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RESEARCH ARTICLE

# FUZZY RETRIAL QUEUE WITH HETEROGENEOUS SERVICE AND GENERALISED VACATION

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## ABSTRACT

This work constructs the membership function of the system characteristics of a single server retrial queues with batch arrivals, two phase of heterogeneous service and a generalised vacation time under Bernouli schedule. Here the batch-arrival rate, batch size, retrial rate, service time, vacation time are all fuzzy numbers. The α-cut approach is used to transform a fuzzy queue into a family of conventional crisp queues in this context. By means of the membership functions of the system characteristics, a set of parametric non-linear programs is developed to describe the family of crisp-single server batch arrival queues. A numerical example is solved successfully to illustrate the validity of the proposed approach. Because of the system characteristics are expressed and governed by the membership function, the single server fuzzy batch arrival retrial queue with heterogeneous service and generalised vacation are represented more accurately and the analytic results are more useful for system designers and practitioners.



## INTRODUCTION

Retrial queues describe operation of many telecommunication networks e.g. the local and wide area networks with the random multiple access protocols, call centres etc. There has been rapid growth in the literature on the queuing systems with repeated attempts which are characterised by the following feature: When an arriving customer finds that all servers are busy and no waiting position is available the customers joins a virtual pool of blocked customer called orbit. The detailed overviews of the related references with retrial queues can be found in the recent book of Falin and Templeton[5] and the survey papers, Artalejo[1,2].

Most of the related studies are based on traditional queuing theory, is that the inter arrival times and service times are assumed to follow certain probability distribution. However, in practice their cases that these parameters may be obtained subjectively [8]. The fuzzy queues are much more realistic than commonly used crisp queues [3,4,8].

In this paper, we focus on single server fuzzy retrial queue with two phase of heterogeneous service under Bernoulli schedule and a fuzzy vacation time, fuzzy varying batch sizes and fuzzy parameters. Clearly when the arrival rate, service times, group size, vacation time and retrial rate are fuzzy the performance measure of the queue also is fuzzy as well. The basic idea is to apply Zadeh's extension principle [9,10,11], two pairs of mixed integer non linear programming models are formulated to calculate the lower and upper bounds of the α-cut of the system performance measure. The membership function of the system performance measure is derived analytically.

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### The Mathematical Model

Consider a queuing system in which customers arrive at single server facility in batches as a Poisson process with group arrival rate $\tilde{\lambda}$, where $\tilde{\lambda}$ is fuzzy number and the two phases of essential services with first essential service $\tilde{S}_1$, and second essential service $\tilde{S}_2$, where $\tilde{S}_1$, $\tilde{S}_2$ are fuzzy numbers. The batch size $\tilde{K}$ of arrival, and the retrial rate $\tilde{R}$ and the server vacation time $\tilde{S}_3$ are represented by fuzzy numbers. Using α-cut, the trapezoidal arrival size can be represented by different intervals of confidence be represented by $[t_{1\alpha}, t_{2\alpha}]$. Since probability distributions for the α-cuts can be represented by uniform distributions, we have

$$P(t_\alpha) = \frac{1}{t_{2\alpha} - t_{1\alpha}}, t_{1\alpha} \le t_\alpha \le t_{2\alpha} \quad \text{………. (1)}$$

Thus the mean of the distribution is

$$E(t_\alpha) = \int_{t_{1\alpha}}^{t_{2\alpha}} \frac{1}{t_{2\alpha} - t_{1\alpha}} t_\alpha dt_\alpha = \frac{1}{2}(t_{2\alpha} + t_{1\alpha}) \quad \text{…….. (2)}$$

Similarly, for the second moment, we have

$$E(t_\alpha{}^2) = \int_{t_{1\alpha}}^{t_{2\alpha}} \frac{1}{t_{2\alpha} - t_{1\alpha}} t_\alpha{}^2 dt_\alpha = \frac{t_{2\alpha}^3 - t_{1\alpha}^3}{3(t_{2\alpha} - t_{1\alpha})} \quad \text{…… (3)}$$

Using the well-known formula $V(t_\alpha) = E(t_\alpha^2) - (E(t_\alpha))^2$, the variance can be obtained as

$$\text{var}(t_\alpha) = \frac{1}{12}(t_{2\alpha} - t_{1\alpha})^2 \quad \text{……………………..… (4)}$$

Expected length of the orbit and expected waiting time of the customer in the orbit in terms of system parameters [7] are given by

$$E(L)=\frac{\lambda^2(E(X))^2\beta_2+\rho(E(X^2)/E(X)}{2(1-\rho)}+\frac{\lambda(\rho+E(X)-1)}{R(1-\rho)} \quad ........ (5)$$

$$E(W) = E(L)/\lambda E(X) \quad ..................(6)$$

In steady state, it is necessary that $\rho=\lambda E(X)(E(S_1)+E(S_2)+pE(S_3))<1$.

