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Research Article

BIANCHI TYPE-III DARK ENERGY MODEL IN SCALE COVARIANT THEORY OF GRAVITATION

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ABSTRACT

In this paper, Bianchi type–III dark energy model with variable equation of state (EoS) has been investigated in scale covariant theory of gravitation formulated by Canuto *et al*. (Phys. Rev. Lett. 39: 429, 1977). Special law of Hubble parameter proposed by Bermann (Nuovo-Cimento 74; 182, 1983) being incorporated to obtain the solutions of field equations. Some physical properties of the model are also discussed.

INTRODUCTION

Although general relativity (GR) passes all present experimental tests with flying colors, still it is very important to study alternative theories of gravitation due to several theoretical and phenomenological reasons. One of them is that at large distance gravity does not behave exactly as Newton and Einstein predicted. Due to this, in recent years there has been an immense interest in alternative theories of gravitation [Brans-Dicke (1961), Nordt-Vedt (1970), Sen and Dunn (1971) and Saez-Ballester (1985)]. Canuto *et al*. (1977) formulated scale covariant theory, which is a viable alternative to GR, by associating the mathematical operation of scale transformation with physics of using different dynamical system to measure space-time distances. In this theory corresponding to each dynamical system of units, there is an arbitrary gauge function depending on gauge condition. The gauge condition is chosen so that for gravitational units the standard Einstein equations are recovered. This gauge condition must be imposed in such a way that gravitational units and atomic units derived from atomic dynamics must be distinct.

Scale covariant theory which admits variable parameter G measure physical quantities in atomic units and Einstein field equations are valid in gravitational units. The conformal transformation which relates metric tensors in two systems of units is given by

$$\bar{g}_{ij} = \phi^2(x^k) g_{ij} \, , \tag{1}$$

where, in Latin, indices takes values 1, 2, 3 and 4. Bar denotes gravitational units and unbar denotes atomic quantities. The gauge function ϕ $(0 < \phi < \infty)$ in its most general formulation is a function of all space-time co-ordinates. Thus, using the conformal transformation of the type given by, Canuto *et al.* (1977) transformed the usual Einstein equation into

$$R_{ij} - \frac{1}{2} R\, g_{ij} + f_{ij}(\phi) = -8\pi G(\phi) T_{ij} + \Lambda(\phi) g_{ij} \, , \tag{2}$$

where

$$\phi^2 f_{ij} = 2\phi\phi_{i;j} - 4\phi_i\phi_j - g_{ij}(\phi\phi^{;k}_{;k} - \phi^{,k}\phi_{,k}) \, . \tag{3}$$

Here R_{ij} is the Ricci tensor, R is Ricci scalar, G is the gravitational constant, T_{ij} is energy momentum tensor. A comma and semicolon denotes partial and covariant derivative respectively. A particular characteristic of this theory is that no independent equation for ϕ exists. Gauge function ϕ can have possible value given by Canuto as

$$\phi(t) = \left(\frac{t_0}{t}\right)^{\varepsilon}, \quad \varepsilon = \pm 1, \pm \frac{1}{2}, \tag{4}$$

where t_0 is constant and $\phi \sim t^{1/2}$ is the most favored form to fit observations (Canuto V. M., Goldman, I., 1983). Reddy *et al.*

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(2012), (2014) have studied dark energy models (DE) in Bianchi type-I and Kaluza-Klein universe. Singh *et al*. (2014) have studied Bianchi type-V dark energy model in a scale covariant theory of gravitation. Reddy (2004) also have obtained a higher dimensional cosmological model in a scale covariant theory of gravitation. Venkateshwarlu and Kumar (Venkateswarlu R., Kumar, K. P., 2005) have obtained higher dimensional string cosmologies in scale-covariant theory of gravitation. Reddy and Venkateshwarlu (2004) studied Einstein-Rosen universe in the scale covariant theory of gravitation.

