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Research Article

DESIGN AND ANALYSIS OF A NONLINEAR CONTROLLER FOR VEHICLE ACTIVE SUSPENSION SYSTEM USING SLIDING MODE CONTROL

Mohammad Mardani* and Majid Mehrabi

Department of Electrical Engineering, Shahab Danesh University, Qom, Iran

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ABSTRACT

One of the roots of vehicle vibrations and shakes is the road roughness. Shocks due to bumps on the road are transferred to the vehicle body through the wheels and not only make discomfort for passengers but also reduce the quality of driving. In this paper, a vehicle suspension system will be designed. Then its performance will be shown through analysis and simulation. The main idea is based on the combination of a nonlinear energy sink and a skyhook. The performance of the proposed suspension system will be evaluated by comparing car body vertical accelerations and suspension deflections with a passive suspension system. The sliding mode control as a robust nonlinear control method has been used to make the system robust to changes and uncertainties in car model. The Lyapunov based stability analysis shows that the tracking error of proposed system will asymptotically approach zero.

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INTRODUCTION

One of the roots of vehicle vibrations and shakes is road roughness. Shocks due to bumps on the road are transferred to the vehicle body through the wheels and make discomfort for passengers and reduce quality of driving. The suspension system is responsible for absorbing these shocks and reducing vehicle shakes as much as possible thereby providing more comfort to passengers.

A vehicle suspension system is used to isolate the car body from the wheels and allow relative motion between the two parts. It is typically rated by its ability to provide good driving quality, isolate passenger from road disturbance, and improve passenger comfort. The disturbance may be caused by various reasons such as road unevenness, aerodynamics forces, non-uniformity of the tire/wheel assembly, and even braking force. The quality of driving especially during cornering and swerving deter mines the active safety of the vehicle. The ability of absorbing vibration from road disturbance is mainly discussed in this research.

In passive suspension systems, the reduction in vertical acceleration results in an increase in suspension travel. The suspension travel has to be limited because the movement of vehicle spring-damper system is constrained.

The car performance factors such as car body acceleration, suspension deflection, and wheel deflection are measured to

compare the proposed control with a passive suspension system.

The *LQR* control method and *H_∞* robust control have been used widely for designing controllers for active suspension systems (Du and Zhang, 2007 and Eslamian *et al*, 2007). Sam *et al*. designed a sliding mode PI controller for the quarter vehicle model and compared their results with those obtained by the *LQR* control method (Sam *et al*, 2004).

The abovementioned methods are based on a linear model and ignore the dynamics of the hydraulic actuator which is highly nonlinear. However, the nonlinearity in the actuator dynamics has a great effect on the overall behavior of the system and cannot be ignored. In the present paper, the studied model is a nonlinear one and a quarter suspension will be designed. The sliding mode control method is then utilized to design a nonlinear and robust controller for the active suspension system.

Quarter Model

Typically, a suspension system consists of the system of springs, shock absorbers, and linkages that connect a vehicle to its wheels. The structure of a simple quarter car model is shown in Figure 1.

In the simplified quarter car model, M_s represents sprung mass (vehicle body), M_u represents unsprung mass (wheel body). According to the component used to generate the control force F to connect the sprung mass (M_s) and unsprung mass (M_u),

*Corresponding author: Mohammad Mardani

Department of Electrical Engineering, Shahab Danesh University, Qom, Iran

the suspension system can be classified as a Passive Suspension System, Semi-active Suspension System, and Active Suspension System.

