ISSN: 0976-3031

Available Online at http://www.recentscientific.com

International Journal of Recent Scientific Research

International Journal of Recent Scientific Research
Vol. 7, Issue, 10, pp. 13777-13781, October, 2016

Research Article

A TRIAL TO SOLVE THE PUZZLES BY MODELING LINEAR EQUATIONS AND USING GAUSS- METHOD

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ARTICLE INFO

Article History:

Received 17th July, 2016
Received in revised form 12th August, 2016
Accepted 04th September, 2016
Published online 28th October, 2016

Key Words:

Gauss elimination, gauss Jordan, gauss seidal, gauss Jacobi, linear equations, puzzles

ABSTRACT

Background/Objectives: our aim is to solve puzzles using a numerical method. So we have used the Gaussian methods to solve the puzzles. For this purpose we are incorporated the linear equation for modeling the puzzles.

Methods/Statistical analysis: We Include the numerical method and adapted the gauss elimination method, gauss Jordan method and gauss seidal to solve the puzzles. So we have modeling the puzzles into linear equations and employ the numerical methods called Gaussian methods for solving the puzzles.

Findings: Here in this article we have trying to solve some critical puzzles using numerical solutions. For solving the puzzles we are employing the Gaussian method is the new one. No one using this method previously we are the first in using the Gaussian method to solve the puzzles. First, we are modeling the puzzles as a set of linear equations, the set of simultaneous equations are easily can be solved using the gauss elimination, and other Gaussian methods. In this article, we use that technique for finding the solution of the puzzles.

Application: This article is one of the real life application of numerical methods, in particularly Gaussian methods. In <30 words. In this similar manner, we can try to solve some other puzzles also. We think that it is the motivation for using the numerical methods for our critical real life situation for solving it.

INTRODUCTION

Researchers in Italy studying the acoustic noise levels from vehicular traffic at a busy three- way intersection on a college campus used a system of linear equations to model the traffic flow at the intersection. To help formulate the system of equations, 'operators' stationed themselves at various locations along the intersection and counted the numbers of vehicles going by[1]. This paper content motivated us to solve the puzzles using the linear equations and we have trying to find the solutions using Gauss elimination and Gauss Jordan elimination methods.

We should generally be faced with much simpler problems daily. What follows are some typical examples.

Example

The token fee at a vehicle stand in a bus stand is $1.50 for cycles and $4.00 for scooters. On a certain day, 2200 vehicles arrive at the stand and $5050 is collected. How many cycles and how many scooters arrived?

We would have set this up by picking a variable for one of the groups (say, "c" for "cycle") and then use "(total) less (what we've already accounted for)" (in this case, "2200 – c") for the other group. Using a system of equations, however, allows us to use two different variables for the two different unknowns.

Number of scooters: a

Number of cycles: c
Total number: $a + c = 2200$

Total income: $4a + 1.5c = 5050$

Now we can solve the system for the number of scooters and the number of cycles. We should solve the first equation for one of the variables, and then substitute the result into the other equation:

$a = 2200 - c$
$4(2200 - c) + 1.5c = 5050$
$8800 - 4c + 1.5c = 5050$
$8800 - 2.5c = 5050$
$-2.5c = -3750$
$c = 1500$
$a = 2200 - (1500) = 700$

There were 1500 cycles and 700 scooters.

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We should probably start out with problems which, like the one above, seem very familiar. But we have then moved on to more complicated problems.

Now we are trying to solve the puzzle using the linear equations as follows

Example

In a shop nearby a school, Vignesh, Murugan, Saraswathi, Lakshmi and Saravanan each of them buys some no of pencils, erasers, scales, sharpners, pens. Each of them pays different amount of money. And they discussed among themselves. Each of the item and the amount paid to the shopkeeper and any one of them must find the cost of each item. Like that, they told and they convert the following equations and asked me how to find the solution. I discussed about the "Gauss – Jordan Elimination" method and find following solution.

Vignesh buys 2 Pencils, 3 erasers, 5 Scales and 5 sharpners which is equal to the Total cost of Rs 23 and a cost of 1 Pen.

Murugan buys 1 Pencil, 4 Erasers and 4 Scales which are equal to Rs 41 less than the total cost of 10 Pens and 2 sharpners.

