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Research Article

AN ANALYTICAL APPROACH TO A PROBLEM ON DISPERSION OF AIR POLLUTANTS

Verma V. S[1]., Uma Srivastava[1] and Bhandari P. S[2]*

[1]Department of Mathematics and Statistics D.D.U. Gorakhpur University, Gorakhpur-273009, U.P., India
*[2]Department of Mathematics, Birendra Multiple Campus, Tribhuvan University, Nepal

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ABSTRACT

In this paper, an analytical approach to a problem on dispersion of air pollutants with constant removal rate and variable wind velocity using power law profile emitted from a point source is proposed to study in steady state condition where eddy diffusivity coefficients are taken as constants. The methods of separation of variables and power series technique have been used for the solution of the problem. It is found that the concentration profile of the air pollutants near the ground is high and it decreases as the distance (vertical or cross-wind or down-wind) from the ground increases.

INTRODUCTION

The word ‘ pollution’ has been derived from a Latin word, ‘pollutionem,’ which means to make dirty (pollution is the process of making the environment i.e., the land , water and air dirty by adding harmful substances to it). Pollution causes imbalance in the environment. This imbalance has threatened the every survival of life. It is a threat to the whole world. Environmental pollution is a serious problem. Nearly 35 percent of India’s total land area is subjected to serious environmental pollution and this percentage is continuously increasing day by day.

In present, we are suffering with four major pollutions. They are (a) Water pollution, (b) Air pollution, (c) Soil pollution and (d) Noise pollution. Air pollution is the most dangerous form of pollution. Land and water pollution have worsened the situation. Pollution causes several types of harmful disease. We must control pollution for our survival. The industrial development and the Green Revolution have adversely affected the environment. People have converted the life supporting of the entire living world into their own resources and have vastly disturbed the natural ecological balance.

Air pollution is the most dangerous form of pollution. The major air pollutants are SO_x, NO_x, CO, Particulate matter, etc. It results from gaseous emission from industry, thermal power stations, domestic combustion, etc. Due to air pollution, the composition of air is changing all over the world. Most of the gases and air pollutants are produced burning fuels. Burning of coal produces Carbon dioxide, Sulpher dioxide etc. which are responsible for acid rain. Chlorofluorocarbons are widely used as propellants and as refrigerants which cause ozone depletion. Noise is also is one of the major pollutants, which includes in air pollution. The general noise level in the cities is rising alarmingly. Nuclear explosions and nuclear tests also pollute the air. The Taj Mahal in Agra is affected by the fumes emitted by the Mathura refinery. Reports estimate that the monument would get defaced with a span of twenty years because of the harmful effluents of the emission from the refinery. The emission of greenhouse gases has led to climate changes. The increase in pollution has resulted in global warming. Global warming is an average increase in the earth’s temperature due to greenhouse effect as a result of both natural and human activity. The term climate is often used interchangeably with the term global warming. The ice-caps in the Polar Regions have begun to melt fast. This has resulted in the rise of the water level of the seas and oceans. Air pollution causes allergies, asthma, lung cancer and bronchitis. Radioactive pollutants cause respiratory problems, paralysis, cancer and other diseases. Excessive noise pollution can lead to deafness, anxiety, stress, increase in the rate of heart beat and other health problems. The depletion of the ozone layer can also result in the skin disease. In order to fight this menace of

*Corresponding author: **Bhandari P. S**
Department of Mathematics, Birendra Multiple Campus, Tribhuvan University, Nepal

pollution, vigorous efforts should be made. The anti-pollution law should be strictly practiced. Tress should be planted everywhere and vehicles should be made eco-friendly.

Modelling or simulation is a process whereby a system is created to simulate a real- life situation. Pollutants are continuously released from numerous sources into the atmosphere. The pollution sources could be point sources (e.g., stacks or vents), area sources (e.g., landfills, ponds, storage piles), or volume sources (e.g., conveyers, structure with multiple vents). The dispersion of air pollutants in the atmosphere emitted from different sources depends on various factors. A dispersion model provides a means of calculating ambient ground level concentration of an emitted substance given information about the emission and nature of the atmosphere. In order to access whether an emission meets the ambient air objective, it is necessary to determine the ground level concentrations that may arise at various distances from the source (Modi *et al*. [1]).

