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SELECTION OF MIXED SAMPLING PLAN USING INTERVENED RANDOM EFFECT POISSON DISTRIBUTION

[1]Sampath Kumar, R., [2]Vijaya Kumar, R and [3]Radhakrishnan, R

[1]Government Arts College, Coimbatore - 18
[2]SSM College of Arts and Science, Komarapalayam
[3]PSG College of Arts and Science, Coimbatore – 14

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ABSTRACT

This paper presents the procedure for the construction and selection of mixed sampling plan (MSP) using Intervened Random Effect Poisson Distribution (IRPD) as a baseline distribution. Having the single sampling plan as attribute plan, the plans are constructed through acceptable quality level (AQL) and maximum allowable percent defective (MAPD). Tables are constructed for easy selection of the plan.

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INTRODUCTION

Mixed sampling plans consist of two stages of rather different nature. During the first stage the given lot is considered as a sample from the respective production process and a criterion by variables is used to check process quality. If process quality is judged to be sufficiently good, the lot is accepted. Otherwise the second stage of the sampling plan is entered and lot quality is checked directly by means of an attribute sampling plan.

There are two types of mixed sampling plans called independent and dependent plans. If the first stage sample results are not utilized in the second stage, then the plan is said to be independent otherwise dependent. The principal advantage of mixed sampling plan over pure attribute sampling plan is a reduction in sample size for a similar amount of protection.

Schiling (1967) proposed a method for determining the operating characteristics of mixed variables – attributes sampling plans, single sided specification and standard deviation known using the normal approximation. The mixed sampling plans have been designed under two cases of significant interest. In the first case, the sample size n_1 is fixed and a point on the OC curve is given. In the second case, plans are designed when two points on the OC curve are given. Devaarul (2003) has studied the mixed sampling plans and reliability based sampling plans. Radhakrishnan and Sampath Kumar (2006, 2007a, b, c, and 2009) have constructed the mixed sampling plans using Poisson distribution as a baseline distribution. Sampath Kumar (2007) has constructed mixed variables – attributes sampling plans indexed through various parameters. Radhakrishnan et.al (2010) has made contributions to mixed sampling plans.

In the product control, the defective units are either rebuilt or replaced by new units during the sampling period. Quality engineers are always interested in improving the quality level of product to enhance the satisfaction of the customers and hence, they keep making changes in the production process. These actions trigger a change in the expected incidence of defective items in the remaining observational period. Any action for reducing the number of defectives during the sampling period is called an intervention and such intervention parameter ranges from 0 to 1.

In Intervened Random effect Poisson Distribution (IRPD), Poisson parameter is modified in two ways: one method is multiplying an intervention parameter ρ(a constant) and secondly, multiplying an unobserved random effect which follows Gamma probability distribution. The IRPD can be very useful to the quality and reliability engineers, who always make changes in the production system in the observational period of quality checking to ensure reliability of the system, because, the failure rate of the components may vary in different time intervals. The other areas of application of IRPD are queuing, demographic studies, and process control and so on.

Shanmugam (1985) has used Intervened Poisson Distribution (IPD) in the place of Zero Truncated Poisson Distribution (ZTPD) for the study on cholera cases. Radhakrishnan and Sekkizhar (2007a, 2007b, and 2007c) introduced intervened random effect Poisson distribution in

* Corresponding author: +91
E-mail address: rkrishnan_cbe@yahoo.com

the place of Poisson distribution for the construction of attribute sampling plans.

In this paper, using the operating procedure of mixed sampling plan (independent case) with single sampling plan as attribute plan, tables are constructed using IRPD as a baseline distribution. The tables are constructed for mixed sampling plan (MSP) indexed through i) AQL ii) MAPD. The plan indexed through MAPD is compared with the plan indexed through AQL.

Conditions for Applications of IRPD - Mixed sampling plan

- Production process is modified during the sampling inspection by an intervention.
- Lots are submitted substantially in the order of their production.
- Inspection is by variable in the first stage and attribute in the second stage with quality defined as the fraction defective. Lot quality variation exists

Glossary of symbols

The symbols used in this paper are as follows:

p : submitted quality of lot or process.