To extend the applicability of the single server batch arrival retrial heterogeneous service with generalised vacation queuing model, we allow for fuzzy specification of system parameters. In this model the group arrival rate λ first essential service time $S_1$, second essential service time $S_2$, server vacation time $S_3$, retrial rate R are represented approximately known and can be represented by convex fuzzy sets.

Let $\phi_{\tilde{\lambda}}(x)$ ,$\phi_{\tilde{S}_1}(u)$, $\phi_{\tilde{S}_2}(v)$, $\phi_{\tilde{S}_3}(w)$, $\phi_{\tilde{R}}(r)$, $\phi_{\tilde{K}}(k)$ denote the membership functions of $\tilde{\lambda}$, $\tilde{S}_1$, $\tilde{S}_2$, $\tilde{S}_3$, $\tilde{R}$ and $\tilde{K}$ respectively. We then have the following fuzzy sets.

$$\tilde{\lambda}=\{(x,\phi_{\tilde{\lambda}}(x))/x\in X\} \quad ..........................(7a)$$

$$\tilde{S}_1=\{(u,\phi_{\tilde{S}_1}(u))/u\in U\} \quad ..............................(7b)$$

$$\tilde{S}_2=\{(v,\phi_{\tilde{S}_2}(v))/v\in V\} \quad ............................ (7c)$$

$$\tilde{S}_3=\{(w,\phi_{\tilde{S}_3}(w))/w\in W\} \quad ........................... (7d)$$

$$\tilde{R}=\{(r,\phi_{\tilde{R}}(r))/r\in R\} \quad ................................ (7e)$$

$$\tilde{K}=\{k,\phi_{\tilde{K}}(k)/k\in K\} \quad ................................ (7f)$$

Where X,U,V,W,R,K are the crisp arrival sets of batch arrival, hetergenous service, vacation time, retrial rate and group size respectively.

Let f(x,u,v,w,r,k) denote the system characteristic of interest. Since $\tilde{\lambda}$, $\tilde{S}_1$, $\tilde{S}_2$, $\tilde{S}_3$, $\tilde{R}$ and $\tilde{K}$ are fuzzy numbers f( $\tilde{\lambda}$, $\tilde{S}_1$, $\tilde{S}_2$, $\tilde{S}_3$, $\tilde{R}$, $\tilde{K}$ ) is also a fuzzy number. Following Zadeh's extension principle (yager[9] and zadeh[10]), the membership function of the system characteristic f( $\tilde{\lambda}$, $\tilde{S}_1$, $\tilde{S}_2$, $\tilde{S}_3$, $\tilde{R}$, $\tilde{K}$ ) is defined as

$$\phi_{f(\tilde{\lambda},\tilde{S}_1,\tilde{S}_2,\tilde{S}_3,\tilde{R},\tilde{K})}(z) = \sup_{x\in X,u\in U,v\in V,w\in w,r\in R,k\in K,\rho<1}\min\{\phi_{\tilde{\lambda}}(x),\phi_{\tilde{S}_1}(u),\phi_{\tilde{S}2}(v),\phi_{\tilde{S}_3}(w),\phi_R(r),\phi_K(k)/ z=f(x,u,v,w,r,k)\} \quad ........................ (8)$$

Assume that the system characteristic of interest is the expected number of customers in the orbit. It follows from (5) that

$$f(x,u,v,w,r,k)=\frac{x^2(E(k))^2\beta_2+\rho(E^2(k))/E(k)}{2(1-\rho)}+\frac{x(\rho+E(k)-1)}{r(1-\rho)} \quad ....(9)$$

The membership function for the expected number of customers in the orbit is

$$\phi_{E(\tilde{L})}(z) = \sup_{x\in X,u\in U,v\in V,w\in w,r\in R,k\in K,\rho<1}\min\{\phi_{\tilde{\lambda}}(x),\phi_{\tilde{S}_1}(u),\phi_{\tilde{S}2}(v),\phi_{\tilde{S}_3}(w),\phi_R(r),\phi_K(k)/ z=f(x,u,v,w,r,k)\} \quad ..... ...........................(10)$$

Unfortunately, the membership function is not expressed in the usual form, making it very difficult to imagine its shape. In this paper we approach the representation problem using a mathematical programming technique. Parametric non linear programs are developed to find the α-cuts of f( $\tilde{\lambda}$, $\tilde{S}_1$, $\tilde{S}_2$, $\tilde{S}_3$, $\tilde{R}$, $\tilde{K}$ ) based on the extension principle.

