e^{-\mu t}\,\frac{(\mu t)^{p-1}}{(p-1)!} u^*(c)^{p-1}\right)dt \sum_{q=1}^{\infty}\int_0^t \left(\theta e^{-\theta t}\,\frac{(\theta t)^{q-1}}{(q-1)!} v^*(c)^{q-1}\right)dt\\
+\sum_{m=1}^{\infty} F_m(t)\, h^*(c)^{m-1} \sum_{n=1}^{\infty}\int_0^t \left(\lambda e^{-\lambda t}\,\frac{(\lambda t)^{n-1}}{(n-1)!} k^*(c)^{n-1}\right)dt\\
\sum_{p=1}^{\infty} \mu e^{-\mu t}\,\frac{(\mu t)^{p-1}}{(p-1)!} u^*(c)^{p-1} \sum_{q=1}^{\infty}\int_0^t \left(\theta e^{-\theta t}\,\frac{(\theta t)^{q-1}}{(q-1)!} v^*(c)^{q-1}\right)dt\\
+\sum_{m=1}^{\infty} F_m(t)\, h^*(c)^{m-1} \sum_{n=1}^{\infty}\int_0^t \left(\lambda e^{-\lambda t}\,\frac{(\lambda t)^{n-1}}{(n-1)!} k^*(c)^{n-1}\right)dt\\
\sum_{p=1}^{\infty}\int_0^t \left(\mu e^{-\mu t}\,\frac{(\mu t)^{p-1}}{(p-1)!} u^*(c)^{p-1}\right)dt \sum_{q=1}^{\infty} \theta e^{-\theta t}\,\frac{(\theta t)^{q-1}}{(q-1)!} v^*(c)^{q-1}
\end{array}\right]
\end{aligned}
\qquad \ldots\ldots(7)
$$

$$
\begin{aligned}
l(t) &= \left(1-h^*(c)\right)\sum_{m=1}^{\infty} f_m(t)\, h^*(c)^{m-1} + \left(1-k^*(c)\right)\lambda e^{-\lambda t\left(1-k^*(c)\right)}\\
&-\left(1-h^*(c)\right)\left(1-k^*(c)\right)\left[\sum_{m=1}^{\infty} f_m(t)\, h^*(c)^{m-1}\left(\frac{1-e^{-\lambda t\left(1-k^*(c)\right)}}{\left(1-k^*(c)\right)}\right) + \sum_{m=1}^{\infty} F_m(t)\, h^*(c)^{m-1}\lambda e^{-\lambda t\left(1-k^*(c)\right)}\right]\\
&+\left(1-u^*(c)\right)\mu e^{-\mu t\left(1-u^*(c)\right)}\\
&-\left(1-h^*(c)\right)\left(1-u^*(c)\right)\left[\begin{array}{l}\sum_{n=1}^{\infty} f_m(t)\, h^*(c)^{m-1}\left(\frac{1-e^{-\mu t\left(1-u^*(c)\right)}}{\left(1-u^*(c)\right)}\right)\\ +\sum_{m=1}^{\infty} F_m(t)\, h^*(c)^{m-1}\mu e^{-\mu t\left(1-u^*(c)\right)}\end{array}\right]\\
&-\left(1-k^*(c)\right)\left(1-u^*(c)\right)\left[\begin{array}{l}\lambda e^{-\lambda t\left(1-k^*(c)\right)}\left(\frac{1-e^{-\mu t\left(1-u^*(c)\right)}}{\left(1-u^*(c)\right)}\right)\\ +\left(\frac{1-e^{-\lambda t\left(1-k^*(c)\right)}}{\left(1-k^*(c)\right)}\right)\mu e^{-\mu t\left(1-u^*(c)\right)}\end{array}\right]
\end{aligned}
$$

$$+\left(1-h^*(c)\right)\left(1-k^*(c)\right)\left(1-u^*(c)\right)\left[\begin{array}{l}\sum_{m=1}^{\infty} f_m(t)\, h^*(c)^{m-1}\left(\dfrac{1-e^{-\lambda t\left(1-k^*(c)\right)}}{\left(1-k^*(c)\right)}\right)\left(\dfrac{1-e^{-\mu t\left(1-u^*(c)\right)}}{\left(1-u^*(c)\right)}\right)\\ +\sum_{m=1}^{\infty} F_m(t)\, h^*(c)^{m-1}\lambda e^{-\lambda t\left(1-k^*(c)\right)}\left(\dfrac{1-e^{-\mu t\left(1-u^*(c)\right)}}{\left(1-u^*(c)\right)}\right)\\ +\sum_{m=1}^{\infty} F_m(t)\, h^*(c)^{m-1}\left(\dfrac{1-e^{-\lambda t\left(1-k^*(c)\right)}}{\left(1-k^*(c)\right)}\right)\mu e^{-\mu t\left(1-u^*(c)\right)}\end{array}\right]$$

$$+\left(1-v^*(c)\right)\theta e^{-\theta t\left(1-v^*(c)\right)}$$

$$-\left(1-h^*(c)\right)\left(1-v^*(c)\right)\left[\begin{array}{l}\sum_{m=1}^{\infty} f_m(t)\, h^*(c)^{m-1}\left(\dfrac{1-e^{-\theta t\left(1-v^*(c)\right)}}{\left(1-v^*(c)\right)}\right)\\ +\sum_{m=1}^{\infty} F_m(t)\, h^*(c)^{m-1}\,\theta e^{-\theta t\left(1-v^*(c)\right)}\end{array}\right]$$

$$-\left(1-k^*(c)\right)\left(1-v^*(c)\right)\left[\begin{array}{l}\lambda e^{-\lambda t\left(1-k^*(c)\right)}\left(\dfrac{1-e^{-\theta t\left(1-v^*(c)\right)}}{\left(1-v^*(c)\right)}\right)\\ +\left(\dfrac{1-e^{-\lambda t\left(1-k^*(c)\right)}}{\left(1-k^*(c)\right)}\right)\theta e^{-\theta t\left(1-v^*(c)\right)}\end{array}\right]$$