In societies that are rapidly developing sufficient resources may not be invested in air pollution control because of other economic and social priorities. The rapid expansion of the industry in these countries has occurred at the same time as increasing traffic from automobiles and trucks, increasing demands for power for the home and concentration of the population in large urban areas. The result has been some of the worst air pollution problem in the world (Admassu and Wubeshet[2]).

The modern science of air pollution modelling began in the decade of 1920's (Macdonald [3]). Several studies on atmospheric dispersion of air pollutants using mathematical models have been conducted by various investigators and researchers, all having the same goal of how atmospheric air quality can be maintained.

Sharan *et al*. [4] have attempted to provide a broad overview of essentials of modeling framework of atmospheric dispersion. Authors have attempted to bring together briefly the essentials and some recent advances in the field of atmospheric dispersion modeling .

Khaled *et al*. [5] have developed an analytical solution of the two dimensional atmospheric diffusion equation using two forms of eddy diffusivities by the method of separation of variables. They used Fourier transform and square compliment method to solve the dispersion equation.

Agarwal *et al*. [6] have proposed an analytical model to the problem of dispersion of an air pollutant with variable wind velocity.

Kumar *et al*. [7] have presented an analytical model for the dispersion of an air pollutant released from a continuous source in the atmospheric boundary layer describing the crosswind- integrated concentration. They have described an analytical scheme to solve the resulting two dimensional steady state advection diffusion equations by taking horizontal wind speed as a generalized function of vertical distance from the source and vertical height.

Verma [8] have presented an analytical approach to the problem of dispersion of an air pollutant with constant wind velocity and constant removal rate by taking eddy diffusivities as constant. He obtained that the concentration profile of air pollutant decreases continuously with increasing downwind distance while it increases with increasing vertical distance.

Sharan *et al*. [9] have given a mathematical model for the dispersion of air pollutants in low wind conditions by assuming constant eddy diffusivity coefficients in the advection diffusion equation. Sharan [10] have given a steady-state mathematical model for the dispersion of air pollutants in low winds by taking into account the diffusion in the three coordinate directions and advection along the mean wind. They obtained analytical solution by assuming constant eddy diffusivities coefficients in the advection diffusion equation.

Srivastava *et al*. [11] have presented a three dimensional atmospheric diffusion model with variable removal rate and variable wind velocity using power law profile. They found that the concentration profile of pollutants becomes high near the ground but as the distance from the ground (either vertical or cross wind or downwind) increases, the concentration profile decreases regularly.

Khaled *et al*. [12] have solved two-dimensional advection diffusion equation to obtain the crosswind integrated concentration.

Verma *et al*. [13] have presented an analytical approach to the problem of dispersion of an air pollutant with variable wind velocity and variable eddy diffusivity. They have used wind velocity in the form of wave function.

Hanna *et al*. [14] have mentioned different types of atmospheric diffusion models in different conditions in Handbook of atmospheric diffusion.

Costa *et al*. [15] has given a three-dimensional solution of the steady state advection diffusion equation considering a vertically inhomogeneous planetary boundary layer. They used the generalized integral transform technique [GITT].

Marrouf and *et al*. [16] have solved the advection diffusion equation analytically using separation of variable techniques, considering first the wind speed and eddy diffusivity as constants, second as variables dependent on vertical height z and have compared between predicted two models observed concentration.

The effect of various parameters on the associated results is given by many researchers, but no attempts have been made by any researcher on the dispersion of air pollutants with constant removal rate and variable wind velocity using power law profile in the steady state condition. Therefore, in this paper, we have made an attempt to the solution of dispersion equation for the concentration of air pollutions emitted from a point source with constant removal rate and variable wind velocity in the steady state condition where eddy diffusivity coefficients are taken as constant. The methods of separation of variables and power series technique have been used for the solution of the problem.