Moreover, there has been considerable interest in cosmological models with dark energy because of the fact that our universe is currently undergoing an accelerated expansion supposedly, driven by an exotic dark energy which has been confirmed by a host of observations, such as type 1a supernovae (1998). Based on these observations, cosmologists have accepted the idea of dark energy, which is a fluid with negative pressure making up around 70% of the present universe energy content to be responsible for this acceleration due to repulsive gravitation. These observational data also suggest that the universe is dominated by two dark components containing dark matter and dark energy. Dark matter, a matter without pressure, is mainly used to explain galastic curves and large scale structure formation, while dark energy, an exotic energy with negative pressure is used to explain the present cosmic accelerating expansion. The most interesting problem in modern astrophysics and cosmology is to know the behavior of dark energy. Several authors such as Ray *et al*. (2010), Saha (2005), Sahni, V. (2004), Singh and Chaubey (2008), Chaubey (2011), Tade *et al*. (2011), Jain *et al.* (2012), Samanta e*t al.* (2013), Ghate *et al*. (2014), Katore and Shaikh (2016), Mishra *et al*. (2017) has studied dark energy models in recent years. Motivated by the above works, in this paper we have investigated Bianchi-III dark energy cosmological model with variable equation of state (E_0S) in scale covariant theory of gravitation.

METRIC AND FIELD EQUATIONS

We consider Bianchi type III metric in the form

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2hx}B^2(t)dy^2 - C^2(t)dz^2, \tag{5}$$

where *A, B, C* are function of *t* only and *h* is constant.

The simplest generalization of equation of state (EoS) parameter of perfect fluid may be used to determine the EoS parameter separately on each spatial axis by preserving the diagonal form of the energy momentum tensor in a way consistent with the considered metric. Therefore, the energy momentum tensor of the fluid is taken as

$$T_j^i = diag\ [T_1^1, T_2^2, T_3^3, T_4^4] \tag{6}$$

One can parameterize this as follows:

$$T_j^i = diag\ [\rho, p_x, p_y, p_z] = diag[1, -\omega_x, -\omega_y, -\omega_z]\rho$$

$$= diag\ [1, -\omega, -(\omega+\delta), -(\omega+\gamma)]\rho\ , \tag{7}$$

where ρ is energy density of the fluid and p_x, p_y, p_z are the pressures along *x, y*, and *z* axes respectively. Here ω_x, ω_y, ω_z are the E_0S parameters in the directions of *x, y*, and *z* axes respectively and ω is the deviation free EoS parameter of the fluid. By setting $\omega_x = \omega$, we have parameterized the deviation from isotropy and then introduce skewness parameters δ and γ which are the deviations from ω on *y* and *z* axes respectively.

In the co-moving coordinate system the field equations (2)-(3) for the metric (Equation (5)) with the help of Equation (7) can be written as

$$\frac{\ddot B}{B} + \frac{\ddot C}{C} + \frac{\dot B}{B}\frac{\dot C}{C} - \left(\frac{\dot A}{A} - \frac{\dot B}{B} - \frac{\dot C}{C}\right)\frac{\dot\phi}{\phi} + \frac{\ddot\phi}{\phi} - \frac{\dot\phi^2}{\phi^2} = -8\pi G\,\omega\rho \tag{8}$$

$$\frac{\ddot A}{A} + \frac{\ddot C}{C} + \frac{\dot A}{A}\frac{\dot C}{C} - \left(\frac{\dot B}{B} - \frac{\dot A}{A} - \frac{\dot C}{C}\right)\frac{\dot\phi}{\phi} + \frac{\ddot\phi}{\phi} - \frac{\dot\phi^2}{\phi^2} = -8\pi G(\omega+\delta)\rho \tag{9}$$

$$\frac{\ddot A}{A} + \frac{\ddot B}{B} + \frac{\dot A}{A}\frac{\dot B}{B} - \frac{h^2}{A^2} - \left(\frac{\dot C}{C} - \frac{\dot A}{A} - \frac{\dot B}{B}\right)\frac{\dot\phi}{\phi} + \frac{\ddot\phi}{\phi} - \frac{\dot\phi^2}{\phi^2} = -8\pi G(\omega+\gamma)\rho$$
(10)