Saraswathi buys 4 Pencils, 2 Erasers, 1 Scale and 3 sharpners which is equal to Rs 10 more than the cost of 3 Pens.

Lakshmi buys 3 Pencils, 1 Eraser, 2 Pens and 7 sharpners which is equal to Rs 33 more than the cost of 4 Scales.

Saravanan buys 2 Pencils, 4 Pens and 1 sharpner which is equal to the total cost of 1 Eraser and 3 Scales more than Rs 22

Then find the cost of 1 Pencil, 1 Eraser, 1 pen, 1 scale and 1 sharpner.

Solution

Let the cost of 1 Pencil be x_1, 1 Eraser be x_2, 1 Pen be x_3, 1 Scale be x_4 and the cost of 1 Sharpner be x_5.
Then

$2x_1 + 3x_2 + 2x_4 + 5x_5 = 23 + x_3$
$\rightarrow 2x_1 + 3x_2 - x_3 + 2x_4 + 5x_5 = 23$
$x_1 + 4x_2 + 4x_4 = 10x_3 + 2x_5 - 41$
$\rightarrow x_1 + 4x_2 - 10x_3 + 4x_4 - 2x_5 = -41$
$4x_1 + 2x_2 + x_4 + 3x_5 = 10 + 3x_3$
$\rightarrow 4x_1 + 2x_2 - 3x_3 + x_4 + 3x_5 = 10$
$3x_1 + x_2 + 2x_3 + 7x_5 = 33 + 4x_4$
$\rightarrow 3x_1 + x_2 + 2x_3 - 4x_4 + 7x_5 = 33$
$2x_1 + 4x_3 + x_5 = 22 + x_2 + 3x_4$
$\rightarrow 2x_1 - x_2 + 4x_3 - 3x_4 + x_5 = 22$

The above equation can be arranged in the matrix form by the following ways

$$\begin{bmatrix} 2 & 3 & -1 & 2 & 5 \\ 1 & 4 & -10 & 4 & -2 \\ 4 & 2 & -3 & 1 & 3 \\ 3 & 1 & 2 & -4 & 7 \\ 2 & -1 & 4 & -3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 23 \\ -41 \\ 10 \\ 33 \\ 22 \end{bmatrix}$$

Shortly $Ax = B$

The augmented matrix

$$(A, B) \sim \left[\begin{array}{ccccc|c} 2 & 3 & -1 & 2 & 5 & 23 \\ 1 & 4 & -10 & 4 & -2 & -41 \\ 4 & 2 & -3 & 1 & 3 & 10 \\ 3 & 1 & 2 & -4 & 7 & 33 \\ 2 & -1 & 4 & -3 & 1 & 22 \end{array}\right]$$

$R_2 \rightarrow 2R_2 - R_1$
$R_3 \rightarrow R_3 - 2R_1$
$R_4 \rightarrow 2R_4 - 3R_1$
$R_5 \rightarrow R_5 - R_1$

$$(A, B) \sim \left[\begin{array}{ccccc|c} 2 & 3 & -1 & 2 & 5 & 23 \\ 0 & 5 & -19 & 6 & -9 & -105 \\ 0 & -4 & -1 & -3 & -7 & -36 \\ 0 & -7 & 7 & -14 & -1 & -3 \\ 0 & -4 & 5 & -5 & -4 & -1 \end{array}\right]$$

$R_3 \rightarrow R_3 - R_5$

$$(A, B) \sim \left[\begin{array}{ccccc|c} 2 & 3 & -1 & 2 & 5 & 23 \\ 0 & 5 & -19 & 6 & -9 & -105 \\ 0 & 0 & -6 & 2 & -3 & -35 \\ 0 & -7 & 7 & -14 & -1 & -3 \\ 0 & -4 & 5 & -5 & -4 & -1 \end{array}\right]$$

$R_4 \rightarrow 5R_5 + 7R_2$
$R_5 \rightarrow 5R_5 + 4R_2$

$$(A, B) \sim \left[\begin{array}{ccccc|c} 2 & 3 & -1 & 2 & 5 & 23 \\ 0 & 5 & -19 & 6 & -9 & -105 \\ 0 & 0 & -6 & 2 & -3 & -35 \\ 0 & 0 & -98 & -28 & -68 & -750 \\ 0 & 0 & -51 & -1 & -56 & -425 \end{array}\right]$$