Mathematical Problem

The dispersion of air pollutants may be described by the following partial differential equation:

$$U(z)\frac{\partial C}{\partial x} = K_y\frac{\partial^2 C}{\partial y^2} + K_z\frac{\partial^2 C}{\partial z^2} - \alpha C \qquad (1)$$

where x, y, z are the Cartesian co-ordinates, C is the concentration of the air pollutant; Ky and K_z are the eddy diffusivities in y- and z- directions respectively which are assumed to be constants , α is the removal rate of the air pollutant due to some natural mechanism like chemical reaction, which is also taken to be constant , U(z) is the variable wind velocity which varies with the vertical distance and taken in the form: U(z) = $\frac{U_H Z^p}{H^p}$, where U_H is wind velocity at a reference height H. For simplicity of the problem, we take $p = \frac{1}{2}$.
Thus, we may take U(z) = $\frac{U_H Z^p}{H^p} = \frac{U_H Z^{1/2}}{H^{1/2}}$.

To solve the dispersion equation (2.1), the following boundary conditions are taken into account:

$$C(x, y, z) = \frac{Q\delta(y)\delta(z-h_s)}{U(z)}, \qquad x=0,\ 0 \le \ _s \le H \qquad (2)$$

$$C(x, y, z) = 0, \qquad y \to \pm\infty \qquad (3)$$

$$C(x, y, z) = 0, \qquad z=0 \qquad (4)$$

$$C(x, y, z) = 0, \qquad z=H \qquad (5)$$

where Q is the emission strength of an elevated point source, δ is the Dirac delta-function, h_s is the stack height . Condition (2) states that the air pollutant is released from the elevated point source of strength Q. Condition (2.3) states that the concentration of the air pollutant is zero for y→ $\pm\infty$.Conditions (4) and (5) state that the concentration of the air pollutant is zero at the ground surface and at the vertical height H from the ground surface.

Using power law profile, the dispersion equation (1) can be written as

$$\frac{U_H Z^p}{H^p}\frac{\partial C}{\partial X} = K_y\frac{\partial^2 C}{\partial y^2} + K_z\frac{\partial^2 C}{\partial z^2} - \alpha C \qquad (6)$$

Method of Solution

The partial differential equations (6) along with boundary conditions (2) – (5) are made dimensionless by considering the following dimensionless quantities:

$$\bar{x} = \frac{K_Z X}{U_H H^2}, \quad \bar{y} = \frac{y}{H}, \quad \bar{z} = \frac{z}{H}, \quad \overline{\ \ }_s = \frac{h_s}{H}, \quad \bar{c} = \frac{U_H H^2 C}{Q},$$

$$\bar{\alpha} = \frac{H^2\alpha}{K_z}, \quad \bar{Q}' = \frac{Q'}{Q}, \quad \delta(\bar{y}) = H\delta(y), \quad \delta(\bar{z} \ \ \overline{\ \ }_s) = H\delta(z \ \ _s).$$

On dropping the bar ( ), the dispersion equation along with boundary conditions become:

$$z^{1/2}\frac{\partial c}{\partial x} = \beta\frac{\partial^2 c}{\partial y^2} + \gamma\frac{\partial^2 c}{\partial z^2} \quad \alpha c \quad \text{, where} \quad \beta = \frac{K_y}{K_z} \qquad (1)$$

$$C = \frac{Q'\delta(y)\delta(z-h_s)}{z^{1/2}}, \qquad x = 0 \qquad (2)$$

$$C = 0 \quad , \qquad y \to \pm\infty \qquad (3)$$

$$C = 0 \quad , \qquad z=0 \qquad (4)$$

$$c = 0 \quad , \qquad z = 1 \qquad (5)$$

To solve equation (1) with boundary conditions (2) (5),we use method of separation of variables for which we assume the following trial solution:

$$C(x, y, z) = L(x)M(y)N(z) \qquad (6)$$

where $L(x)$,$M(y)$ *and* $N(z)$ are lonely functions of x, y and z respectively.
Using the above assumption (3.6) in (3.1), we get

$$z^{1/2}\frac{\partial}{\partial x}\{L(x)M(y)N(z)\} = \beta\frac{\partial^2}{\partial y^2}\{L(x)M(y)N(z)\} + \frac{\partial^2}{\partial z^2}\{L(x)M(y)N(z)\} \quad \alpha\{L(x)M(y)N(z)\}$$

or $\quad z^{1/2}L'MN = \beta LM''N + LMN'' \quad \alpha\ LMN$

or $\quad z^{1/2}L'MN = L(\beta M''N + MN'' \quad \alpha\ MN)$

or $\quad \frac{L'}{L} = \frac{\beta M''N + MN'' - \alpha\ MN}{z^{1/2}MN} = \lambda^2$,say