$$\frac{\dot A\dot B}{AB} + \frac{\dot B}{B}\frac{\dot C}{C} + \frac{\dot A\dot C}{AC} - \frac{h^2}{A^2} + \left(\frac{\dot A}{A} + \frac{\dot B}{B} + \frac{\dot C}{C}\right)\frac{\dot\phi}{\phi} - \frac{\ddot\phi}{\phi} + 3\frac{\dot\phi^2}{\phi^2} = 8\pi G\rho \tag{11}$$

$$\frac{\dot A}{A} - \frac{\dot B}{B} = 0\ . \tag{12}$$

Integrating equation (12), we get
$B = \mu A$,
where μ is a constant of integration which can be taken to unity without loss of generality, so that we have

$$B = A\,. \tag{13}$$

Using equation (13) in equations (8) and (9), we obtained

$$\delta = 0\,. \tag{14}$$

Now, using equation (13) and (14), field equations (8)-(11) reduces to

$$\frac{\ddot B}{B} + \frac{\ddot C}{C} + \frac{\dot B}{B}\frac{\dot C}{C} + \frac{\dot C}{C}\frac{\dot\phi}{\phi} + \frac{\ddot\phi}{\phi} - \frac{\dot\phi^2}{\phi^2} = -8\pi G\omega\rho, \tag{15}$$

$$2\frac{\ddot B}{B} + \frac{\dot B^2}{B^2} - \frac{h^2}{B^2} - \left(\frac{\dot C}{C} - 2\frac{\dot B}{B}\right)\frac{\dot\phi}{\phi} + \frac{\ddot\phi}{\phi} - \frac{\dot\phi^2}{\phi^2} = -8\pi G(\omega+\gamma)\rho, \tag{16}$$

$$\frac{\dot B^2}{B^2} + 2\frac{\dot B}{B}\frac{\dot C}{C} - \frac{h^2}{B^2} + \left(2\frac{\dot B}{B} + \frac{\dot C}{C}\right)\frac{\dot\phi}{\phi} - \frac{\ddot\phi}{\phi} + 3\frac{\dot\phi^2}{\phi^2} = 8\pi G\rho, \tag{17}$$

where overhead dots represent differentiation with respect to t. The spatial volume *V* and the average scale factor a for the metric (5) are defined as

$$V = a^3 = A^2C\ , a = \sqrt[3]{A^2C}\ . \tag{18}$$

The constant deceleration parameter for the models of the universe is defined as

$$q = -\frac{a\ddot a}{\dot a^2} = \text{Constant.} \tag{19}$$

The generalized mean Hubble parameter H is given by

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right), \tag{20}$$

where $H_1 = \frac{\dot A}{A} = H_2, H_3 = \frac{\dot C}{C}$ are the directional Hubble's parameters in the directions of *x, y* and *z* axes respectively. Using equation (18) and (20), we obtained

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right) = \frac{\dot{a}}{a}. \tag{21}$$

The expansion scalar θ and shear scalar σ are respectively given by

$$\theta = 2\frac{\dot{A}}{A} + \frac{\dot{C}}{C} \tag{22}$$

and

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right)^2. \tag{23}$$

Also, the average anisotropic parameter is defined as

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2. \tag{24}$$

SOLUTIONS OF THE FIELD EQUATIONS

The field equations (15)-(17) are three independent equations in five unknowns *B, C, ω, γ, ρ*. Hence to solve this system of highly nonlinear equations, we need two more conditions:

1. We assume that scalar expansion θ is proportional to shear scalar *σ, w*hich gives (Collins *et. al*., 1980)
   $$C = A^m, \tag{25}$$
   where $m > 1$ is a constant.
2. The E_0S parameter ω is proportional to skewness parameter γ (mathematical condition) such that
   $$\omega + \gamma = 0. \tag{26}$$