$$+\left(1-h^*(c)\right)\left(1-k^*(c)\right)\left(1-v^*(c)\right)$$

$$\left[\begin{array}{l}\sum_{m=1}^{\infty} f_m(t)\, h^*(c)^{m-1}\left(\dfrac{1-e^{-\lambda t\left(1-k^*(c)\right)}}{\left(1-k^*(c)\right)}\right)\left(\dfrac{1-e^{-\theta t\left(1-v^*(c)\right)}}{\left(1-v^*(c)\right)}\right)\\ +\sum_{m=1}^{\infty} F_m(t)\, h^*(c)^{m-1}\lambda e^{-\lambda t\left(1-k^*(c)\right)}\left(\dfrac{1-e^{-\theta t\left(1-v^*(c)\right)}}{\left(1-v^*(c)\right)}\right)\\ +\sum_{m=1}^{\infty} F_m(t)\, h^*(c)^{m-1}\left(\dfrac{1-e^{-\lambda t\left(1-k^*(c)\right)}}{\left(1-k^*(c)\right)}\right)\theta e^{-\theta t\left(1-v^*(c)\right)}\end{array}\right]$$

$$-\left(1-u^*(c)\right)\left(1-v^*(c)\right)\left[\begin{array}{l}\mu e^{-\mu t\left(1-u^*(c)\right)}\left(\dfrac{1-e^{-\theta t\left(1-v^*(c)\right)}}{\left(1-v^*(c)\right)}\right)\\ +\left(\dfrac{1-e^{-\mu t\left(1-u^*(c)\right)}}{\left(1-u^*(c)\right)}\right)\theta e^{-\theta t\left(1-v^*(c)\right)}\end{array}\right]$$

$$+\left(1-h^*(c)\right)\left(1-u^*(c)\right)\left(1-v^*(c)\right)$$

$$\left[\begin{array}{l}\sum_{m=1}^{\infty} f_m(t)\,h^*(c)^{m-1}\left(\frac{1-e^{-\mu t\left(1-u^*(c)\right)}}{\left(1-u^*(c)\right)}\right)\left(\frac{1-e^{-\theta t\left(1-v^*(c)\right)}}{\left(1-v^*(c)\right)}\right)\\ +\sum_{m=1}^{\infty} F_m(t)\,h^*(c)^{m-1}\mu e^{-\mu t\left(1-u^*(c)\right)}\left(\frac{1-e^{-\theta t\left(1-v^*(c)\right)}}{\left(1-v^*(c)\right)}\right)\\ +\sum_{m=1}^{\infty} F_m(t)\,h^*(c)^{m-1}\left(\frac{1-e^{-\mu t\left(1-u^*(c)\right)}}{\left(1-u^*(c)\right)}\right)\theta e^{-\theta t\left(1-v^*(c)\right)}\end{array}\right]$$

$$+\left(1-k^*(c)\right)\left(1-u^*(c)\right)\left(1-v^*(c)\right)$$

$$\left[\begin{array}{l}\lambda e^{-\lambda t\left(1-k^*(c)\right)}\left(\frac{1-e^{-\mu t\left(1-u^*(c)\right)}}{\left(1-u^*(c)\right)}\right)\left(\frac{1-e^{-\theta t\left(1-v^*(c)\right)}}{\left(1-v^*(c)\right)}\right)\\ +\left(\frac{1-e^{-\lambda t\left(1-k^*(c)\right)}}{\left(1-k^*(c)\right)}\right)\mu e^{-\mu t\left(1-u^*(c)\right)}\left(\frac{1-e^{-\theta t\left(1-v^*(c)\right)}}{\left(1-v^*(c)\right)}\right)\\ +\left(\frac{1-e^{-\lambda t\left(1-k^*(c)\right)}}{\left(1-k^*(c)\right)}\right)\left(\frac{1-e^{-\mu t\left(1-u^*(c)\right)}}{\left(1-u^*(c)\right)}\right)\theta e^{-\theta t\left(1-v^*(c)\right)}\end{array}\right]$$

$$-\left(1-h^*(c)\right)\left(1-k^*(c)\right)\left(1-u^*(c)\right)\left(1-v^*(c)\right)$$

$$\left[\begin{array}{l}\sum_{m=1}^{\infty} f_m(t)\,h^*(c)^{m-1}\left(\frac{1-e^{-\lambda t\left(1-k^*(c)\right)}}{\left(1-k^*(c)\right)}\right)\left(\frac{1-e^{-\mu t\left(1-u^*(c)\right)}}{\left(1-u^*(c)\right)}\right)\left(\frac{1-e^{-\theta t\left(1-v^*(c)\right)}}{\left(1-v^*(c)\right)}\right)\\ +\sum_{m=1}^{\infty} F_m(t)\,h^*(c)^{m-1}\lambda e^{-\lambda t\left(1-k^*(c)\right)}\left(\frac{1-e^{-\mu t\left(1-u^*(c)\right)}}{\left(1-u^*(c)\right)}\right)\left(\frac{1-e^{-\theta t\left(1-v^*(c)\right)}}{\left(1-v^*(c)\right)}\right)\\ +\sum_{m=1}^{\infty} F_m(t)\,h^*(c)^{m-1}\left(\frac{1-e^{-\lambda t\left(1-k^*(c)\right)}}{\left(1-k^*(c)\right)}\right)\mu e^{-\mu t\left(1-u^*(c)\right)}\left(\frac{1-e^{-\theta t\left(1-v^*(c)\right)}}{\left(1-v^*(c)\right)}\right)\\ +\sum_{m=1}^{\infty} F_m(t)\,h^*(c)^{m-1}\left(\frac{1-e^{-\lambda t\left(1-k^*(c)\right)}}{\left(1-k^*(c)\right)}\right)\left(\frac{1-e^{-\mu t\left(1-u^*(c)\right)}}{\left(1-u^*(c)\right)}\right)\theta e^{-\theta t\left(1-v^*(c)\right)}\end{array}\right] \quad \ldots\ldots(8)$$