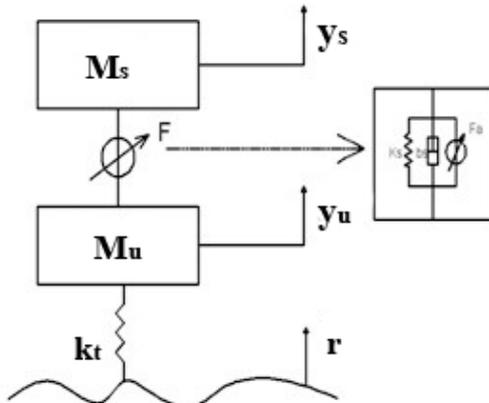


Figure 1 Simplified Quarter Car Model

Active Suspension System

Active suspension system can improve passengers' safety and riding comfort of the vehicle significantly. In an active suspension system, a force actuator is used instead of a passive damper or the combination of passive damper and spring. A significant difference between a passive suspension system and an active one is that active suspension system can add and dissipate energy to/from the system via a force actuator, unlike a passive suspension system which can only dissipate energy via a passive damper. However, the introduction of active force actuator causes other problems too. The first problem is the requirement of larger power which decreases the overall performance of the vehicle. The second problem is that the force actuator also increases the complexity of the whole system which leads to a more extensive range of control problems. The third one is the requirement of more sensors which increases the cost of the suspension system.

Combined Suspension System

The suspension analysis is based on the simplified quarter car model which has been shown in Figure 1.

The simplified model consists of the sprung mass M_s which represents car body and unsprung mass M_u which includes the mass of the tire and axles. The tire is modeled as a linear spring with stiffness k_t .

The suspension system is controlled by force F which is designed by the engineer. Typically, F is generated by a linear spring, a damper and another force actuator which may be an active actuator or a semi-active actuator. r represents the road disturbance which is modeled as a sine function and can be showed as follows:

$$r \quad (1)$$

where A_1 represents the amplitude of the road disturbance signal.

Usually one focuses on the car body acceleration y_s , suspension deflection $y_s - y_u$ and wheel deflection $y_u - r$ which determine the vehicle suspension system performance. The main idea of the active suspension system is to design an active element to

generate force F_a which can adjust itself continuously to changing road conditions.

The sprung mass acceleration y_s is utilized to represent the passenger's comfort. The lower the sprung mass acceleration, the better the passenger's comfort.

The model of the suspension system with the nonlinear energy sink and skyhook is shown in Figure 2 (Chen, 2009).

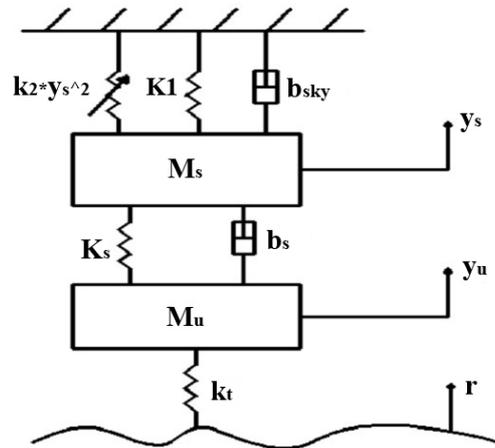


Figure 2 Suspension System Combined with Nonlinear Energy Sink and Skyhook

Obviously, the combined suspension system has two parts: the nonlinear energy sink part and skyhook part. Based on the forces generated by the nonlinear energy sink and skyhook parts, the force generated by the combined suspension system can be presented as

where L_{01} and L_{02} represent length of springs and K_1 and K_2 represent stiffness of springs.

The corresponding quarter car model together with the schematics for the hydraulic system is showed in Figure 3.

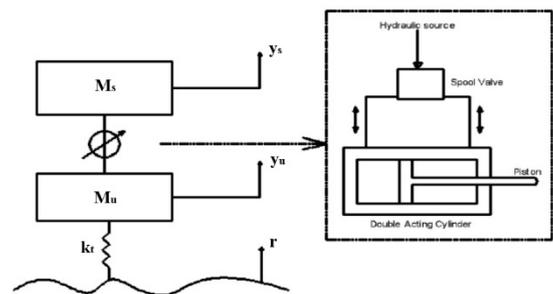


Figure 3 Quarter Car Model with Hydraulic System

Typically, hydraulic servomechanisms are used to control hydraulically actuated suspensions. The hydraulic pressure to the servos is provided by a high pressure radial piston hydraulic pump.

Sensors are used to monitor body movement and vehicle ride level continuously and transfer gathered data to the computer. As the computer receives and processes the data, it operates the hydraulic servos, mounted beside each wheel. The servo-regulated suspension generates counter forces to body lean, dive, and squat during driving maneuvers.

Hydraulic actuators are one of the most viable choices thanks to their high power-to-weight ratio, low cost, and the fact that force can be generated over a prolonged period of time without overheating. The hydraulic system consists of a source of hydraulic pressure, a spool valve, and a hydraulic cylinder.