Now integrating equation (19), we get

$$a = (ct + d)^{\frac{1}{1+q}}, \tag{27}$$

where $c \neq o$ and d are constants of integration. Further on solving field equations (15)-(17) by using equations (18) and (27) with the choice of suitable constants and coordinates, we obtained the expression for metric coefficient as:

$$A = B = t^{\frac{3}{\alpha\beta}} \tag{28}$$

and

$$C = t^{\frac{3m}{\alpha\beta}}, \tag{29}$$

where *α= (q+1)* and *β= (m+2).*

So the metric (5), in view of (28) and (29), can be written as

$$ds^2 = dt^2 - t^{\frac{6}{\alpha\beta}}dx^2 - t^{\frac{6}{\alpha\beta}}e^{2x}dy^2 - t^{\frac{6m}{\alpha\beta}}dz^2. \tag{30}$$

Equation (30) represents Bianchi-III radiating cosmological model in presence of dark energy with negative constant deceleration parameter in scale covariant theory.

SOME PHYSICAL PROPERTIES

Spatial volume by using equation (18) is,

$$V = t^{\frac{3}{\alpha}} \tag{31}$$

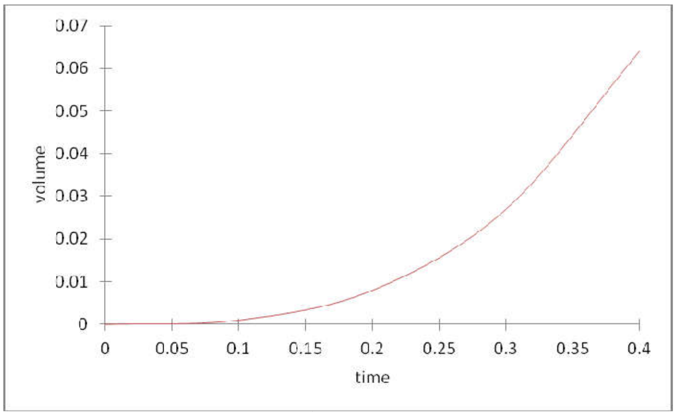

Figure 1: The plot of volume (V) verses time *(t)*, α=1

From the figure 1, it is observed that at an initial epoch, spacial volume is zero and increases with increase in cosmic time *t*, which shows that universe start evolving with zero volume and expands with cosmic time *t* showing late time accelerated expansion of the universe.

Hubble parameter is

$$H = \frac{1}{\alpha t}, \tag{32}$$

whose variation with respect to time t is shown in figure 2 as follows:

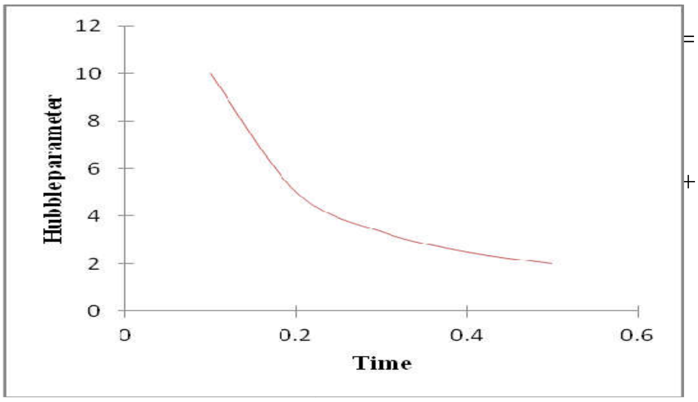

Figure 2: The plot of Hubble parameter (H) verses time (t), α=1

Expansion scalar is,

$$\theta = \frac{3}{\alpha t}. \tag{33}$$

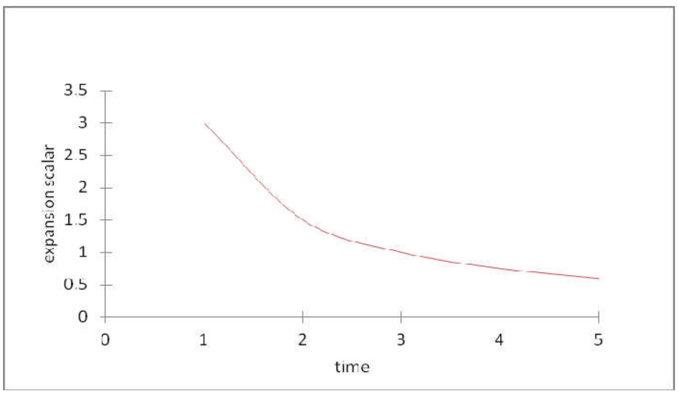

Figure 3: The plot of Expansion scalar (θ) verses time (t)