$R_4 \rightarrow 6R_4 - 98R_3$
$R_5 \rightarrow 6R_5 - 51R_3$

$$(A, B) \sim \left[\begin{array}{ccccc|c} 2 & 3 & -1 & 2 & 5 & 23 \\ 0 & 5 & -19 & 6 & -9 & -105 \\ 0 & 0 & -6 & 2 & -3 & -35 \\ 0 & 0 & 0 & -364 & -114 & -1070 \\ 0 & 0 & 0 & -108 & -183 & -765 \end{array}\right]$$

$R_4 \rightarrow R_4 / 2, \quad R_5 \rightarrow R_5 / 3$

$$(A, B) \sim \left[\begin{array}{ccccc|c} 2 & 3 & -1 & 2 & 5 & 23 \\ 0 & 5 & -19 & 6 & -9 & -105 \\ 0 & 0 & -6 & 2 & -3 & -35 \\ 0 & 0 & 0 & -182 & -57 & -535 \\ 0 & 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

$R_5 \rightarrow \underline{X_5 = 3}$

$R_4 \rightarrow 182X_4 + 57X_5 = 535$

$182X_4 + 171 = 535$

$182X_4 = 535-171$

$182X_4 = 364$

$\underline{X_4 = 2}$

$R_3 \rightarrow -6X_3 + 2X_4 - 3X_5 = -35$

$-6X_3 + 4 - 3(3) = -35$

$-6X_3 = -35 + 5$

$-6X_3 = -30$

$\underline{X_3 = 5}$

$R_2 \rightarrow 5X_2 - 19X_3 + 6X_4 - 9X_5 = -105$

$5X_2 - 19(5) + 6(2) - 9(3) = -105$

$5X_2 - 95 + 12 - 27 = -105$

$5X_2 - 110 = -105$

$5X_2 = 5$

$\underline{X_2 = 1}$

$R_1 \rightarrow 2X_1 + 3X_2 - X_3 + 2X_4 + 5X_5 = 23$

$2X_1 + 3(1) - 5 + 2(2) + 5(3) = 23$

$2X_1 + 3 - 5 + 4 + 15 = 23$

$2X_1 + 17 = 23$

$2X_1 = 6$

$\underline{X_1 = 3}$

In the following example, we use the linear equations to solve the puzzle

Example

Garland to vinayakar

Three different temples are placed in three different places in a pond. The first temple is Vinayaka temple and the second temple is Murugan temple and the third temple is a Shiva temple.

The Gurukkal took the garland and must submerge into the water then came out near Vinayaka temple. When he submerged into the water the garland in his hand was doubled. He put some number of garlands to the Vinayakar.

Again he must submerge into the water and want to go to the next temple Murugan. When he submerged the remaining garland in his hand was doubled.

The Gurukkal wanted to put the same number of garland like Vinayakar.

Then he took the remaining garland and submerged into the water. At the time the garland in his hand doubled. Then he came out near the Shiva temple and he should put the same number of garland like Vinayakar and Murugan.

Finally, when he came out from the Shiva temple, there should be no garland in his hand.

Then find the number of garland first he took and find the number of garland put to the Vinayakar, Murugan and Shiva.

Solution

Let the number of garland first he took = x

When he submerged into the water, the garland in his hand is doubled therefore 2x

He put y number of garland to the god Vinayakar, therefore, remaining garland in his hand = 2x –y

When he submerged into the water the garland was doubled.

That is 2(2x-y) = 4x-2y

Again he put y number of garland to the god Murugar

Therefore, remaining garland = 4x-2y-y = 4x-3y
Again he submerged into the water the garland in his hand was doubled.

Therefore 2(4x-3y) = 8x-6y

He put y number of garland to the god Shiva and there is no garland in his hand

Therefore 8x-6y-y = 0
Therefore 8x-7y = 0
8x = 7y

The no of garland and the number of put the garland to the god are integer

Therefore the solutions are
x = 7, y = 8
And also
x = 14, y = 16
And other solutions are
x = 21, y = 24
x = 28, y = 32 and so on.