**Parametric Non Linear Programming**

To re express the membership function $\phi_{E(\tilde{L})}(z)$ of E( $\tilde{L}$ ) in an understandable and usable form, we adopt Zadeh's approach which relies on α-cuts of $\tilde{L}$. Definitions of α-cuts of $\tilde{\lambda}$, $\tilde{S}_1$, $\tilde{S}_2$, $\tilde{S}_3$, $\tilde{R}$ and $\tilde{K}$ as crisp intervals as follows:

$$\lambda(\alpha)=[x_\alpha^L,x_\alpha^U]=[\min_{x\in X}\{x/\phi_{\tilde{\lambda}}(x)\geq\alpha\},\max_{x\in X}\{x/\phi_{\tilde{\lambda}}(x)\geq\alpha\}] \quad ..(11a)$$

$$S_1(\alpha)=[u_\alpha^L,u_\alpha^U]=[\min_{u\in U}\{u/\phi_{\tilde{S}_1}(u)\geq\alpha\},\max_{u\in U}\{u/\phi_{\tilde{S}_1}(u)\geq\alpha\}] \quad ...... (11b)$$

$$S_2(\alpha)=[v_\alpha^L,v_\alpha^U]=[\min_{v\in V}\{v/\phi_{\tilde{S}_2}(v)\geq\alpha\},\max_{v\in V}\{v/\phi_{\tilde{S}_2}(v)\geq\alpha\}] \quad .... (11c)$$

$$S_3(\alpha)=[w_\alpha^L,w_\alpha^U]=[\min_{w\in W}\{w/\phi_{\tilde{S}_3}(w)\geq\alpha\},\max_{w\in W}\{w/\phi_{\tilde{S}_3}(w)\geq\alpha\}] \quad .... (11d)$$

$$R(\alpha)=[r_\alpha^L,r_\alpha^U]=[\min_{r\in R}\{r/\phi_R(r)\geq\alpha\},\max_{r\in R}\{r/\phi_R(r)\geq\alpha\}] \quad ..... (11e)$$

$$K(\alpha)=[k_\alpha^L,k_\alpha^U]=[\min_{k\in K}\{k/\phi_K(k)\geq\alpha\},\max_{k\in K}\{k/\phi_K(k)\geq\alpha\}] \quad ... (11f)$$

The arrival rate, service time, vacation time ,retrial rate, and group size are shown as intervals when the membership functions are no less than a given possibility level for α. As a result, the bounds of these intervals can be described as functions of α and can be obtained as:

$$x_\alpha^L=\min\phi_{\tilde{\lambda}}^{-1}(\alpha), x_\alpha^U=\max\phi_{\tilde{\lambda}}^{-1}(\alpha) \quad ................. (12a)$$

$$u_\alpha^L=\min\phi_{\tilde{S}_1}^{-1}(\alpha), u_\alpha^U=\max\phi_{\tilde{S}_1}^{-1}(\alpha) \quad ................ (12b)$$

$$v_\alpha^L=\min\phi_{\tilde{S}_2}^{-1}(\alpha), v_\alpha^U=\max\phi_{\tilde{S}_2}^{-1}(\alpha) \quad ................ (12c)$$

$$w_\alpha^L=\min\phi_{\tilde{S}_3}^{-1}(\alpha), w_\alpha^U=\max\phi_{\tilde{S}_3}^{-1}(\alpha) \quad ............ (12d)$$

$$r_\alpha^L=\min\phi_R^{-1}(\alpha), r_\alpha^U=\max\phi_R^{-1}(\alpha) \quad ............... (12e)$$

$$k_\alpha^L=\min\phi_K^{-1}(\alpha), k_\alpha^U=\max\phi_K^{-1}(\alpha) \quad ................ (12f)$$

Therefore, we can use the α-cuts of $E(\tilde{L})$ to construct its membership function since the membership function defined in (10) is parameterised by α

Using Zadeh's extension principle, $\phi_{E(\tilde{L})}(z)$ is the minimum of $\phi_{\tilde{\lambda}}(x), \phi_{\tilde{S}_1}(u), \phi_{\tilde{S}2}(v), \phi_{\tilde{S}_3}(w), \phi_R(r), \phi_K(k)$. To derive the membership function of $\phi_{E(\tilde{L})}(z)$, we need at least one of the following cases to hold such that