Now, by taking first and third ratios of the above, we have

$\frac{L'}{L} = \lambda^2$ or $L' = \lambda^2 L$ (7)

Similarly, by taking second and third ratios of the above, we have

$\frac{\beta M''N + MN'' - \alpha MN}{z^{1/2}MN} = \lambda^2$ or $\beta M''N + MN'' \quad \alpha MN = \lambda^2 z^{1/2} MN$

or $\beta M''N + MN'' \quad \alpha MN + \lambda^2 z^{1/2} MN = 0$

or $\beta M''N + M(N'' \quad \alpha N + \lambda^2 z^{1/2} N) = 0$

or $\beta \frac{M''}{M} = (\frac{N'' - \alpha N + \lambda^2 z^{1/2} N}{N}) = \eta^2$,say

Again, by taking first and third ratios of the above, we have

or $\frac{M''}{M} = \frac{\eta^2}{\beta}$ or $M'' = \left(\frac{\eta^2}{\beta}\right) M$ (8)

Similarly, by taking second and third ratios of the above, we have

$\left(\frac{N'' - \alpha N + \lambda^2 z^{1/2} N}{N}\right) = \eta^2$ or $N'' \quad \alpha N + \lambda^2 z^{1/2} N = N\eta^2$

or $N'' \quad \alpha N + \lambda^2 z^{1/2} N + N\eta^2 = 0$

or $N'' \quad \left(\alpha \quad \lambda^2 z^{1/2} \quad \eta^2\right) N = 0$ (9)

where λ^2 *and* η^2 both are constants and are known as separation constants.
The solution of equations (3.7) and (3.8) are given by

$L(x) = c_1 \, e^{(-\lambda^2 x)}$ (10)

$M(y) = c_2 \, (e^{-\left(\frac{\eta}{\sqrt{\beta}}\right)y} + e^{\left(\frac{\eta}{\sqrt{\beta}}\right)y})$ (11)

where c_1 *and* c_2 are arbitrary constants of integration.

Now, using the boundary condition (3) in (11), we get

$M(y) = c_2[H(y)e^{-\left(\frac{\eta}{\sqrt{\beta}}\right)y} + H(\quad y)e^{\left(\frac{\eta}{\sqrt{\beta}}\right)y}]$ (12)

where H(y) is the unit step function given by H(y) = $\begin{cases} 0, & if\ y < 0 \\ 1, & if\ y > 0 \end{cases}$

To find the solution of (3.9), the method of power series is employed and so rearranging

$N'' \quad \left(\alpha \quad \lambda^2 z^{1/2} \quad \eta^2\right) N = 0$, we have

$N'' \quad [(\alpha \quad \eta^2) \quad \lambda^2 z^{1/2}] N = 0$

Now, let $A = \alpha \quad \eta^2$ and $B = \lambda^2$, therefore, the above equation (9) becomes

$N'' \quad [A \quad Bz^{1/2}] N = 0$

or $\sum_{n=0}^{\infty} (n+2)(n+1)c_{n+2} z^n \quad \left(A \quad Bz^{1/2}\right) \sum_{n=0}^{\infty} c_n z^n = 0$, $w \; ere$ $N = \sum_{n=0}^{\infty} c_n z^n$

or $\sum_{n=0}^{\infty} [(n+2)(n+1)c_{n+2} \quad \left(A \quad Bz^{1/2}\right) c_n]z^n = 0$

which gives us the recurrence formula $c_{n+2} = \frac{\left(A - Bz^{1/2}\right)c_n}{(n+2)(n+1)}$