A hydraulic pump which is typically augmented with accumulators to reduce pressure fluctuations and supply additional fluid for peak demands is used to supply hydraulic pressure. The hydraulic cylinder is a double acting cylinder. The position of the piston can be changed by modulating the oil flow into and out of the cylinder chambers, which are connected to the spool valve through cylindrical ports. The modulation is provided by the spool valve. The dynamic function of the hydraulic actuator and the spool valve are as follows (Hashemipour, 2013):

$$P \quad (3)$$

$$x \quad (4)$$

$$F \quad (5)$$

Where A is the pressure area in the actuator, P_L is the load pressure, $v_p=d$ is the actuator piston velocity, and F is the output force generated by the hydraulic actuator.

The parameters α, β, γ are determined by actuator pressure area, effective system oil volume, effective oil bulk modulus, oil density, hydraulic load flow, total leakage coefficient of the cylinder, discharge coefficient of the cylinder, and servo valve area gradient. x_v is the spool valve position, τ is the actuator electrical time constant, $K=1$ is the DC gain of the four-way spool valve, and u is the input current to the servo valve.

Dynamic Equations

According to the simplified quarter car model in Figure 1, the states and dynamic system of the simplified quarter car model are given as follows:

$$\begin{aligned} x_1 &= y_s \\ x_2 &= \dot{y}_s \\ x_3 &= y_u \\ x_4 &= \dot{y}_u \\ x_5 &= P_L \\ x_6 &= x_v \\ \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{k_s}{m_s} x_1 + x_2 - \frac{b_s}{m_s} x_2 + \frac{k_s}{m_s} x_3 + \frac{b_s}{m_s} x_4 + \frac{A}{m_s} x_5 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{k_s}{m_u} x_1 + \frac{b_s}{m_u} x_2 - \left(\frac{k_t}{m_u} + \frac{k_s}{m_u} \right) x_3 - \frac{b_s}{m_u} x_4 - \frac{A}{m_u} x_5 + \frac{k_t}{m_u} \\ \dot{x}_5 &= -\beta x_5 - A(x_2 - x_4) + x_6 \sqrt{P_s - \text{sgn}(x_6)x_5} \\ \dot{x}_6 &= \frac{1}{\tau} (-x_6 + K * u) \end{aligned} \quad (6)$$

Sliding mode control is a class of nonlinear control. The goal of the switching control law is to drive the nonlinear plant’s state trajectory onto a designed surface in the state space and maintain the plant’s state trajectory on that surface all the time. The surface defined is called a sliding surface (sliding manifold). A control input is designed so that the plant state can slide to the surface and stay on it. Moreover, a Lyapunov approach is also utilized to prove the stability of the control system.

Sliding Mode Control

The desired dynamics of the active system consisting of a passive suspension system with a nonlinear energy sink and skyhook has been shown in Figure 2.

In order to simplify the problem, the following first order system is taken into consideration first. Assume the nonlinear system has the form:

$$\dot{x} = f(x) + g(x)u \quad (7)$$

Where x is the state vector of the system and u is the control input to the system. If the control input is designed based on feedback linearization, then the control input should be

where x is the state vector of the system and u is the control input to the system. If the control input is designed based on feedback linearization, then the control input should be

$$u = \frac{1}{g(x)} (\dot{f}(x) + v) \quad (8)$$

Substitute Equation (8) into Equation (7), results $\dot{x} = v$. If the feedback is designed as $v = k * x$ then asymptotically stable system can be obtained. The Lyapunov function V may be chosen as

$$V = \frac{1}{2} x^T x \quad (9)$$

Then taking the first derivative of the Lyapunov function, asymptotically stable or even exponentially stable system can be achieved by designing an appropriate V . However, feedback linearization control is based on exact knowledge of the system model, which means $f(x)$ and $g(x)$ should be known. In order to design a robust controller, the sliding mode control method is utilized. Firstly, the first sliding surface which is the error between the actual state and ideal state can be presented as follows

$$s_1(x, t) = x_{actual} - x_{desired} \quad (10)$$

The Lyapunov function is chosen as:

$$V = \frac{1}{2} s_1^T s_1 \quad (11)$$

If the first derivative of the Lyapunov function satisfies the following inequality, then the system can satisfy the robustness or sliding condition:

$$V = \frac{d}{dt} \left(\frac{1}{2} s^2 \right) = \dot{s} s \leq -k s^2 \quad (12)$$

Where k is a positive constant which is designed by the engineer. According to the Lyapunov analysis, exponential stability can be proved, which means the error between the actual state and ideal state will exponentially converge to zero as time tends to infinity.