$$
\begin{aligned}
&=\left(1-k^{*}(c)\right)\lambda e^{-\lambda t\left(1-k^{*}(c)\right)-\mu t\left(1-u^{*}(c)\right)-\theta t\left(1-v^{*}(c)\right)} \\
&+\left(1-u^{*}(c)\right)\mu e^{-\lambda t\left(1-k^{*}(c)\right)-\mu t\left(1-u^{*}(c)\right)-\theta t\left(1-v^{*}(c)\right)} \\
&+\left(1-v^{*}(c)\right)\theta e^{-\lambda t\left(1-k^{*}(c)\right)-\mu t\left(1-u^{*}(c)\right)-\theta t\left(1-v^{*}(c)\right)} \\
&+\left(1-h^{*}(c)\right)\sum_{m=1}^{\infty} f_{m}(t)h^{*}(c)^{m-1}e^{-\lambda t\left(1-k^{*}(c)\right)-\mu t\left(1-u^{*}(c)\right)-\theta t\left(1-v^{*}(c)\right)} \\
&-\left(1-h^{*}(c)\right)\left(1-k^{*}(c)\right)\lambda\sum_{m=1}^{\infty} F_{m}(t)h^{*}(c)^{m-1}e^{-\lambda t\left(1-k^{*}(c)\right)-\mu t\left(1-u^{*}(c)\right)-\theta t\left(1-v^{*}(c)\right)} \\
&-\left(1-h^{*}(c)\right)\left(1-u^{*}(c)\right)\mu\sum_{m=1}^{\infty} f_{m}(t)h^{*}(c)^{m-1}e^{-\lambda t\left(1-k^{*}(c)\right)-\mu t\left(1-u^{*}(c)\right)-\theta t\left(1-v^{*}(c)\right)} \qquad \ldots(9)\\
&-\left(1-h^{*}(c)\right)\left(1-v^{*}(c)\right)\theta\sum_{m=1}^{\infty} f_{m}(t)h^{*}(c)^{m-1}e^{-\lambda t\left(1-k^{*}(c)\right)-\mu t\left(1-u^{*}(c)\right)-\theta t\left(1-v^{*}(c)\right)}
\end{aligned}
$$

Taking Laplace Transform on both side L[f(t)] = F(s)

$$
\begin{aligned}
l^{*}(s)&=\frac{\lambda\left(1-k^{*}(c)\right)}{s+\lambda\left(1-k^{*}(c)\right)-\mu\left(1-u^{*}(c)\right)-\theta\left(1-v^{*}(c)\right)} \\
&+\frac{\mu\left(1-u^{*}(c)\right)}{s+\lambda\left(1-k^{*}(c)\right)-\mu\left(1-u^{*}(c)\right)-\theta\left(1-v^{*}(c)\right)} \\
&+\frac{\theta\left(1-v^{*}(c)\right)}{s+\lambda\left(1-k^{*}(c)\right)-\mu\left(1-u^{*}(c)\right)-\theta\left(1-v^{*}(c)\right)} \\
&+\left(1-h^{*}(c)\right)\sum_{m=1}^{\infty} f^{*}{}_{m}\left(s+\lambda\left(1-k^{*}(c)\right)-\mu\left(1-u^{*}(c)\right)-\theta\left(1-v^{*}(c)\right)\right)h^{*}(c)^{m-1} \\
&-\frac{\lambda\left(1-h^{*}(c)\right)\left(1-k^{*}(c)\right)}{s+\lambda\left(1-k^{*}(c)\right)-\mu\left(1-u^{*}(c)\right)-\theta\left(1-v^{*}(c)\right)} \\
&\sum_{m=1}^{\infty} f^{*}{}_{m}\left(s+\lambda\left(1-k^{*}(c)\right)-\mu\left(1-u^{*}(c)\right)-\theta\left(1-v^{*}(c)\right)\right)h^{*}(c)^{m-1} \\
&-\frac{\mu\left(1-h^{*}(c)\right)\left(1-u^{*}(c)\right)}{s+\lambda\left(1-k^{*}(c)\right)-\mu\left(1-u^{*}(c)\right)-\theta\left(1-v^{*}(c)\right)} \\
&\sum_{m=1}^{\infty} f^{*}{}_{m}\left(s+\lambda\left(1-k^{*}(c)\right)-\mu\left(1-u^{*}(c)\right)-\theta\left(1-v^{*}(c)\right)\right)h^{*}(c)^{m-1}
\end{aligned}
$$

$$-\frac{\theta\left(1-h^*(c)\right)\left(1-v^*(c)\right)}{s+\lambda\left(1-k^*(c)\right)-\mu\left(1-u^*(c)\right)-\theta\left(1-v^*(c)\right)}$$
$$\sum_{m=1}^{\infty} f^*{}_m\left(s+\lambda\left(1-k^*(c)\right)-\mu\left(1-u^*(c)\right)-\theta\left(1-v^*(c)\right)\right) h^*(c)^{m-1} \qquad \ldots(10)$$

$$l^*(s)=\frac{\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right)}{s+\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right)}$$
$$+\sum_{m=1}^{\infty} f^*{}_m\left(s+\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right)\right) h^*(c)^{m-1}$$
$$\left[\left(1-h^*(c)\right)-\left(1-h^*(c)\right)\left\{\frac{\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right)}{s+\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right)}\right\}\right] \qquad \ldots\ldots(11)$$

$$\frac{d}{ds}\left[l^*(s)\right]=\frac{-\left[\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right)\right]}{\left[s+\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right)\right]^2}$$
$$+\sum_{m=1}^{\infty} f^*{}_m\left(s+\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right)\right) h^*(c)^{m-1} \qquad \ldots\ldots(12)$$
$$\left[0+\left(1-h^*(c)\right)\left\{\frac{\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right)}{\left[s+\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right)\right]^2}\right\}\right]$$

$$-\frac{d}{ds}\left[l^*(s)\right]\Big|_{S=0}=\frac{1}{\left[\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right)\right]}$$
$$-\sum_{m=1}^{\infty} f^*{}_m\left(s+\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right)\right) h^*(c)^{m-1}$$
$$\left[\frac{\left(1-h^*(c)\right)}{\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right)}\right] \qquad \ldots\ldots(13)$$

$$E(T)=\frac{1}{\left[\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right)\right]}$$
$$+\left[\frac{\left(1-h^*(c)\right)}{\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right)}\right]$$
$$\sum_{m=1}^{\infty} f^*{}_m\left(s+\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right)\right) h^*(c)^{m-1} \qquad \ldots\ldots(14)$$

The expression for the cumulative distribution function of the partial sum $S_m=u_1+u_2+\ldots+u_m$. when the random variables u_i, i=1, 2, 3, m are exchangeable, exponentially distributed and are with construct with correlation has derived that the cumulative distribution function of

$$f_m^*(s)=\frac{1}{(1+bs)^m\left(1+\dfrac{mRbs}{(1-R)(1+bs)}\right)} \qquad \text{……(15)}$$

Where b=a(1-R), 'a' being the parameter of the exponential distribution. Noting that the interarrival times between decision making epochs are constantly correlated exponential variables with parameter 'a'.