$P_a(p)$: probability of acceptance for given quality ' p '

p_1 : submitted quality such that $P_a(p_1)$ = 0.95 (also called AQL)

p_* : maximum allowable percent defective (MAPD)

n : sample size for each lot.

c : sample acceptance number.

d : number of defectives in the sample.

n_1 : sample size for variable sampling plan.

n_2 : sample size for attribute sampling plan.

β_j : probability of acceptance for the lot quality ' p_j '

β_j' : probability of acceptance assigned to first stage for percent defective ' p_j '

β_j'' : probability of acceptance assigned to second stage for percent defective ' p_j '

z (j) : 'z' value for the j^{th} ordered observation.

k : variable factor such that a lot is accepted if $\bar{X} \leq A = U - k\sigma$

Operating procedure of Mixed Sampling Plan having Single Sampling Plan as attribute plan

Bowker and Goode (1952)' has been incorporate, Schilling (1967) suggested the following procedure for the independent mixed sampling plan.

- Select a random sample of size n_1 from the lot assumed to be large.
- If a sample average $\bar{X} \leq A$, accept the lot
- If a sample average $\bar{X} > A$, take a second sample of size n_2.
- Inspect and find the number of defectives in the second sample.
  (i) If the number of defectives $d \leq c$, accept the lot.
  (ii) If the number of defectives $d > c$, reject the lot.

Construction of Mixed Sampling Plan having Single Sampling Plan as attribute plane using IRPD

The operation of Mixed Sampling plans can be properly assessed by the OC curve for given values of the fraction defective. The development of mixed sampling plans and the subsequent discussions are limited only to the upper specification limit 'U'. By symmetry, a parallel discussion can be made for lower specification limits.

The procedure for the construction of mixed variables – attributes sampling plans is provided by Schilling (1967) for a given 'n_1' and a point ' p_j ' on the OC curve is given below.

- Assume that the mixed sampling plans are independent.
- Split the probability of acceptance (β_j) determining the probability of acceptance that will be assigned to the first stage. Let it be β_j'.
- Decide the sample size n_1 (for variable sampling plan) to be used
- Calculate the acceptance limit for the variable sampling plan as

$A = U - k\sigma = U - [z(p_j) + \{z(\beta_j')/\sqrt{n_1}\}]\sigma$, where U is the upper specification limit and z(t) is the standard normal variate corresponding to 't' such that t

$$= \int_{z(t)}^{\infty} \left(\frac{1}{\sqrt{2\pi}}\right) e^{-u^2/2}\, du$$

- Determine the sample average $\bar{X}$. If a sample average $\bar{X} > A = U - k\sigma$, take a second stage sample size 'n_2' using attribute sampling plan.
- Now determine β_j'', the probability of acceptance assigned to the attributes plan associated with the second stage sample as $\beta_j'' = (\beta_j - \beta_j')/(1 - \beta_j')$
- Determine the appropriate second stage sample size 'n_2' from
- $P_a(p) = \beta_j''$ for $p = p_j$
- Using the above procedure, tables can be constructed to facilitate easy selection of mixed sampling plan with single sampling plan as attribute plans using IRPD as a baseline distribution indexed through AQL and MAPD

Radhakrishnan and Sekkizhar (2007a, 2007b, and 2007c) suggested the probability mass function of the IRPD as

$$P_a(p) = \sum_{x=0}^{c}\left[\frac{e^{-\theta}\theta^x}{(1+\rho\theta)^{\alpha}} \sum_{l=0}^{x}\left(\frac{\rho}{1+\rho}\right)^{l} \frac{(\alpha+l-1)!}{l!(x-l)!(\alpha-1)!}\right],$$

where $\theta = \left(\frac{np}{1+\rho}\right)$

when α=1, $P_a(p) = \sum_{x=0}^{c}\left[\frac{e^{-\theta}\theta^x}{(1+\rho\theta)}\sum_{l=0}^{x}\left(\frac{\rho}{1+\rho}\right)^l\frac{1}{(x-l)!}\right]$,

where $\theta = \left(\frac{np}{1+\rho}\right)$ (1)

Using the above procedure, tables can be constructed to facilitate easy selection of MSP using IRPD as a baseline distribution. The tables furnished in this paper are for the case when α=1.