The value of k , determines the convergence rate of the error. The larger the value of k , the faster will be the convergence rate. However, k is also constrained by the control input which is determined by the physical system.

In the suspension system, we expect that the force generated by the hydraulic actuator can track the ideal force generated by the combined suspension system. We assume the road disturbance of the suspension system is unknown but bounded by 0.1 m.

Based on the state space in Equation (6), the first sliding surface can be defined as

$$s_1(x, t) = F_{actual} - F_{desired} \quad (13)$$

Where F_{actual} can be calculated as:

$$F_{actual} = AP_L = Ax_5 \quad (14)$$

Where A represents the area of the valve, and P_L represents the pressure. Thus the force generated by the hydraulic actuator can be obtained by multiplying the area and pressure of the valve.

The desired force can also be presented as

$$F_{desired} = K_1 \begin{pmatrix} L_{01} & Y_s \\ b_{sky} & Y_u \end{pmatrix} K_2 \begin{pmatrix} L_{02} & Y_s \end{pmatrix}^3 = A x_{5desired} \quad (15)$$

The derivative of $F_{desired}$ can be written as

$$\dot{F}_{desired} = K_1 \begin{pmatrix} L_{01} & Y_s \\ b_{sky} & Y_u \end{pmatrix} K_2 \begin{pmatrix} L_{02} & Y_s \end{pmatrix}^3 \quad (16)$$

Substituting the state variables which have been shown in dynamic equation results:

$$\dot{F}_{desired} = (x) Hx_r \quad (17)$$

x_r represents the unknown road disturbance and is bounded by 0.1m and H is a positive constant determined by parameters b_{sky}, k_r, μ . Then according to the dynamic system, \dot{x}_5 is

$$\dot{x}_5 = \beta x_5 - A(x_2 - x_4) + \gamma x_6 \sqrt{P_s} \operatorname{sgn}(x_6)x_5 \quad (18)$$

Then let

$$f(x) = -\beta x_5 - A(x_2 - x_4) \quad (19)$$

$$g(x) = \gamma \sqrt{P_s} - \operatorname{sgn}(x_6)x_5$$

Taking the first derivative of s_1 will give

$$\dot{s}_1 = \dot{F}_{actual} - \dot{F}_{desired} = A\dot{x}_{5actual} - A\dot{x}_{5desired} = A(f(x) + g(x)x_{6desired} - \dot{x}_{5desired}) \quad (20)$$

According to the design principle mentioned above, $\dot{x}_{6desired}$ can be designed as:

$$\dot{x}_{6desired} = \frac{1}{g(x)} (f(x) + x_{5desired} k_3 - s_1 - k_5 \operatorname{sgn}(s_1)) \quad (21)$$

where $f(x)$ and $g(x)$ have been defined in Equation (19) and k_3 and k_5 are positive constants. The value of k_3 will affect the convergence rate of the error between the actual force and ideal force. The variable $x_{5desired}$ can be obtained as follows

$$\dot{x}_{5desired} = \frac{d\left(\frac{F_{desired}}{A}\right)}{dt} \quad (22)$$

Based on the state x_6 , the second sliding surface can be defined

$$s_2 = x_{6actual} - x_{6desired} \quad (23)$$

where $x_{6actual}$ has been defined in Equation (6), and $x_{6desired}$ has been defined from the first sliding surface in Equation (21). Taking the first derivative of the second sliding surface, gives

$$\begin{aligned} \dot{s}_2 &= \dot{x}_{6actual} - \dot{x}_{6desired} \\ &= \frac{1}{\tau} (x_{6actual} + u) - \dot{x}_{6desired} \end{aligned} \quad (24)$$

In order to let the system satisfy robustness condition and for the nonlinear plant's state variables to slide along the two surfaces, the control input may be designed as:

$$u = x_{6actual} + \tau(x_{6desired} - k_4 s_2) \quad (25)$$

where $\dot{x}_{6desired}$ can be obtained by taking the derivative of Equations 4 to 15, and k_4 is a positive constant which is designed by the engineer (Kim, 2015).