Now, using this formula in the power series

$N(z) = c_0 + c_1 z + c_2 z^2 + c_3 z^3 + c_4 z^4 + c_5 z^5 + c_6 z^6 + \ldots\ldots\ldots$, by taking n = 0, 1, 2, 3….., we get

$$\mathrm{N}(\mathrm{z}) = c_0 + c_1 z + \frac{\left(A-Bz^{1/2}\right)c_0}{2!}\mathrm{z}^2 + \frac{\left(A-Bz^{1/2}\right)c_1}{3!}\mathrm{z}^3 + \frac{\left(A-Bz^{1/2}\right)c_0}{4!}\mathrm{z}^4 + \frac{\left(A-Bz^{1/2}\right)c_1}{5!}\mathrm{z}^5 + \frac{\left(A-Bz^{1/2}\right)c_0}{6!}\mathrm{z}^6 + \ \ldots$$

$$= c_0\left[1 + \frac{\left(A-Bz^{1/2}\right)c_0}{2!}\mathrm{z}^2 + \frac{\left(A-Bz^{1/2}\right)c_0}{4!}\mathrm{z}^4 + \frac{\left(A-Bz^{1/2}\right)c_0}{6!}\mathrm{z}^6 + \ \ldots.\right]$$

$$+ c_1\left[z + \frac{\left(A-Bz^{1/2}\right)c_1}{3!}\mathrm{z}^3 + \frac{\left(A-Bz^{1/2}\right)c_1}{5!}\mathrm{z}^5 + \ \ldots\ldots.\right]$$

$$= c_0\left[1 + \frac{A}{2}\mathrm{z}^2 \quad \frac{B}{2}\mathrm{z}^{5/2} + \frac{\mathrm{A}^2}{24}\mathrm{z}^4 \quad \frac{AB}{12}\mathrm{z}^{9/2} + \frac{\mathrm{B}^2}{24}\mathrm{z}^5 + \quad .\right]$$

$$+ c_1\left[z + \frac{A}{6}\mathrm{z}^3 \quad \frac{B}{6}\mathrm{z}^{7/2} + \frac{\mathrm{A}^2}{120}\mathrm{z}^5 \quad \frac{AB}{60}\mathrm{z}^{11/2} + \frac{\mathrm{B}^2}{120}\mathrm{z}^6 + \quad \right]$$

$$= k_1\left[1 + \frac{\left(\alpha - \eta^2\right)}{2}\mathrm{z}^2 \quad \frac{\lambda^2}{2}\mathrm{z}^{5/2} + \frac{\left(\alpha - \eta^2\right)^2}{24}\mathrm{z}^4 \quad \frac{\left(\alpha - \eta^2\right)\lambda^2}{12}\mathrm{z}^{9/2} + \frac{\mathrm{B}^2}{24}\mathrm{z}^5 + \quad .\right]$$

$$+ k_2\left[z + \frac{(\alpha - \eta^2)}{6}\mathrm{z}^3 \quad \frac{\lambda^2}{6}\mathrm{z}^{7/2} + \frac{(\alpha - \eta^2)^2}{120}\mathrm{z}^5 \quad \frac{(\alpha - \eta^2)\lambda^2}{60}\mathrm{z}^{11/2} + \frac{\lambda^4}{120}\mathrm{z}^6 + \quad \right]$$

where $c_0 = k_1$ and $c_1 = k_2$ are constants.

The above value of N (z) can also be put in the following form:

$$\mathrm{N}(z) = k_1[\,1 + 0.5\left(\alpha \quad \eta^2\right)z^2 \quad 0.5\lambda^2 z^{5/2} + 0.041\left(\alpha \quad \eta^2\right)^2 z^4 \qquad]$$
$$+ \ k_2\,[z + 0.166\left(\alpha \quad \eta^2\right)z^3 \quad 0.166\lambda^2 z^{7/2} + 0.0083\left(\alpha \quad \eta^2\right)^2 z^5 \qquad] \tag{13}$$

where k_1 and k_2 are constants.
Now, using boundary condition C = 0 at z = 0, we get $k_1 = 0$
Therefore, the above solution becomes

$$\mathrm{N}(z) = k_2\,[z + 0.166\left(\alpha \quad \eta^2\right)z^3 \quad 0.166\lambda^2 z^{7/2} + 0.0083\left(\alpha \quad \eta^2\right)^2 z^5 \qquad] \tag{14}$$