Execution

First Gurukkal took 7 garlands

And he submerged into the water, the garland in his hand was doubled

Now number of garland = 14
He put 8 garlands to the god Vinayakar
The remaining garlands = 14 – 8 = 6
Again he submerged into the water, the garlands was doubled
Number of garland = 12
He put 8 garlands to Murugar
The remaining garland is 12 -8 = 4
When he submerged into the water the 4 garlands was doubled
Therefore total garlands in his hand = 8
He put 8 garlands to the god Shiva
Therefore no garland in his hand at last.

Next we want to solve one more puzzle using the Gauss Jordan elimination method

Example

Gauss – Jordan elimination method *(Direct method)*

This method is a modification of the above gauss elimination method. In this method, the coefficient matrix A of the system AX = B is brought to a diagonal matrix or unit matrix by making the matrix A not only upper triangular matrix.

$$\begin{bmatrix} 2 & 3 & 5 & 1 & 2 & 1 \\ 3 & 1 & 12 & 3 & 1 & 5 \\ 6 & 2 & 4 & 2 & 3 & 3 \\ 8 & 2 & 4 & 5 & 2 & 5 \\ 5 & 1 & 3 & 3 & 3 & 3 \\ 4 & 2 & 2 & 8 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 58 \\ 105 \\ 77 \\ 100 \\ 72 \\ 86 \end{bmatrix}$$

Shortly Ax = B

The Augumented matrix

$$(A,B) \sim \left[\begin{array}{cccccc|c} 2 & 3 & 5 & 1 & 2 & 1 & 58 \\ 3 & 1 & 12 & 3 & 1 & 5 & 105 \\ 6 & 2 & 4 & 2 & 3 & 3 & 77 \\ 8 & 2 & 4 & 5 & 2 & 5 & 100 \\ 5 & 1 & 3 & 3 & 3 & 3 & 72 \\ 4 & 2 & 2 & 8 & 2 & 0 & 86 \end{array}\right]$$

$R_2 \rightarrow 2R_2 - 3R_1$
$R_3 \rightarrow R_3 - 3R_1$
$R_4 \rightarrow R_4 - 4R_1$
$R_5 \rightarrow 2R_5 - 5R_1$
$R_6 \rightarrow R_6 - 2R_1$

$$(A,B) \sim \left[\begin{array}{cccccc|c} 2 & 3 & 5 & 1 & 2 & 1 & 58 \\ 0 & -7 & 9 & 3 & -4 & 7 & 36 \\ 0 & -7 & -11 & -1 & -3 & 0 & -97 \\ 0 & -10 & -16 & 1 & -6 & 1 & -132 \\ 0 & -13 & -19 & 1 & -4 & 1 & -146 \\ 0 & -4 & -8 & 6 & -2 & -2 & -30 \end{array}\right]$$

$R_3 \rightarrow R_3 - R_2$
$R_4 \rightarrow 7R_4 - 10R_2$
$R_5 \rightarrow 7R_5 - 13R_2$
$R_6 \rightarrow 7R_6 - 4R_2$

$$(A,B) \sim \left[\begin{array}{cccccc|c} 2 & 3 & 5 & 1 & 2 & 1 & 58 \\ 0 & -7 & 9 & 3 & -4 & 7 & 36 \\ 0 & 0 & -20 & -4 & 1 & -7 & -133 \\ 0 & 0 & -202 & -23 & -2 & -63 & -1284 \\ 0 & 0 & -250 & -32 & 24 & -84 & -1490 \\ 0 & 0 & -92 & 30 & 2 & -42 & -354 \end{array}\right]$$

$R_4 \rightarrow 10R_4 - 101R_3$
$R_5 \rightarrow 2R_5 - 25R_3$
$R_6 \rightarrow 5R_6 - 23R_3$

$$(A,B) \sim \left[\begin{array}{cccccc|c} 2 & 3 & 5 & 1 & 2 & 1 & 58 \\ 0 & -7 & 9 & 3 & -4 & 7 & 36 \\ 0 & 0 & -20 & -4 & 1 & -7 & -133 \\ 0 & 0 & 0 & 174 & -121 & 77 & 593 \\ 0 & 0 & 0 & 36 & 23 & 7 & 345 \\ 0 & 0 & 0 & 242 & -13 & -49 & 1289 \end{array}\right]$$