Construction of mixed sampling plans indexed through MAPD

MAPD, introduced by Mayer (1967) and studied by Soundararajan (1975) is the quality level corresponding to the inflection point of the OC curve. The degree of sharpness of inspection about this quality level 'p' is measured by 'p_t', the point at which the tangent to the OC curve at the inflection point cuts the proportion defectives. For designing, Soundararajan (1975) proposed a selection procedure for SSP indexed with MAPD and $R = \frac{p_t}{p_*}$

Using the probability mass function of the IRPD, given in expression (1), the inflection point (p_*) is obtained by using $\frac{d^2P_a(p)}{dp^2} = 0$ and $\frac{d^3P_a(p)}{dp^3} \neq 0$. The n_2MAPD values are calculated for different values of c and ρ for $\beta_*' = 0.30$ using c++ program and presented in Table 1.

Table 1 n_2MAPD values for different values of ρ and c when $\beta_*' = 0.30$

c	ρ 0.9	0.8	0.7	0.6	0.5	0.4
1	1.1808	1.2797	1.0339	1.5292	1.6883	1.8771
2	2.3559	2.5579	2.7913	3.0673	3.4021	3.8262
3	3.5609	3.8660	4.2210	4.6476	5.1793	5.9062
4	4.7942	5.2065	5.6880	6.2703	7.0264	8.1704
5	6.0552	6.5785	7.1902	7.9392	8.9577	10.7296
6	7.3378	7.9774	8.7217	9.6521	10.9822	13.8965
7	8.6401	9.3954	10.2833	11.4045	13.1171	19.6675
8	9.9557	10.8378	11.8694	13.2008	15.3849	-
9	11.2826	12.2913	13.4787	14.5615	17.8467	-
10	12.6195	13.7627	15.1068	16.9206	20.5885	-

The MAAOQ (Maximum Average Outgoing Quality) of a Sampling Plan is designed as the Average Outgoing Quality (AOQ) at the MAPD.

By definition AOQ = $p\ P_a(p)$ and

$$\text{MAAOQ} = p_*\ P_a(p_*)$$

The values of MAPD and MAAOQ are calculated for different values of c and ρ for $\beta_*' = 0.30$ and the ratio $R = \frac{MAAOQ}{MAPD}$ is presented in Table 2.

Table 2 R values for different values of ρ and c when $\beta_*' = 0.30$

c	ρ 0.9	0.8	0.7	0.6	0.5	0.4
1	0.6821	0.6492	0.6114	0.5670	0.5161	0.4581
2	0.6111	0.5652	0.5124	0.4514	0.3812	0.3010
3	0.5721	0.5181	0.4557	0.3834	0.3008	0.2010
4	0.5451	0.4848	0.4150	0.3347	0.2430	0.1397
5	0.5240	0.4585	0.3828	0.2960	0.1971	0.0871
6	0.5068	0.4367	0.3562	0.2637	0.1590	0.0431
7	0.4922	0.4184	0.3332	0.2361	0.1264	0.0072
8	0.4798	0.4021	0.3131	0.2117	0.0981	-
9	0.4691	0.3881	0.2952	0.1898	0.0728	-
10	0.4597	0.3754	0.2792	0.1072	0.0502	-

Selection of the plan

Tables 1 and 2 are used to construct the plan when ρ, MAPD and MAAOQ are given. For any given values of ρ, MAPD and MAAOQ one can find the ratio R. From Table 2, for a given value of ρ the nearest value of 'R' is found out and c value is noted. Using the values of 'c' and ρ, one can find the value of 'n_2' from Table 1 as $n_2 = \frac{n_2 MAPD}{MAPD}$.

Example 1: Given ρ=0.5, MAPD=0.028 and MAAOQ=0.010. Find the ratio $R = \frac{MAAOQ}{MAPD}$ =0.3571. Select the nearest value of R from Table 2 as 0.3812 which is associated with c=2. For the values of c=2, ρ=0.5 and MAPD=0.028, from Table 1, the second stage sample size $n_2 = \frac{n_2 MAPD}{MAPD} = \frac{3.4021}{0.028} = 121$. Thus n_2=121, c=2 and ρ=0.5 are the parameters selected for the mixed sampling plan having SSP as attribute plan for a specified ρ=0.5, MAPD=0.028 and MAAOQ =0.010 by taking IRPD as a baseline distribution.