From the figure 3, it is observed that Expansion scalar at an initial epoch diverges. Cosmic time t increases gradually, expansion scalar decreases and finally vanishes when $t \to \infty$, which shows that it possess initial singularity. Further model has non-zero expansion rate i.e. universe start with an infinite rate of expansion. This behaves like big bang model of the universe.

Finaly, shear scalar is

$$\sigma^2 = \frac{3(3-\beta)^2}{\alpha^2\beta^2 t^2} \tag{34}$$

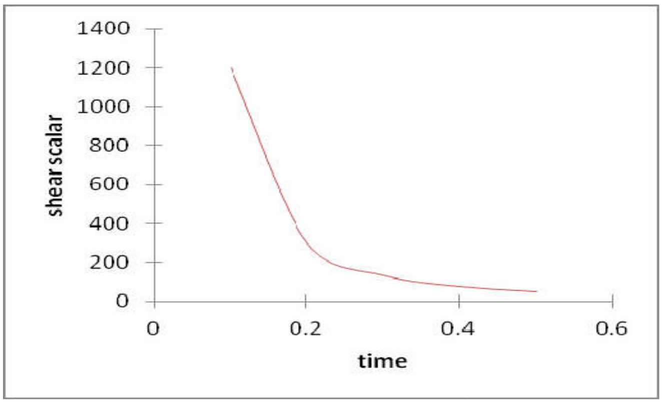

Figure 4: The plot of Shear scalar (σ) verses time *(t)*.

From the figure 4, it is observed that Shear scalar has initial singularity at $t = 0$ and it dies out for large value of t.

Also, the anisotropic parameter is,

$$A_\alpha = \frac{4(\beta-2)(\beta-4)}{\beta^2}. \tag{35}$$

The mean anisotropy parameter is uniform throughout the evolution of the universe, as it does not depend on the cosmic time t.

The energy density for the model is

$$\rho = \frac{\alpha^2\beta^2\varepsilon(\varepsilon-1) + 3\alpha\beta(1-m\varepsilon) + 9(m-1)}{4\pi G\alpha^2\beta^2 t^2}. \tag{36}$$

The EoS and skewness parameters in the model are

$$\omega = \frac{3\alpha\beta[1+m(\varepsilon+1)] - \alpha^2\beta^2\varepsilon - 9(m^2+m+1)}{2[\alpha^2\beta^2\varepsilon(\varepsilon-1) + 3\alpha\beta(1-m\varepsilon) + 9(m-1)]} \tag{37}$$

$$\gamma = -\frac{3\alpha\beta[1+m(\varepsilon+1)] - \alpha^2\beta^2\varepsilon - 9(m^2+m+1)}{2[\alpha^2\beta^2\varepsilon(\varepsilon-1) + 3\alpha\beta(1-m\varepsilon) + 9(m-1)]} \tag{38}$$

and $\delta = 0$. (39)

Also, the ratio of anisotropy is

$$\frac{\sigma^2}{\theta^2} = \frac{(3-\beta)^2}{3\beta^2} \neq 0. \tag{40}$$

As this ratio of anisotropy is not equal to zero which means that the model does not attain isotropy at large time t.

CONCLUSION

In this paper, we have investigated dark energy model with Bianchi type -III metric in scale covariant theory with the help of special law of Hubble parameter. Dark energy model plays a vital role in the discussion of accelerated model of the universe. It is observed that at an initial epoch, model has no singularity and all the physical parameters get infinite and all are decreases with increase in time. Further, the model is expanding and accelerating and does not attain isotropy for large value of t i.e. the model remains anisotropic throughout the evolution. We hope that the model we investigated in this work will be useful for better understanding of dark energy concept in scale covariant theory of gravitation.

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