$R_5 \rightarrow 29R_5 - 6R_4$
$R_6 \rightarrow 87R_6 - 121R_4$
$R_6 \rightarrow R_6 / 70$

$$(A,B) \sim \left[\begin{array}{cccccc|c} 2 & 3 & 5 & 1 & 2 & 1 & 58 \\ 0 & -7 & 9 & 3 & -4 & 7 & 36 \\ 0 & 0 & -20 & -4 & 1 & -7 & -133 \\ 0 & 0 & 0 & 174 & -121 & 77 & 593 \\ 0 & 0 & 0 & 0 & 1393 & -259 & 6447 \\ 0 & 0 & 0 & 0 & 193 & -194 & 577 \end{array}\right]$$

$R_6 \rightarrow 1393R_6 - 193R_5$
$R_6 \rightarrow R_6 / 220255$
$R_5 \rightarrow R_5 + 259R_6$
$R_5 \rightarrow R_5 / 1393$
$R_4 \rightarrow R_4 - 77R_6$
$R_3 \rightarrow R_3 + 7R_6$
$R_2 \rightarrow R_2 - 7R_6$
$R_1 \rightarrow R_1 - R_6$

$$(A,B) \sim \left[\begin{array}{cccccc|c} 2 & 3 & 5 & 1 & 2 & 0 & 56 \\ 0 & -7 & 9 & 3 & -4 & 0 & 22 \\ 0 & 0 & -20 & -4 & 1 & 0 & -119 \\ 0 & 0 & 0 & 174 & -121 & 0 & 439 \\ 0 & 0 & 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

$R_4 \rightarrow R_4 + 121R_5$
$R_4 \rightarrow R_4 / 174$
$R_3 \rightarrow R_3 - R_5$
$R_2 \rightarrow R_2 + 4R_5$
$R_1 \rightarrow R_1 - 2R_5$

$$(A,B) \sim \left[\begin{array}{cccccc|c} 2 & 3 & 5 & 1 & 0 & 0 & 46 \\ 0 & -7 & 9 & 3 & 0 & 0 & 42 \\ 0 & 0 & -20 & -4 & 0 & 0 & -124 \\ 0 & 0 & 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

$R_3 \rightarrow R_3 + 4R_4$
$R_3 \rightarrow R_3 / -20$
$R_2 \rightarrow R_2 - 3R_4$
$R_1 \rightarrow R_1 - R_4$

$$(A,B) \sim \left[\begin{array}{cccccc|c} 2 & 3 & 5 & 0 & 0 & 0 & 40 \\ 0 & -7 & 9 & 0 & 0 & 0 & 24 \\ 0 & 0 & 1 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

$R_2 \rightarrow R_2 - 9R_3$
$R_2 \rightarrow R_2 / -7$
$R_1 \rightarrow R_1 - 5R_3$

$$(A,B) \sim \left[\begin{array}{cccccc|c} 2 & 3 & 0 & 0 & 0 & 0 & 15 \\ 0 & 1 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

$R_1 \rightarrow R_1 - 3R_2$
$R_1 \rightarrow R_1 / 2$

$$(A,B) \sim \left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

The Solutions are

$X_1 = 3$ The cost of 1 A = Rs 3
$X_2 = 3$ The cost of 1 B = Re 3
$X_3 = 5$ The cost of 1 C = Rs 5
$X_4 = 6$ The cost of 1 D = Rs 6
$X_5 = 5$ The cost of 1 E = Rs 5
$X_6 = 2$ The cost of 1 F = Rs 2

CONCLUSION

The Linear equations and numerical methods, which could be gainfully employed by scientists and engineers to solve the problems, arising in research and industry. This paper shows that the above methods how we have used to solve some puzzle in real life.

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How to cite this article:

Sakthivel, A and Kavitha, T. N.2016, A Trial To Solve The Puzzles By Modeling Linear Equations and Using Gauss- Method. *Int J Recent Sci Res.* 7(10), pp. 13777-13781.