Stability Analysis

In this section, a stability analysis based on Lyapunov analysis is made. According to the previous section, two sliding surfaces are defined. Thus let the Lyapunov function be:

$$V = \frac{1}{2} s_1^2 + \frac{1}{2} s_2^2 \quad (26)$$

Then take the first derivative of the Lyapunov function. According to Equation (18) through (24), it can be derived as:

$$\dot{V} = s_1 \dot{s}_1 + s_2 \dot{s}_2 = k_3 s_1^2 - k_4 s_2^2 - H s_1 x_r + k_5 |s_1| \quad (27)$$

If the value of k_5 satisfies:

$$0.1H \leq k_5$$

then:

$$\dot{V} \leq -C$$

where C is a positive constant determined by the values of k_3 and k_4 . After solving the first derivative equation, V can be presented as:

$$V(t) \leq e^{-\frac{t}{\tau}}$$

Obviously, according to Equation (30), V will be exponentially convergent to zero as time goes to infinity. Because V is defined in Equation (26), thus the s_1 and s_2 should also converge to zero exponentially as time tends to infinity (Zhang, 2009).

If s_1 goes to zero as time goes to infinity, it means the force generated by the hydraulic actuator can perfectly track the ideal force defined by the suspension system combined with the nonlinear energy sink and skyhook as time goes to infinity.

Simulation

In the simulation, assume the vehicle travels at a steady horizontal speed of 40mph which is same as before. Also there is a road bump with amplitude 0.1 m and effecting at $t=5$. Block diagram of controller is shown in Figure 4.

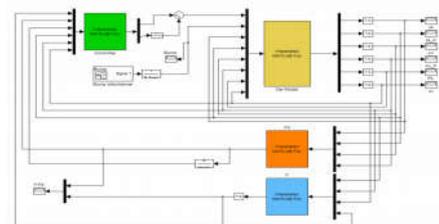


Figure 4 Block diagram of controller

Based on the simulation, several observations are taken into consideration. The first one is to verify that the force generated by the hydraulic actuator can track the desired force designed using sliding mode control. The desired force and actual force are shown in Figure 5. It is seen in Figure 5 that the actual force can track the ideal force perfectly.

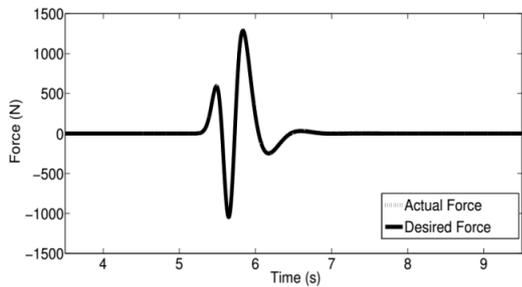


Figure 5. Force Tracking by Sliding Mode Control

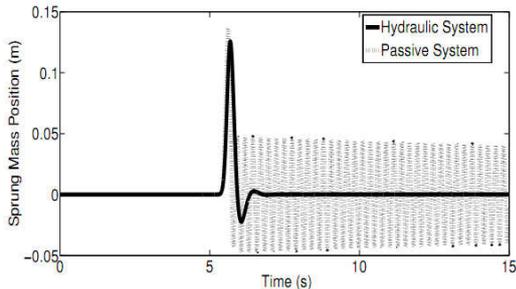


Figure 6. Car Body Position

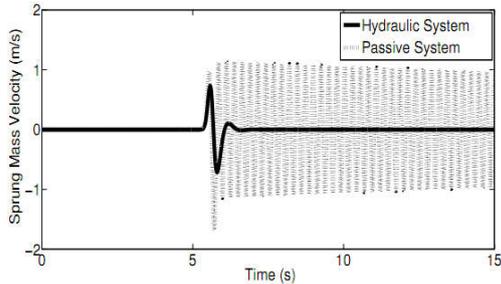


Figure 7. Car Body Velocity

Figures 6 through 9 show a comparison between the passive suspension system and the suspension system with the hydraulic actuator. According to Figures the car body settles down very quickly which is preferred.