Construction of mixed sampling plans indexed through AQL

The procedure given in section 5 is used for constructing the mixed sampling plan indexed through AQL (p_1). By assuming the probability of acceptance of the lot be β_1 =0.95 and β_1' =0.30, the $n_2 p_1$ values are calculated for different values of 'c' and 'ρ' using c++ program and is presented in Table 3.

Selection of the plan for a given AQL, c and ρ

Table 3 is used to construct the plans when AQL (p_1) and c are given. For any given values of p_1, c and ρ one can determine n_2 value using $n_2 = \frac{n_2 p_1}{p_1}$.

Example 2: Let the probability of acceptance of the lot be β_1=0.95 and β_1'=0.30. For the given values of p_1 = 0.00763 ρ=0.9 and c=4 from Table 3, the second stage sample size

Table 3 n_2AQL values for different values of ρ and c when β_1=0.95 and β_1' =0.30

c	ρ								
	0.9	**0.8**	**0.7**	**0.6**	**0.5**	**0.4**	**0.3**	**0.2**	**0.1**
1	0.4161	0.4182	0.4205	0.4229	0.4254	0.4280	0.4306	0.4331	0.4352
2	0.8720	0.8800	0.8885	0.8977	0.9075	0.9179	0.9284	0.9385	0.9470
3	1.3744	1.3910	1.4090	1.4285	1.4496	1.4720	1.4951	1.5177	1.5368
4	1.8997	1.9268	1.9565	1.9891	2.0245	2.0626	2.1024	2.1417	2.1751
5	2.4376	2.4766	2.5198	2.5675	2.6198	2.6764	2.7362	2.7958	2.8471
6	2.9830	3.0349	3.0928	3.1572	3.2283	3.3060	3.3886	3.4718	3.5441
7	3.5331	3.5987	3.6722	3.7544	3.8460	3.9467	4.0547	4.1643	4.2604
8	4.0863	4.1660	4.2558	4.3569	4.4702	4.5957	4.7313	4.8698	4.9923
9	4.6416	4.7358	4.8424	4.9631	5.0992	5.2509	5.4159	5.5856	5.7371
10	5.1984	5.3074	5.4312	5.5720	5.7317	5.9109	6.1070	6.3101	6.4926

$n_2 = \frac{n_2 p_1}{p_1}$ = $\frac{1.8997}{0.00763}$ = 410. Thus n_2 = 410, ρ=0.9 and c =4 are the parameters selected for the mixed sampling plan for a specified p_1 = 0.00763, ρ=0.9 and c =4.

Table 4 Comparison of plans

Given values			INDEXED THROUGH MAPD		INDEXED THROUGH AQL	
MAPD	**MAAOQ**	**ρ**	**n_2**	**c**	**n_2**	**c**
0.022	0.012	0.9	218	4	410	4
0.072	0.03	0.8	130	7	146	7
0.038	0.02	0.6	40	1	50	1
0.028*	0.01	0.5	121	2	142	2

***** OC curves are drawn

Comparison of mixed sampling plan indexed through MAPD and AQL

In this section MSP indexed through MAPD is compared with MSP indexed through AQL by fixing the parameters c and β_j'.

For the specified values of ρ, MAPD and MAAOQ with the assumption $\beta_*' = 0.30$ one can find the values of c and n_2 indexed through MAPD. By fixing the values of c and n_2, find the value of p_1 by equating $P_a(p) = \beta_1$=0.95. Using β_1'=0.30, c and n_2 one can find the values of n_2 using $n_2 = \frac{n_2 p_1}{p_1}$ from Table 3. For different combinations of ρ, MAPD and MAAOQ the values of c, n_2 (indexed through MAPD) and c, n_2 (indexed through AQL) are calculated and presented in Table 4.

