



ISSN: 0976-3031

Available Online at <http://www.recentscientific.com>

CODEN: IJRSFP (USA)

International Journal of Recent Scientific Research  
Vol. 9, Issue, 1(H), pp. 23395-23397, January, 2018

**International Journal of  
Recent Scientific  
Research**

DOI: 10.24327/IJRSR

## Research Article

# CONTROL CHART FOR VARIABLES WITH SPECIFIED WEIGHTED PROCESS CAPABILITY INDEX

Livingston Thiraviya Kumar J<sup>1</sup> and Devaraj Arumainayagam S<sup>2</sup>

<sup>1</sup>Department of Economics and Statistics, Government of TamilNadu, India

<sup>2</sup>Department of Statistics, Govt. Arts College, Coimbatore, Tamil Nadu, India

DOI: <http://dx.doi.org/10.24327/ijrsr.2018.0901.1459>

### ARTICLE INFO

#### Article History:

Received 15<sup>th</sup> October, 2017

Received in revised form 25<sup>th</sup>

November, 2017

Accepted 23<sup>rd</sup> December, 2017

Published online 28<sup>th</sup> January, 2018

### ABSTRACT

Statistical Quality Control is a method of effective use of data for studying causes of variation in quality. Control Charts are used to confirm whether the process is statistically control. For measuring process performance, the four basic process capability indices  $C_p$ ,  $C_{pk}$ ,  $C_{pm}$  and  $C_{pmk}$  were developed. A weighted process capability index  $C_{pw}$  was introduced to unify the aforementioned capability indices. Once the process is statistically control the capability indices can be used to assess the capability of the process. This paper proposes the control chart for variables namely  $\bar{X}$  chart and R chart with specified weighted process capability index. A simulated process data from normal distribution is presented to demonstrate the application of the proposed control chart.

#### Key Words:

Process Capability Index, Control Chart,  
Variation, Normal Distribution,  
Assignable Cause.

**Copyright © Livingston Thiraviya Kumar J and Devaraj Arumainayagam S, 2018**, this is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution and reproduction in any medium, provided the original work is properly cited.

## INTRODUCTION

The term Statistical Quality Control is a method of planned collection and effective use of data for studying causes of variation in quality either in between processes, methods, materials, machines and management etc over certain period of time. The concept of process control was pioneered by W. A. Shewhart of Bell telephone laboratories in 1924 by using the concept of theory of probability and sampling. Statistical process control deals with controlling the process. Historically, Feigenbaum and Juran first proposed  $6\sigma$  as a measure of process capability. Juran compared  $6\sigma$  to the tolerance width as a method of determining the need for process improvement activities. Finally, Juran & Gryna proposed the first metric that directly compared process variability to customer specifications.  $C_p$  is the reciprocal of Juran & Gryna's capability ratio and it does not consider location of mean. To deal with violations of the centering assumptions, the index  $C_{pk}$  was developed. Chan et al. (1988) introduced so called Taguchi capability index  $C_{pm}$  that is measurable & directly related to the quadratic loss of the measured feature.

### Control Chart

Control charts are the basic tools to monitor the process. Control chart was developed by Dr. A. Shewhart. The

variations in the process are due to two types of causes, chance cause and assignable cause. The variation due to chance cause is called allowable variation. The range of such variation is called natural tolerance. Assignable causes arise due to non-random causes and are rectifiable. Control charts are the basic tools of statistical process control. A typical control chart consists of three horizontal lines. They are, a central line (CL), Upper control limit (UCL), Lower control limit (LCL). The  $\bar{X}$  and R control charts are widely used to monitor the mean and variability. A process that is operating with only chance causes of variation present is said to be in statistical control and the presence of assignable causes is said to be out of control. Control chart is an on-line process monitoring technique widely used to detect the occurrence of assignable causes.