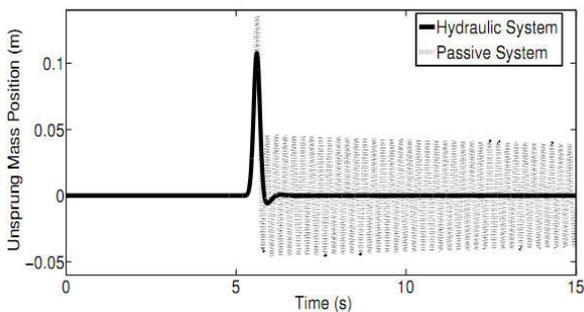


Figure 8. Unsprung Mass Position

The control input of the suspension system with the hydraulic actuator is shown in Figure 10. And the control input in the system is the input current to the servo valve. Figures 12 through 14 show the tracking performance between the original

nonlinear energy sink and skyhook suspension system and the suspension system with hydraulic actuator using sliding mode control.

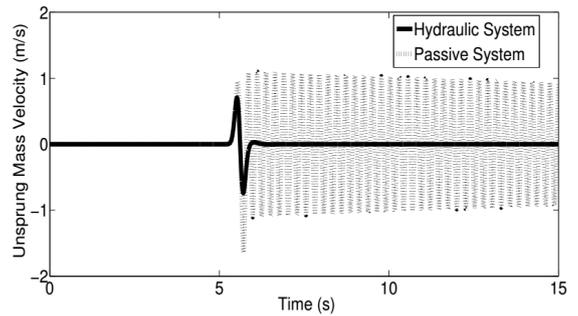


Figure 9. Unsprung Mass Velocity

There are some small differences between the original suspension system with the nonlinear energy sink and skyhook and the suspension system with the hydraulic actuator. However the trends are almost same. Based on these Figures, the tracking performance by designing the sliding mode control is quite acceptable.