$$f_m^*(s)=\left(\frac{(1-R)\ (1+bs)^{1-m}}{[1-R+s(b-Rb+mRb)]}\right)$$

$$f_m^*\big(\lambda(1-k^*(c))+\mu(1-u^*(c))+\theta(1-v^*(c))\big)$$

$$=\left(\frac{(1-R)\ \big(1+b\big(\lambda(1-k^*(c))+\mu(1-u^*(c))\big)+\theta(1-v^*(c))\big)^{1-m}}{\big\{1-R+\big(\lambda(1-k^*(c))+\mu(1-u^*(c))+\theta(1-v^*(c))\big)[b-Rb+mRb]\big\}}\right) \qquad \text{……(16)}$$

$$E(T)=\frac{1}{\left[\lambda(1-k^*(c))+\mu(1-u^*(c))+\theta(1-v^*(c))\right]}$$

$$-\left[\frac{(1-h^*(c))}{\lambda(1-k^*(c))+\mu(1-u^*(c))+\theta(1-v^*(c))}\right] \qquad \text{……(17)}$$

$$\sum_{m=1}^{\infty}\left[\frac{(1-R)\left[1+b\big(\lambda(1-k^*(c))+\mu(1-u^*(c))+\theta(1-v^*(c))\big)\right]^{1-m}}{1-R+\big(\lambda(1-k^*(c))+\mu(1-u^*(c))+\theta(1-v^*(c))\big)b[1-R+mR]}h^*(c)^{m-1}\right]$$

h(.), k(.), u(.) and v(.) are exponentially distributed with parameters α,β, γ, and η.

$$h^*(c)=\frac{\alpha}{\alpha+c},\qquad k^*(c)=\frac{\beta}{\beta+c},\qquad u^*(c)=\frac{\gamma}{\gamma+c},\qquad v^*(c)=\frac{\eta}{\eta+c},$$

$$1-h^*(c)=\frac{c}{c+\alpha},\quad 1-k^*(c)=\frac{c}{c+\beta},\quad 1-u^*(c)=\frac{c}{c+\gamma},\quad 1-v^*(c)=\frac{c}{c+\eta}$$

$$E(T)=\frac{1}{\dfrac{\lambda c}{c+\beta}+\dfrac{\mu c}{c+\gamma}+\dfrac{\theta c}{c+\eta}}$$

$$-\frac{\left(\dfrac{c}{c+\alpha}\right)}{\dfrac{\lambda c}{c+\beta}+\dfrac{\mu c}{c+\gamma}+\dfrac{\theta c}{c+\eta}}\left(\sum_{m=1}^{\infty}\frac{(1-R)\left(1+b\left(\dfrac{\lambda c}{c+\beta}+\dfrac{\mu c}{c+\gamma}+\dfrac{\theta c}{c+\eta}\right)\right)^{1-m}\left(\dfrac{\alpha}{\alpha+c}\right)^{m-1}}{\left\{(1-R)+\left(\dfrac{\lambda c}{c+\beta}+\dfrac{\mu c}{c+\gamma}+\dfrac{\theta c}{c+\eta}\right)b[1-R+mR]\right\}}\right) \qquad \text{……(18)}$$

Put m=4,

$$E(T)=\frac{1}{\dfrac{\lambda c}{c+\beta}+\dfrac{\mu c}{c+\gamma}+\dfrac{\theta c}{c+\eta}}$$

$$-\frac{\left(\frac{c}{c+\alpha}\right)}{\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}}\left(\begin{array}{l}\frac{(1-R)}{\left\{(1-R)+\left(\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}\right)b\right\}}\\ +\frac{(1-R)\left[1+b\left(\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}\right)\right]^{-1}\left(\frac{\alpha}{\alpha+c}\right)}{\left\{(1-R)+\left(\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}\right)b[1+R]\right\}}\\ +\frac{(1-R)\left[1+b\left(\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}\right)\right]^{-2}\left(\frac{\alpha}{\alpha+c}\right)^2}{\left\{(1-R)+\left(\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}\right)b[1+2R]\right\}}\\ +\frac{(1-R)\left[1+b\left(\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}\right)\right]^{-3}\left(\frac{\alpha}{\alpha+c}\right)^3}{\left\{(1-R)+\left(\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}\right)b[1+3R]\right\}}\end{array}\right) \quad \text{……….(19)}$$

$$E\left(T^2\right)=\frac{d^2}{ds^2}\left[l^*(s)\right]\Big|_{S=0}=\frac{2}{\left[\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right)\right]^2}$$

$$-\left[\frac{2\left(1-h^*(c)\right)}{[\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right)]^2}\right] \quad \text{……(20)}$$

$$\sum_{m=1}^{\infty} f^*{}_m(\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right))\,h^*(c)^{m-1}$$

$V(T)=E(T^2)-[E(T)]^2$

$$=\frac{1}{\left[\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right)\right]^2}$$

$$-\left[\frac{\left(1-h^*(c)\right)}{\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right)}\right]^2$$

$$\left[\sum_{m=1}^{\infty} f^*{}_m(\lambda\left(1-k^*(c)\right)+\mu\left(1-u^*(c)\right)+\theta\left(1-v^*(c)\right))\,h^*(c)^{m-1}\right]^2$$

$$= \frac{1}{\left[\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}\right]^2}$$

$$-\left[\frac{\frac{c}{c+\alpha}}{\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}}\right]^2$$

$$\left(\sum_{m=1}^{\infty}\frac{(1-R)\left(1+b\left(\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}\right)\right)^{1-m}\left(\frac{\alpha}{\alpha+c}\right)^{m-1}}{\left\{(1-R)+\left(\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}\right)b\left[1-R+mR\right]\right\}}\right)$$

Put m=4,

$$V(T)=\frac{1}{\left[\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}\right]^2}$$