CONCLUSION

In this paper the construction of mixed sampling plan with single sampling plan as attribute plan indexed through the parameters MAPD and AQL are presented by taking IRPD as a baseline distribution. Further the plan indexed through MAPD is compared with the plan indexed through AQL. It is concluded from the study that the second stage sample size required for single sampling plan indexed through MAPD is

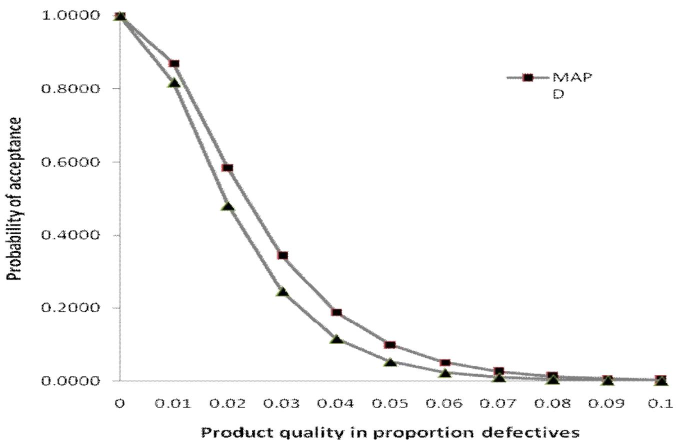

Fig 1 OC Curves for the plans (121, 2) and (142,2)

References

Bowker, A.H. and Goode, H.P,1952. Sampling Inspection by Variables, McGrawHill, Newyork.

Devaarul, S, 2003. Certain Studies Relating to Mixed Sampling Plans and Reliability Based Sampling Plans, Ph.D., Dissertation, Bharathiar University, Coimbatore, Tamil Nadu, India.

Mayer, P.L, 1967. A note on sum of Poisson probabilities and an application, Annals of Institute of Statistical Mathematics, Vol.19, pp.537-542.

Radhakrishnan, R. and Sampath Kumar, R, 2006. Construction of mixed sampling plan indexed through MAPD and AQL with chain sampling plan as attribute plan, STARS, Vol.7, No.1, June 2006, pp.14-22. .

Radhakrishnan, R. and Sampath Kumar, R, 2007a, b, c. Construction and comparison of mixed sampling plans having repetitive group sampling plan as attribute plan, National Journal of Technology, Vol. 4, No.3. pp. 1-6.

Radhakrishnan, R. and Sampath Kumar, R, 2009. Construction and comparison of mixed sampling plans having ChSP-(0,1) plan as attribute plan, *The* International Journal of Statistics and Management System, Vol.4, No.1-2, pp. 134-149.

Radhakrishnan, R. Sampath Kumar, R. and Malathi, M, 2010. Selection of mixed sampling plan with TNT-(n_1, n_2; 0) plan as attribute plan indexed through MAPD and

MAAOQ, International Journal of Statistics and System, Vol.5, No.4, pp. 477-484.

Radhakrishnan, R. and Sekkizhar, J, 2007a. Construction of sampling plans using intervened random effect Poisson distribution. The International Journal of Statistics and Management Systems, Vol.2, 1-2, pp 88-97.

Radhakrishnan, R. and Sekkizhar, J, 2007b. Construction of conditional double sampling plans using intervened random effect Poisson distribution, Proceedings volume of *SJYSDNS-2005*, Acharya Nagarjuna University, Guntur. pp.57-61.

Radhakrishnan, R. and Sekkizhar, J, 2007c. Application of intervened random effect Poisson distribution in process control plans, International Journal of Statistics and Systems. Vol. 2. No.1. pp.29-39.

Sampath Kumar, R, 2007. Construction and Selection of Mixed Variables – Attributes Sampling Plans, Ph.D., Dissertation, Bharathiar University, Coimbatore, Tamil Nadu, India.

Schilling, E.G, 1967. A General Method for Determining the Operating Charateristics of Mixed Variables – Attribute Sampling Plans Single Side Specifications, S.D. known, Ph.D Dissertation – Rutgers – The State University, New Brunswick, New Jersy.

Shanmugam, R, 1985. An intervened Poisson distribution and its medical applications, Biometrics, 41, 1025-1029.

Soundararajan, V, 1975. Maximum allowable percent defective (MAPD) single sampling inspection by attributes plan, Journal of Quality Technology, Vol.7, No.4, pp.173-182.

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