Lai.Ka Chan *et. al* (1988), analyzed the effect of non-normality on  $\bar{X}$  and R charts. Effect of departure from normality was examined by comparing the probabilities that  $\bar{X}$  and R lie outside their three standard deviation and two standard deviation control limits. Control charts based on the assumption of normality gave inaccurate results when the tails of the underlying distribution are thin or thick. Janacek *et. al* (1997), proposed a modified control chart based on median to overcome the problem of non-normality. Subramani *et. al* (2012) proposed  $\bar{X}$  and R control chart based on process

\*Corresponding author: Livingston Thiraviya Kumar J

Department of Economics and Statistics, Government of TamilNadu, India

capability indices Cp and Cpk. The control limits for  $\bar{X}$  and R-control chart are given below:

**The control limit for the R chart**

$$UpperControlLimit = D_4\bar{R} \tag{1}$$

$$Centrl Line = \bar{R} \tag{2}$$

$$Lower Control Limit = D_3\bar{R} \tag{3}$$

**The control limit for the  $\bar{X}$  chart**

$$Upper Control Limit = \bar{\bar{X}} + A_2\bar{R} \tag{4}$$

$$Centrl Line = \bar{\bar{X}} \tag{5}$$

$$Lower Control Limit = \bar{\bar{X}} - A_2\bar{R} \tag{6}$$

**Weighted Process Capability Index-based Control Chart**

A weighted process capability index  $C_{pw}$  was introduced by Spiring *et. al* (1997). A weight is used in the definition in an attempt to unify all the aforementioned process capability indices.

$$C_{pw} = \frac{USL - LSL}{6\sqrt{\sigma^2 + w(\mu - T)^2}} \tag{7}$$

- where, USL – Upper Specification limit
- LSL – Lower Specification limit
- $\sigma^2$  - Variance
- $\mu$  - Process mean
- T - Target

Varying the weight function (w) to different values permits  $C_{pw}$  to assume equivalent computational algorithms to those for Cp, Cpk and Cpm. The average range is derived by solving (7),

$$\bar{R} = |d_2| \sqrt{-w^2(\mu_0 - T)^2 + \frac{(USL - LSL)^2}{6C_{pw}^2}} \tag{8}$$

where,  $d_2$  – Control chart constant

Thus (8) can be substituted in (1), (3), (4), (6) to obtain the respective control limits. The control limits based on this new method can be used to monitor the process for any specified value of the basic capability indices Cp, Cpk and Cpm.

**Simulation Results**

We consider the following parameters USL = 28, LSL = 22 and T = 25 for a simulated process that follow normal distribution with mean  $\mu = 25$  and standard deviation  $\sigma = 1$ . The sample size of 25 with subgroup size 5 has the following values:

$$\bar{\bar{X}} = 24.8062 ; \bar{R} = 2.495$$

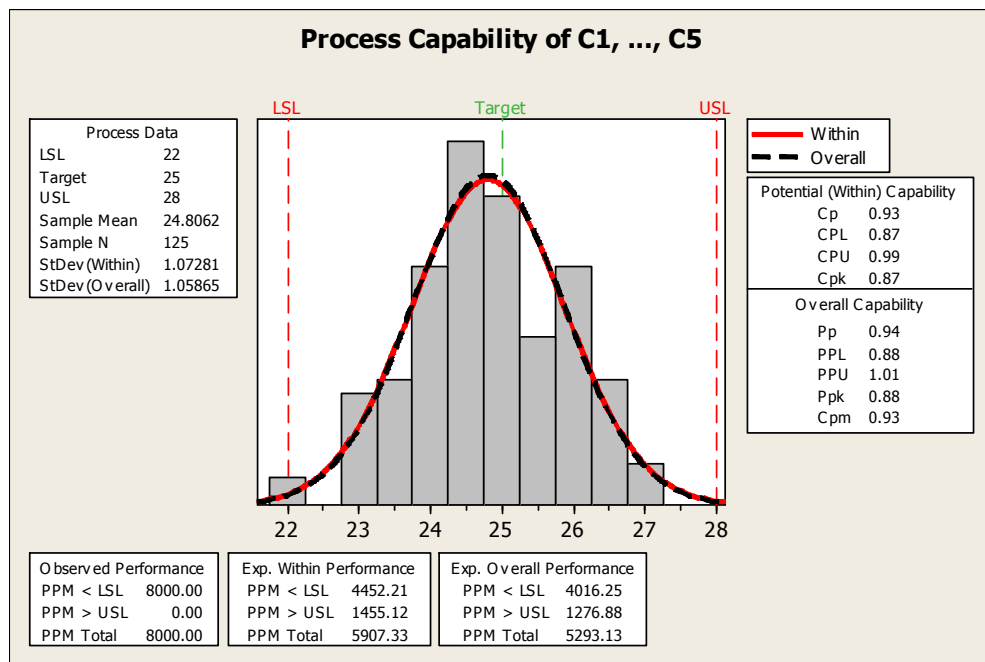
Figure 1 represents the capability analysis results. Cp=0.93, Cpk = 0.87 and Cpm = 0.93.