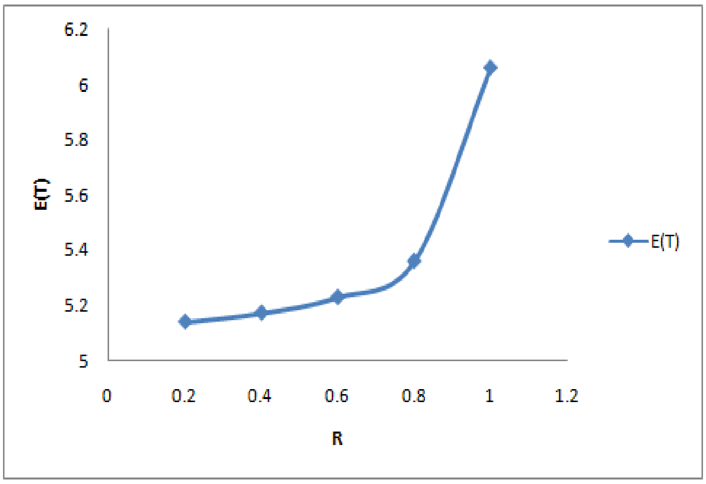
$$-\left[\frac{\frac{c}{c+\alpha}}{\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}}\right]^2\left(\begin{array}{l}\frac{(1-R)}{\left\{(1-R)+\left(\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}\right)b\right\}}\\+\frac{(1-R)\left[1+b\left(\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}\right)\right]^{-1}\left(\frac{\alpha}{\alpha+c}\right)}{\left\{(1-R)+\left(\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}\right)b\left[1+R\right]\right\}}\\+\frac{(1-R)\left[1+b\left(\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}\right)\right]^{-2}\left(\frac{\alpha}{\alpha+c}\right)^2}{\left\{(1-R)+\left(\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}\right)b\left[1+2R\right]\right\}}\\+\frac{(1-R)\left[1+b\left(\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}\right)\right]^{-3}\left(\frac{\alpha}{\alpha+c}\right)^3}{\left\{(1-R)+\left(\frac{\lambda c}{c+\beta}+\frac{\mu c}{c+\gamma}+\frac{\theta c}{c+\eta}\right)b\left[1+3R\right]\right\}}\end{array}\right)^2 \quad \text{....(21)}$$

Numerical Example

The parameters such as R, α, β,γ, η and λ, μ, θ are taken in different inputted data values and keeping c=0.24 and b=0.24 are fixed. The expected time to recruitment and its variance are shown in table 5.1 to table 5.8 and figure 5.1 to figure 5.8 respectively.

Table 1 Variations in E(T) and V(T) for the variations in R
and λ=1, μ=1.2,θ=1.4, α=5, β=10,γ=15,η=2.5 andc=0.24,b=0.24

R	E(T)	V(T)
0.2	5.1392	35.8959
0.4	5.1717	35.9548
0.6	5.2291	36.0539
0.8	5.3596	36.2541
1	6.0620	36.7475



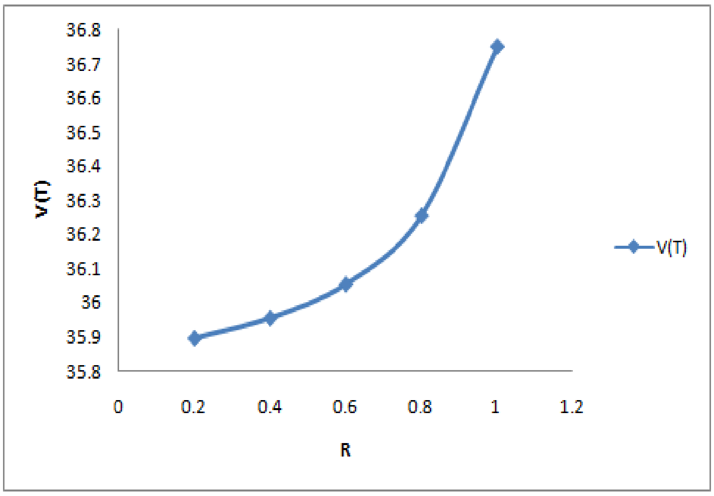
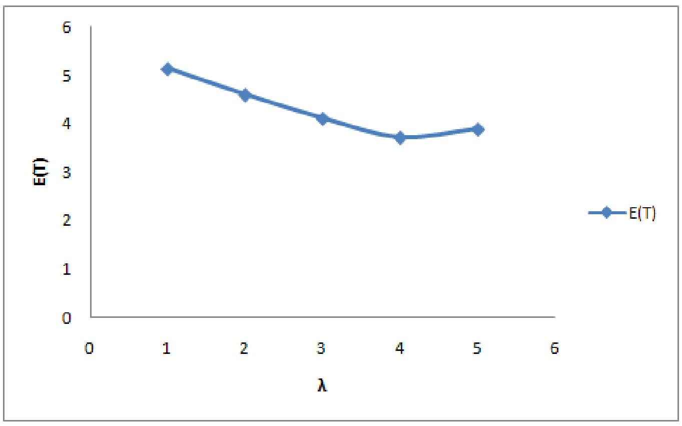


Fig 1 Variations in E(T) and V(T) for the variations in R

Table 2 Variations in E(T) and V(T) for the variations in λ and $R=0.2$, $\mu=1.2, \theta=1.4, \alpha=5, \beta=10, \gamma=15, \eta=2.5$ and $c=0.24, b=0.24$

λ	E(T)	V(T)
1.0	5.1392	35.8959
2.0	4.5987	27.6704
3.0	4.1069	21.9074
4.0	3.7123	17.7760
5.0	3.8863	14.7133



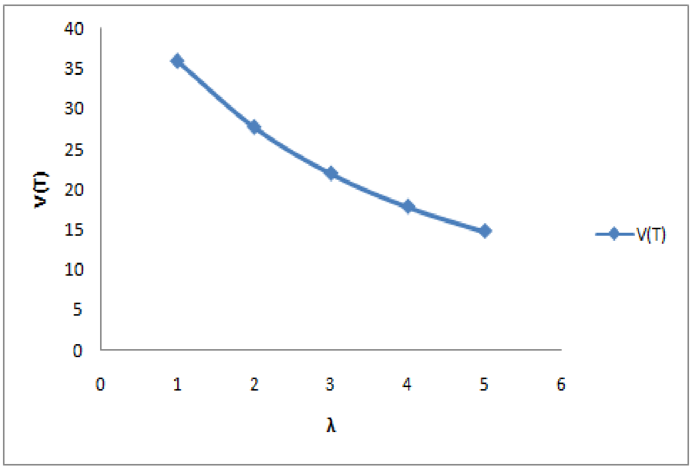
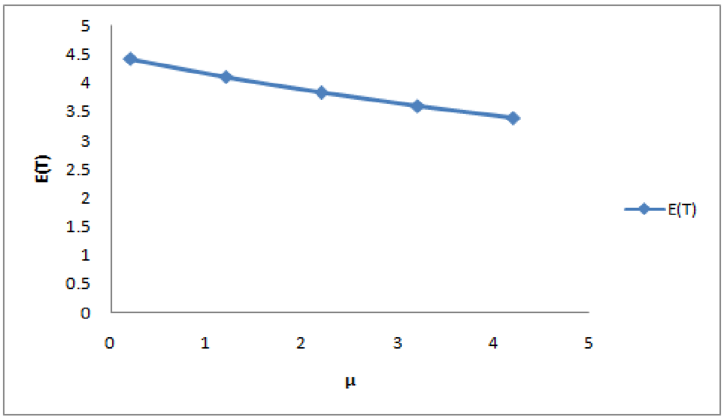