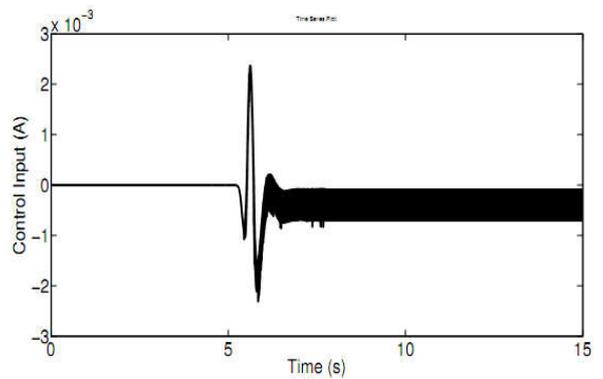


Figure 10. Control Input for Hydraulic System

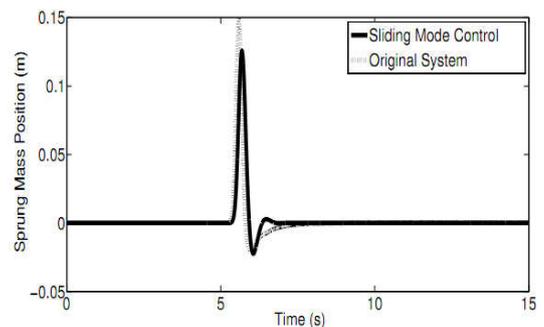


Figure 11. Car Body Position

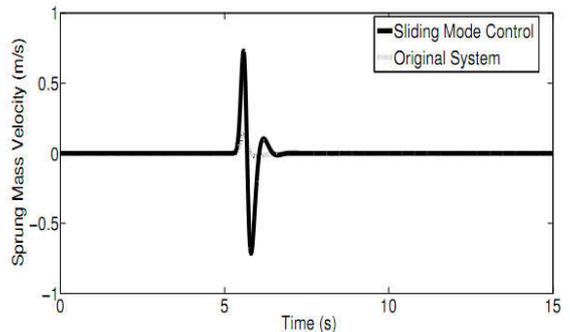


Figure 12. Car Body Velocity

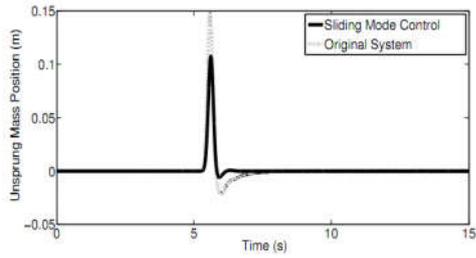


Figure 13 Unsprung Mass Position

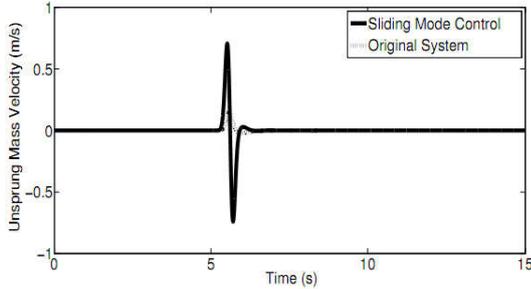


Figure 14 Unsprung Mass Velocity

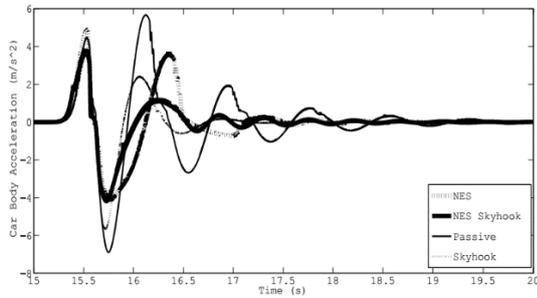


Figure 15 Car Body Acceleration

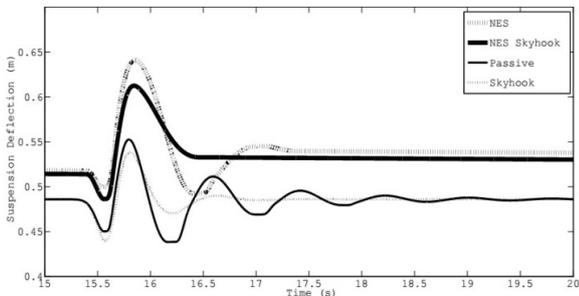


Figure 16 Suspension Deflection

Figure 17 shows the car body acceleration. The car body acceleration is the acceleration of the sprung mass which can be presented as Y_s which should be minimized for passenger's comfort.

According to Figure 15 it is obvious that the combined suspension system shows the lowest car body acceleration which means the best performance for passenger comfort.

The suspension deflection is measured and shown in Figure 16. Suspension deflection is the vertical distance between the mass centers of the sprung mass and unsprung mass which can be expressed as $Y_s - Y_u$. Thus if the passenger comfort is the most significant requirement, then the suspension system with nonlinear energy sink part will be better choice.

Now two other input which are shown in Figure 17 and 18 are applied to the system.