Again, using the condition C = 0 at z = 1, we get the following eigen value equation:

$$1 + 0.166\left(\alpha \quad \eta^2_m\right) \quad 0.166\lambda_m^2 + 0.0083\left(\alpha \quad \eta^2_m\right)^2 \quad 0.0166\left(\alpha \quad \eta^2_m\right)\lambda_m^2 + 0.0083\left(\alpha \quad \eta^2_m\right)^2 + \quad] = 0$$

Since the above equation is uniformly convergent in [0, 1] for $\lambda^2 \leq \eta^2$, therefore, we take the value of separation constants so as $\frac{\lambda^2}{\eta^2} \leq 1$.

Putting the values from (3.10), (3.12) and (3.14) in equation (3.6), we get

$$\mathrm{C} = c_1\,\mathrm{e}^{(-\lambda^2 x)}c_2[H(y)\mathrm{e}^{-\left(\frac{\eta}{\sqrt{\beta}}\right)y} + H(\ \ y)\mathrm{e}^{\left(\frac{\eta}{\sqrt{\beta}}\right)y}]\,k_2\,[z + 0.166\left(\alpha \quad \eta^2\right)z^3 \quad 0.166\lambda^2 z^{7/2} + 0.0083\left(\alpha \quad \eta^2\right)^2 z^5 \qquad]$$

$$= c_1 c_2 k_2 \mathrm{e}^{(-\lambda_m{}^2 x)}[H(y)\mathrm{e}^{-\left(\frac{\eta}{\sqrt{\beta}}\right)y} + \ H(\ \ y)\mathrm{e}^{\left(\frac{\eta}{\sqrt{\beta}}\right)y}]\ [z + 0.166\left(\alpha \quad \eta^2\right)z^3 \quad 0.166\lambda^2 z^{7/2} + 0.0083\left(\alpha \quad \eta^2\right)^2 z^5 \qquad]$$

$$or\ \ \mathrm{C} = \sum_{m=1}^{\infty} k_m\,\mathrm{e}^{(-\lambda_m{}^2 x)}[H(y)\mathrm{e}^{-\left(\frac{\eta}{\sqrt{\beta}}\right)y} + H(\ \ y)\mathrm{e}^{\left(\frac{\eta}{\sqrt{\beta}}\right)y}]\ f_m(z)\ , \quad \text{where } k_m = c_1.c_2.k_2 \tag{15}$$

and $f_m(z) = z + 0.166\left(\alpha \quad \eta^2\right)z^3 \quad 0.166\lambda^2 z^{7/2} + 0.0083\left(\alpha \quad \eta^2\right)^2 z^5$

Again, using the boundary condition (3 .2), i.e., $C = \frac{Q'\delta(y)\delta(z-h_s)}{\mathrm{z}^{1/2}}$, $x = 0$, we have

$$Q'\delta(y)\delta(z \qquad _s) = \sum_{m=1}^{\infty} k_m\,\mathrm{e}^{(-\lambda^2 x)}[H(y)\mathrm{e}^{-\left(\frac{\eta_m}{\sqrt{\beta}}\right)y} + H(\ \ y)\mathrm{e}^{\left(\frac{\eta_m}{\sqrt{\beta}}\right)y}]\ f_m(z)\mathrm{z}^{1/2} \tag{16}$$