Fig 2 Variations in E(T) and V(T) for the variations in λ

Table 3 Variations in E(T) and V(T) for the variations in μ and R=0.2, λ=1,θ=1.4, α=5, β=10,γ=15,η=2.5 andc=0.24,b=0.24

μ	E(T)	V(T)
0.2	4.4245	25.5511
1.2	4.1069	21.9074
2.2	3.8329	18.9920
3.2	3.5941	16.6229
4.2	3.3840	14.6716



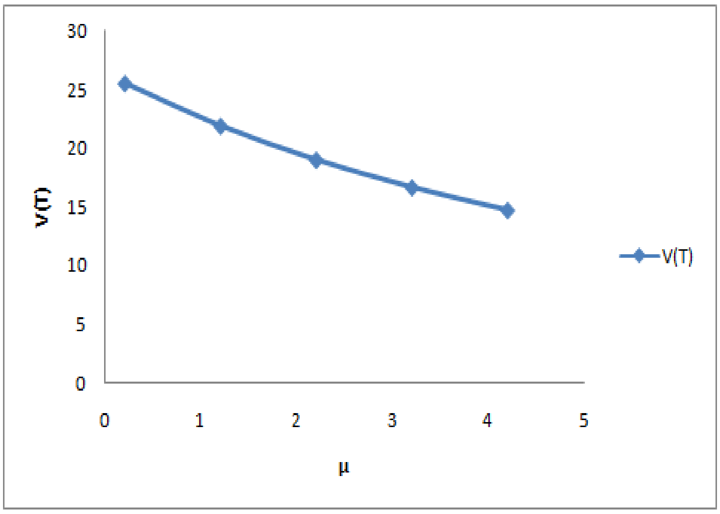
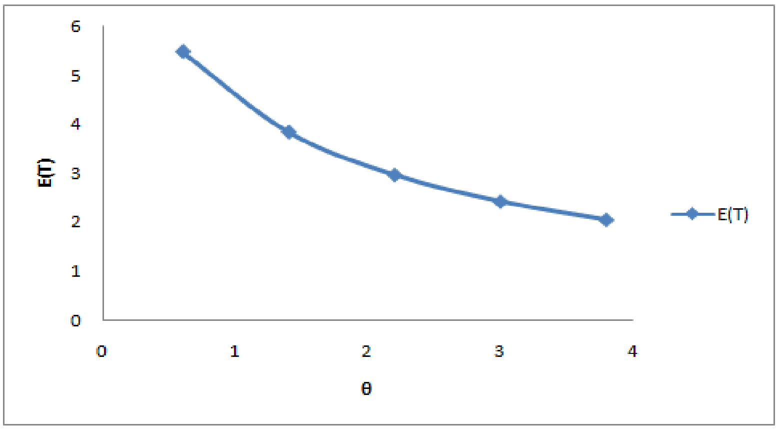


Fig 3 Variations in E(T) and V(T) for the variations in μ

Table 4 Variations in E(T) and V(T) for the variations in θ and R=0.2, λ=1,μ=1.2, α=5, β=10,γ=15,η=2.5 andc=0.24,b=0.24

θ	E(T)	V(T)
0.6	5.4686	39.5312
1.4	3.8329	18.9920
2.2	2.9623	11.1288
3.0	2.4203	7.3055
3.8	2.0494	5.1621



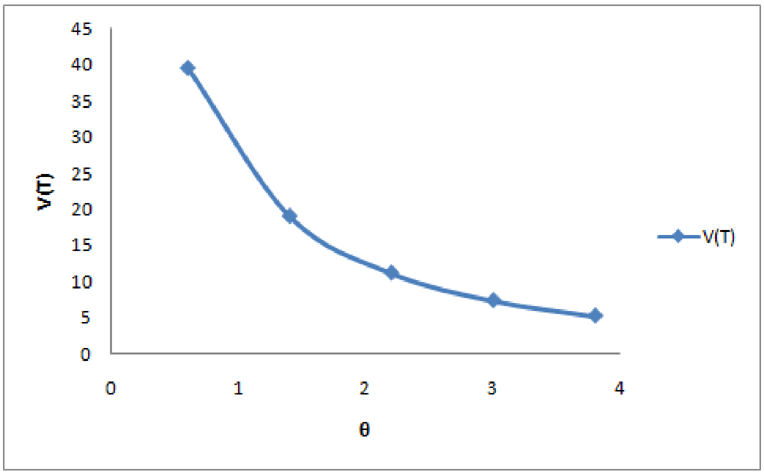
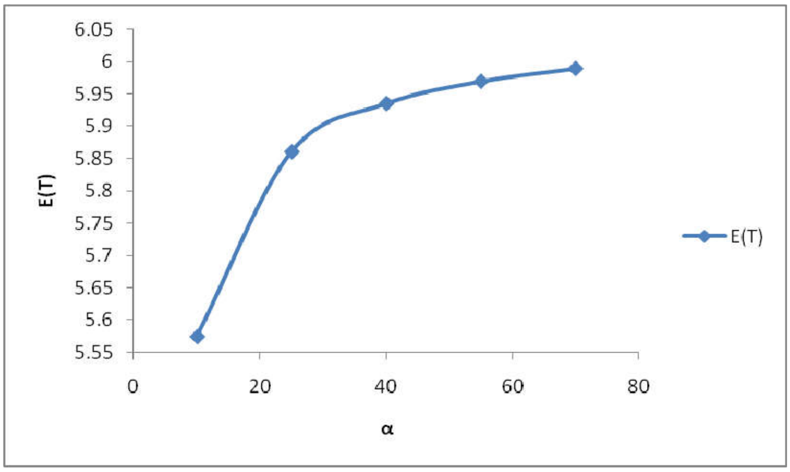


Fig 4 Variations in E(T) and V(T) for the variations in θ

Table 5 Variations in E(T) and V(T) for the variations in α and R=0.2, λ=1,μ=1.2, θ=1.4, β=10,γ=15,η=2.5 andc=0.24,b=0.24

α	E(T)	V(T)
10	5.5742	36.5096
25	5.8601	36.7067
40	5.9347	36.7313
55	5.9690	36.7388
70	5.9888	36.7426