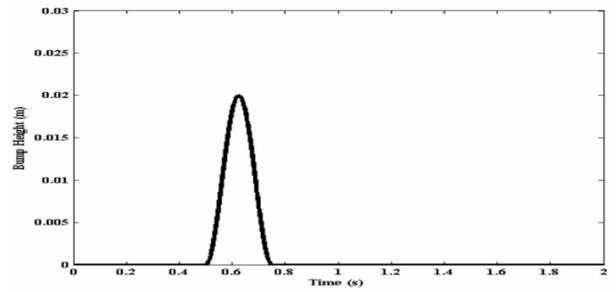


Figure 17 The first road input

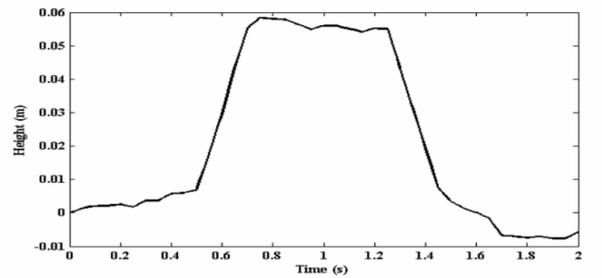


Figure 18 The second road input

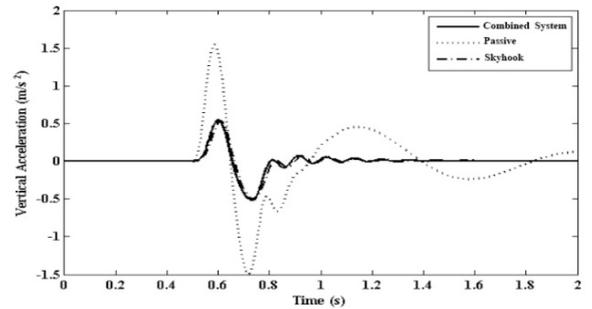


Figure 19 Vertical acceleration for first road input

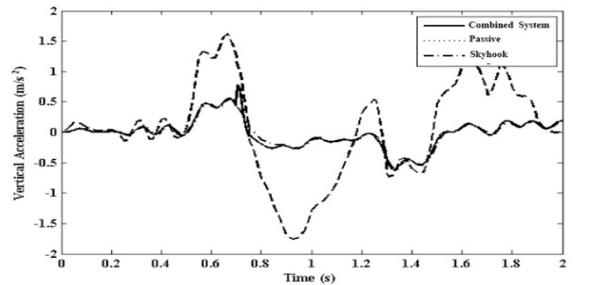


Figure 20 Vertical acceleration for second road input

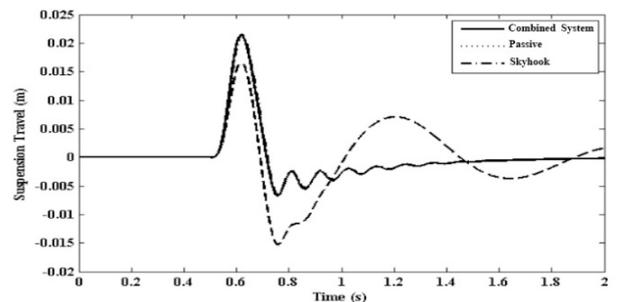


Figure 21 Suspension deflection for first road input

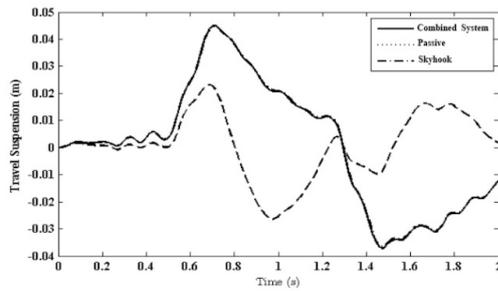


Figure 22 Suspension deflection for second road input

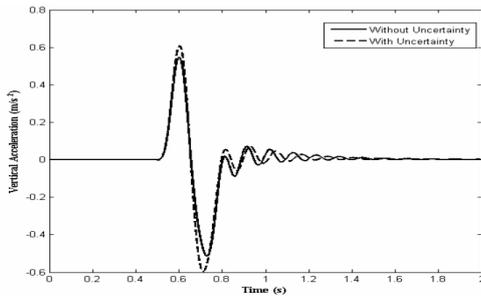


Figure 23 Robustness analysis for the sliding mode controller

Figures 19 through 22 show that combined system is better choice for both inputs based on car body acceleration and suspension system deflection.

On the other hand, in the controller design, nominal parameter values were assumed. Such an assumption cannot be made in practice and deviations from nominal values are inevitable. For example, the vehicle mass varies based on factors such as its load, number of passengers, and the amount of fuel. Also, the elasticity of vehicle tires changes based on vehicle speed, temperature increase, and road quality

Figure 23 presents the robustness analysis of the sliding mode controller. It can be seen that the sliding mode controller is robust against up to 20% changes in elasticity of vehicle tires and vehicle mass.

CONCLUSION

The design and control of a suspension system is developed in this paper. The sliding mode control is applied to let the suspension system with hydraulic actuator track the ideal force generated from the combined suspension system of nonlinear energy sink and skyhook. Simulations show that the sliding mode control can indeed control the hydraulic actuator to generate the expected force.

Based on the results, the proposed controller suggests effective control of suspension system based on car body acceleration and suspension deflection factors.

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