Multiplying both sides of the above by $f_n(z)$ and integrating over [0, 1], we get

$$\int_0^1 Q'\delta(y)\delta(z \qquad _s)\,f_n(z)dz = \sum_{m=1}^{\infty} k_m\,[H(y)\mathrm{e}^{-\left(\frac{\eta_m}{\sqrt{\beta}}\right)y} + H(\ \ y)\mathrm{e}^{\left(\frac{\eta_m}{\sqrt{\beta}}\right)y}]\ \int_0^1 f_m(z)\mathrm{z}^{1/2}\,f_n(z)dz$$

$$\text{Or}\ \ Q'\delta(y)\,f_n(\ \ _s) = \sum_{m=1}^{\infty} k_m\,[H(y)\mathrm{e}^{-\left(\frac{\eta_m}{\sqrt{\beta}}\right)y} + H(\ \ y)\mathrm{e}^{\left(\frac{\eta_m}{\sqrt{\beta}}\right)y}]\ \int_0^1 f_m{}^2(z)\mathrm{z}^{1/2}\,dz$$

Again, integrating it with respect to y from ∞ to ∞ , we get

$\int_{-\infty}^{\infty} Q'\delta(y)\, f_n(\ _s)dy = \sum_{m=1}^{\infty} k_m \int_{-\infty}^{\infty}[H(y)\, e^{-\left(\frac{\eta_m}{\sqrt{\beta}}\right)y} + H(\ \ y)e^{\left(\frac{\eta_m}{\sqrt{\beta}}\right)y}]\, dy \int_0^1 f_m{}^2(z)z^{1/2}\, dz$

Or $Q' f_n(\ _s).1 = k_m\, .2\frac{\sqrt{\beta}}{\eta_m}. \int_0^1 f_m{}^2(z)z^{1/2}\, dz$

Or $Q' f_m(\ _s)\ = k_m\, .2\frac{\sqrt{\beta}}{\eta_m}. \int_0^1 f_m{}^2(z)z^{1/2}\, dz$, where n=m

Therefore, we have $k_m = \left(\frac{\eta_m}{\sqrt{\beta}}\right) \frac{Q' f_m(h_s)}{2\int_0^1 f_m{}^2(z)z^{1/2}\, dz}$ (17)

Putting this value of k_m in equation (15) with $Q' = 1$, we get

$C(x, y, z) = \sum_{m=1}^{\infty} e^{(-\lambda^2 x)}\ [H(y)e^{-\left(\frac{\eta_m}{\sqrt{\beta}}\right)y} + H(\ \ y)e^{\left(\frac{\eta_m}{\sqrt{\beta}}\right)y}] \left(\frac{\eta_m}{\sqrt{\beta}}\right) \frac{f_m(z) f_m(h_s)}{2\int_0^1 f_m{}^2(z)z^{1/2}\, dz}$ (18)

where $f_m(z) = z + 0.166\left(\alpha \quad \eta^2\right)z^3 \quad 0.166\lambda^2 z^{7/2} + 0.0083\left(\alpha \quad \eta^2\right)^2 z^5$

RESULTS AND DISCUSSION

In order to study the dispersion of air pollutants considered in the model, concentration profiles have been calculated for the various conditions using equation (3.18). The parametric values used in the analysis are taken as follows:

$\alpha = 2, \quad \beta = 10, \quad \gamma = 1, \quad _s = 0.2, \quad H = 1, \quad \varepsilon = 0.005, \quad \lambda = 1$

In figure 1, the concentration profile is plotted against the downwind distance (x=0.1, 0.2, 0.3…) keeping the crosswind distance fixed at the value (y=±1) for different values of vertical distances (z=0.2, 0.3, and 0.4). From the graph, it is found that the concentration profile increases upto certain downwind distance and then decreases and it is seen that the concentration level of the air pollutants become high near the ground.

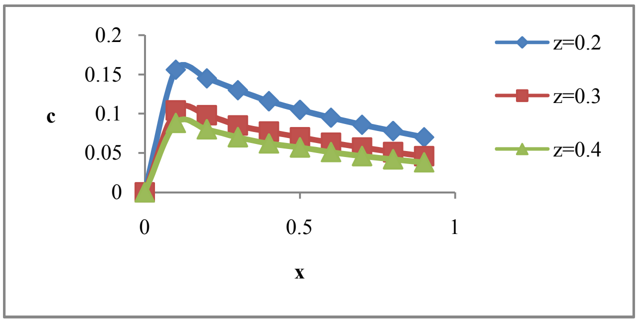

Fig-1 Variation of dimensionless concentration C(x, ±1, z) with dimensionless downwind distance x for different z.