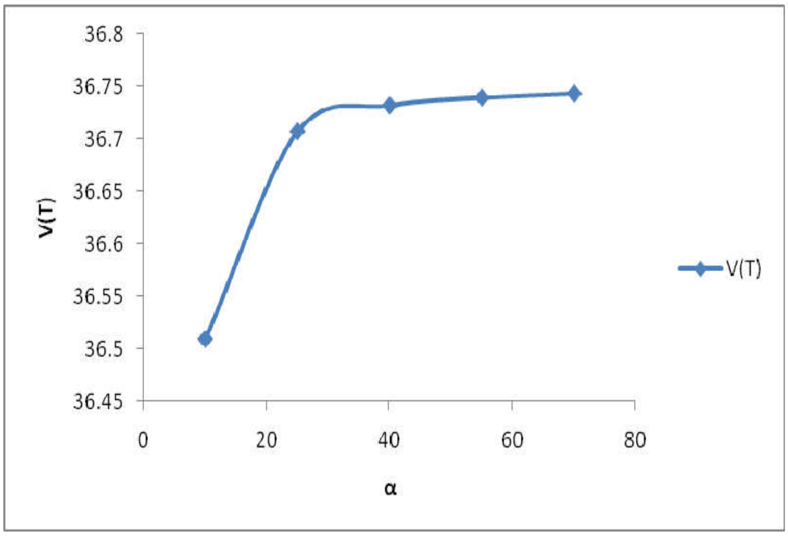
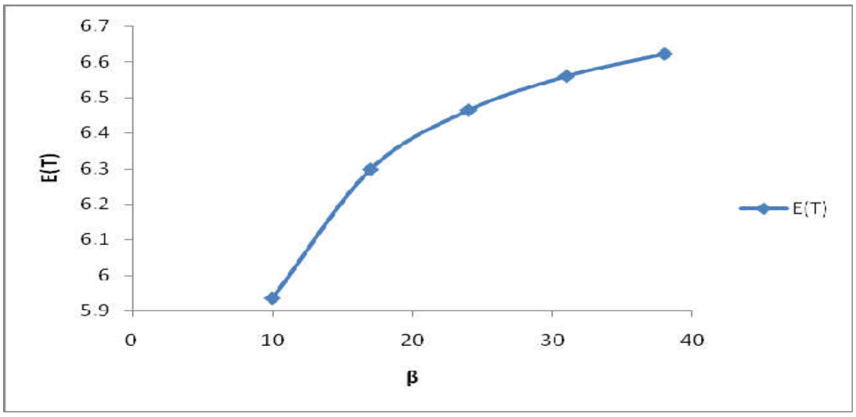


Fig 5 Variations in E(T) and V(T) for the variations in α

Table 6 Variations in E(T) and V(T) for the variations in β and R=0.2, λ=1,μ=1.2, θ=1.4, α=10,γ=15,η=2.5 andc=0.24,b=0.24

β	E(T)	V(T)
10	5.9347	36.7313
17	6.2971	41.3661
24	6.4639	43.5915
31	6.5598	44.8974
38	6.6221	45.7558



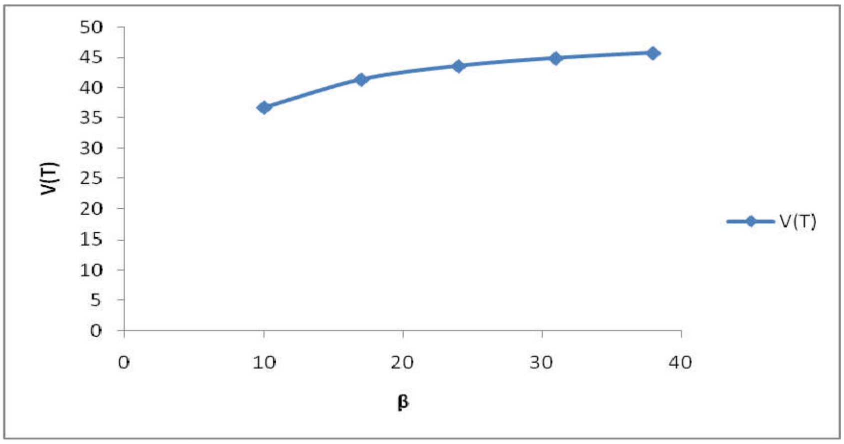
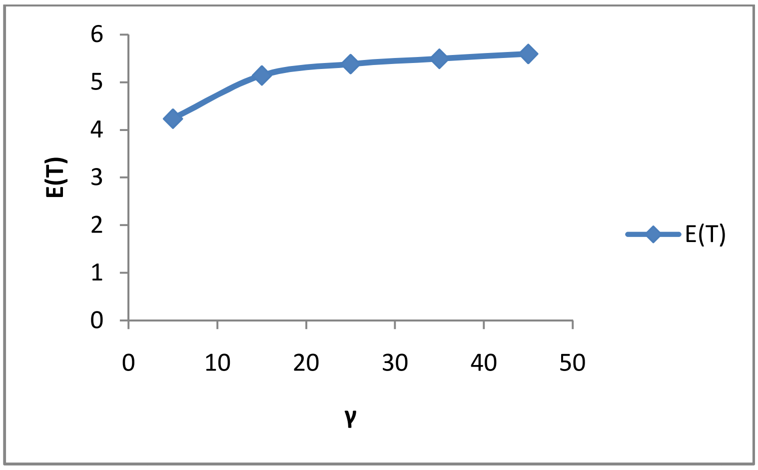


Fig 6 Variations in E(T) and V(T) for the variations in β

Table 7 Variations in E(T) and V(T) for the variations in γ and R=0.2, λ=1,μ=1.2, θ=1.4, α=10,β=24,η=2.5 andc=0.24,b=0.24

γ	E(T)	V(T)
5	4.2354	24.1989
15	5.1392	35.8959
25	5.3786	39.3808
35	5.4893	41.0473
45	5.5912	44.0214