The control limits of the proposed capability index based R chart for the specified capability index (1, 1.5, 2) is given in Table 1 and it is observed that when the capability index value increased, the width of the control limits are reduced.

**Table 1** Control limits of R Chart for specified capability index

Capability Index	w	UCL	CL	LCL
1	0	4.919	2.326	0
	1	4.919	2.326	0
1.5	0	3.280	1.551	0
	1	3.280	1.551	0
2	0	2.460	1.163	0
	1	2.460	1.163	0

Table 2 represents the control limits of the proposed capability index based  $\bar{X}$  chart for the specified capability index (1, 1.5, 2) and it is observed that both the control limits UCL and LCL converge towards CL when the capability index value increased.



**Figure 1** Process capability Analysis

**Table 2** Control limits of  $\bar{X}$  Chart for specified capability index

Capability Index	w	UCL	CL	LCL
1	0	26.1483	24.8062	23.4641
	1	26.1483	24.8062	23.4641
1.5	0	25.7009	24.8062	23.9115
	1	25.701	24.8062	23.9114
2	0	25.4773	24.8062	24.1351
	1	25.4773	24.8062	24.1351

## CONCLUSION

In this paper we have proposed a unified approach to capability index based control chart. Various capability indices are used to assess the capability of a process. The proposed method detects the presence of assignable causes and assesses the capability of the process simultaneously. In the simulation study, the control limit for the specified value of the indices Cp and Cpm were presented. For both the variable control charts, the width of the control limit reduces when the capability value increased. Thus practitioners can apply this method to calculate the control limit of R chart and  $\bar{X}$  chart for any specified values of the capability indices.

## Reference

1. Chang L.K, Cheng.S.W and Spiring F.A (1988). New Measure of Process Capability, *Cpm Journal of Quality Technology*, 20 (3), 162-175.

2. Douglas C. Montgomery, Introduction to Statistical Quality Control, 4<sup>th</sup> Edition, Wiley India.
3. G.J.Janacek and S.E.Meikle (1997). Control Charts based on median, *The Statistician*, 46, No.1, pp. 19-31.
4. Kerstin Vannman (1995). A Unified Approach to Capability Indices, *Statistica Sinica*, 805-820.
5. Lai K. Chan, K P. Hapuarachchi and B D. Macpherson (1988). Robustness of X-bar and R charts, *Transactions on Reliability*, Vol. 37, No.1.
6. Samuel Kotz and Norman L. Johnson (2002). Process Capability Indices – A Review, 1992-2000, *Journal of Quality Technology*, Vol. 34, No.1.
7. Spiring F.A. (1997). A Unifying Approach to Process Capability Indices. *Journal of Quality Technology*, 18, 49-58.
8. Subramani.J and Balamurali.S (2012). Control Charts for Variables with Specified Process Capability Indices, *Int.J.Prob. & Statistics*, 1(4): 101-110..
9. Sullivan, L.P (1984). Reducing Variability: A new approach to quality, *Qual. Prog.*, 17(7), 15-21.
10. Victor E. Kane (1986). Process Capability Indices, *Journal of Quality Technology*, Vol. 18, No.1.

### How to cite this article:

Livingston Thiraviya Kumar J and Devaraj Arumainayagam S.2018, Control Chart for Variables with Specified Weighted Process Capability Index. *Int J Recent Sci Res.* 9(1), pp. 23395-23397.  
DOI: <http://dx.doi.org/10.24327/ijrsr.2018.0901.1459>

\*\*\*\*\*