In figure 2, the concentration profile is plotted with respect to crosswind distance (0≤ $y \leq 1$) for different values of downwind distances (x=0.1, 0.3, 0.5).From the graph, it is found that the concentration profile is slightly changed upto certain x and thereafter the concentration profile is almost following uniform distribution with increasing downwind distance($x \geq 0.5$).

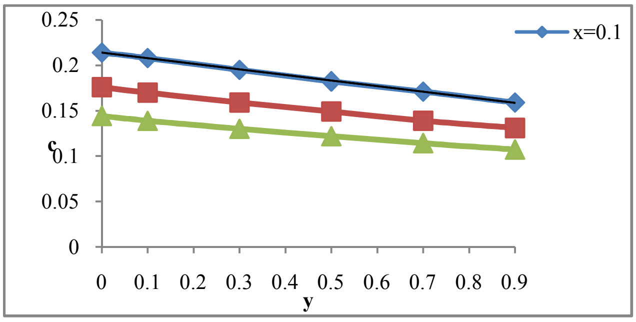

Fig-2 Variation of dimensionless concentration C(x, y, 0.2) with dimensionless cross-wind distance 0≤ $y \leq 1$ for different x.

In figure 3, the concentration profile is plotted with respect to crosswind distance (-1≤ $y \leq 0$) for different values of downwind distances (x=0.1, 0.3, 0.5). From the graph, it is seen that the concentration profile is slightly changed upto certain x and thereafter the concentration profile is almost following uniform distribution with increasing downwind distance ($x \geq 0.5$).

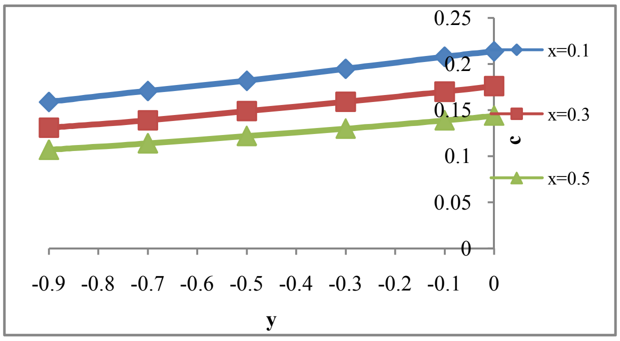

Fig-3 Variation of dimensionless concentration C(x, y, 0.2) with dimensionless Cross-wind distance $-1 \leq y \leq 0$ for different x.

In figure 4, the concentration profile is plotted against the downwind distance (x=0.1, 0.2, 0.3…) for different values of crosswind distances (y=0.1, 0.3, and 0.5). From the graph, it is found that the concentration profile increases upto certain downwind distance and then decreases and it is seen that the concentration level of the air pollutants become high near the cross-wind distance.

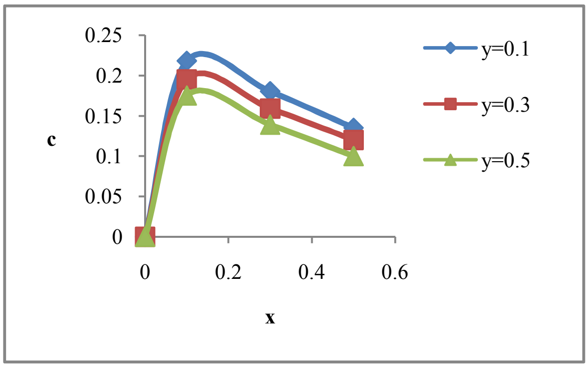

Fig-4 Variation of dimensionless concentration C(x, y, 0.2) with dimensionless downwind distance x for different y.

CONCLUSION

In this study, an analytical approach is used for dispersion of air pollutants for constant removal rate and variable wind velocity. The wind velocity is assumed to vary with the power of vertical height .The model is solved by using power series technique and the method of separation of variables. It is found that the concentration profile of the air pollutants near the ground is high and it decreases as the distance (vertical or cross-wind or down-wind) from the ground increases.

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