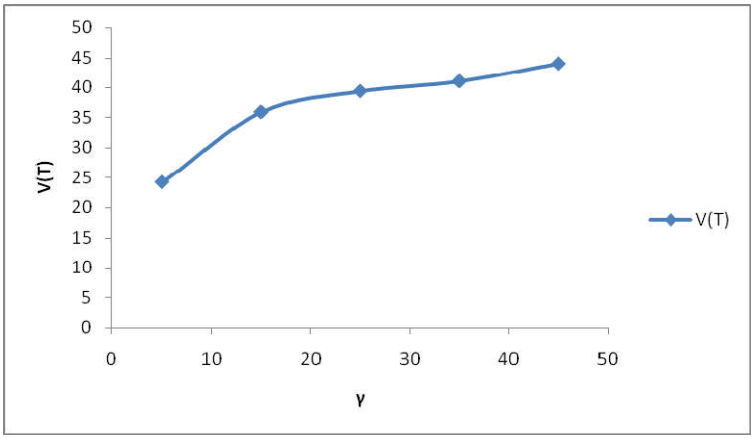
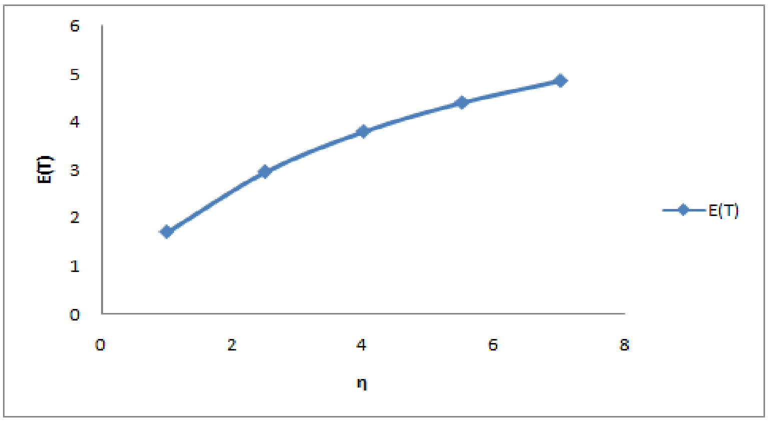


Fig 7 Variations in E(T) and V(T) for the variations in γ

Table 8 Variations in E(T) and V(T) for the variations in η and R=0.2, λ=1,μ=1.2, θ=1.4, α=10,β=24,γ=25 andc=0.24,b=0.24

η	E(T)	V(T)
1.0	1.7062	3.5181
2.5	2.9623	11.1288
4.0	3.8024	18.6801
5.5	4.4060	25.3308
7.0	4.8611	31.0237



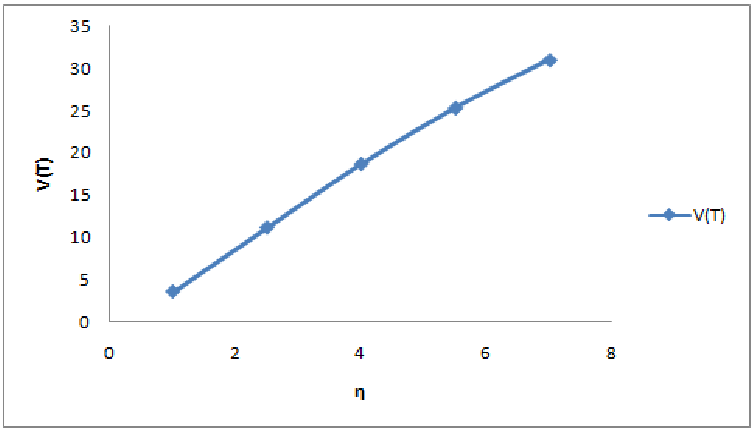


Fig 8 Variations in E(T) and V(T) for the variations in η

CONCLUSION

From table 5 it is observed that, if the value of R, which is the parameter of the correlation between the inter-decision time increases and other inputted data values λ=1, μ=1.2,θ=1.4, α=5, β=10,γ=15,η=2.5 and c=0.24,b=0.24 are kept fixed, the expected time to recruitment and its variance increases, as shown in table 1 and figure 1 respectively. From table 2 it is observed that, if the value of λ, which is the parameter of the random variable representing the amount of diminution at every epoch of policy decision increases and other inputted data values R=0.2, μ=1.2,θ=1.4, α=5, β=10,γ=15,η=2.5 and c=0.24,b=0.24 are kept fixed, the expected time to recruitment and its variance decreases, as shown in table 2 and figure 2 respectively. In table 3, if the value of μ, which is the parameter of the random variable representing the amount of diminution at every epoch of transfer of personnel increases and other inputted data values R=0.2, λ=1,θ=1.4, α=5, β=10,γ=15,η=2.5 and c=0.24,b=0.24 are kept fixed, the results shows that the expected time to recruitment and its variance decreases, which is also depicted in table 3 and figure 3 respectively. In table 4, if the value of θ, which is the parameter of the interarrival times between n^{th} transfer of personnel increases and other inputted data values R=0.2, λ=1,μ=1.2, α=5, β=10,γ=15,η=2.5 and c=0.24,b=0.24 are kept fixed, the results shows that the expected time to recruitment and its variance decreases, which is also depicted in table 4 and figure 4 respectively. In table 5, if the values of α, which is the parameter of the successive epochs of first transfer increases and other values R=0.2, λ=1,μ=1.2, θ=1.4, β=10,γ=15,η=2.5 and c=0.24,b=0.24 are kept fixed, the results shows that the expected time to recruitment and its variance increases, which is also depicted in table 5 and figure 5 respectively. In table 6, if the values of β, which is the parameter of the successive epochs of n^{th} transfer increases and other values R=0.2, λ=1,μ=1.2, θ=1.4, α=10,γ=15,η=2.5 and c=0.24,b=0.24 are kept fixed, the results shows that the E(T) and V(T) increases, which is also depicted in table 6 and figure 6 respectively. In table 7, if the values of γ, which is the parameter of the successive epochs of n^{th} transfer increases and other values R=0.2, λ=1,μ=1.2, θ=1.4, α=10,β=24,η=2.5 and c=0.24,b=0.24 are kept fixed, the results shows that the E(T) and V(T) increases, which is also depicted in table 7 and figure 7 respectively. In table 8, if the values of η, which is the parameter of the successive epochs of n^{th} transfer increases and other values R=0.2, λ=1,μ=1.2, θ=1.4,

$\alpha=10, \beta=24, \gamma=25$ and c=0.24,b=0.24 are kept fixed, the results shows that the E(T) and V(T) increases, which is also depicted in table 8 and figure 8 respectively.

These are many areas of an organization or industry in which the application of Stochastic models is quite necessary. It would be very much useful in every sector of human activity. First of all it is imperative to identify those areas of human activity where the demand for manpower and supply are at disequilibrium. Especially in the area of specialist skill, it becomes necessary to identify where the disequilibrium exists and also were there is interruption in the work schedule due to shortage of manpower. The identification of such areas, the type of problems involved and the conversion of a real life situation into a Stochastic model are essential to develop human resource management, which will yield profits not only to the management but also to the society itself.

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