Z= $\frac{x^2(E(k))^2\beta_2+\rho(E^2(k))/E(k)}{2(1-\rho)}+\frac{x(\rho+E(k)-1)}{r(1-\rho)}$ satisfies $\phi_{E(\tilde{L})}(z)$ =α:

Case1:
$(\phi_{\tilde{\lambda}}(x)=\alpha, \phi_{\tilde{S}_1}(u)\ge\alpha, \phi_{\tilde{S}_2}(v)\ge\alpha, \phi_{\tilde{S}_3}(w)\ge\alpha, \phi_R(r)\ge\alpha, \phi_K(k)\ge\alpha)$

Case2:
$(\phi_{\tilde{\lambda}}(x)\ge\alpha, \phi_{\tilde{S}_1}(u)=\alpha, \phi_{\tilde{S}_2}(v)\ge\alpha, \phi_{\tilde{S}_3}(w)\ge\alpha, \phi_R(r)\ge\alpha, \phi_K(k)\ge\alpha)$

Case3:
$(\phi_{\tilde{\lambda}}(x)\ge\alpha, \phi_{\tilde{S}_1}(u)\ge\alpha, \phi_{\tilde{S}_2}(v)=\alpha, \phi_{\tilde{S}_3}(w)\ge\alpha, \phi_R(r)\ge\alpha, \phi_K(k)\ge\alpha)$

Case4:
$(\phi_{\tilde{\lambda}}(x)\ge\alpha, \phi_{\tilde{S}_1}(u)\ge\alpha, \phi_{\tilde{S}_2}(v)\ge\alpha, \phi_{\tilde{S}_3}(w)=\alpha, \phi_R(r)\ge\alpha, \phi_K(k)\ge\alpha)$

Case5:
$(\phi_{\tilde{\lambda}}(x)\ge\alpha, \phi_{\tilde{S}_1}(u)\ge\alpha, \phi_{\tilde{S}_2}(v)\ge\alpha, \phi_{\tilde{S}_3}(w)\ge\alpha, \phi_R(r)=\alpha, \phi_K(k)\ge\alpha)$

Case6:
$(\phi_{\tilde{\lambda}}(x)\ge\alpha, \phi_{\tilde{S}_1}(u)\ge\alpha, \phi_{\tilde{S}_2}(v)\ge\alpha, \phi_{\tilde{S}_3}(w)\ge\alpha, \phi_R(r)\ge\alpha, \phi_K(k)=\alpha)$

This can be accomplished using parametric NLP techniques. The NLP to find the lower and upper bounds of the α-cut of $\phi_{E(\tilde{L})}(z)$ for case 1 are:

$$(E(L))_\alpha^{L_1}=\min\left(\frac{x^2(E(k))^2\beta_2+\rho(E^2(k))/E(k)}{2(1-\rho)}+\frac{x(\rho+E(k)-1)}{r(1-\rho)}\right)$$

$$(E(L))_\alpha^{U_1}=\max\left(\frac{x^2(E(k))^2\beta_2+\rho(E^2(k))/E(k)}{2(1-\rho)}+\frac{x(\rho+E(k)-1)}{r(1-\rho)}\right)$$

For case 2 are

$$(E(L))_\alpha^{L_2}=\min\left(\frac{x^2(E(k))^2\beta_2+\rho(E^2(k))/E(k)}{2(1-\rho)}+\frac{x(\rho+E(k)-1)}{r(1-\rho)}\right)$$

$$(E(L))_\alpha^{U_2}=\max\left(\frac{x^2(E(k))^2\beta_2+\rho(E^2(k))/E(k)}{2(1-\rho)}+\frac{x(\rho+E(k)-1)}{r(1-\rho)}\right)$$

For case 3 are

$$(E(L))_\alpha^{L_3}=\min\left(\frac{x^2(E(k))^2\beta_2+\rho(E^2(k))/E(k)}{2(1-\rho)}+\frac{x(\rho+E(k)-1)}{r(1-\rho)}\right)$$

$$(E(L))_\alpha^{U_3}=\max\left(\frac{x^2(E(k))^2\beta_2+\rho(E^2(k))/E(k)}{2(1-\rho)}+\frac{x(\rho+E(k)-1)}{r(1-\rho)}\right)$$

For case 4 are

$$(E(L))_\alpha^{L_4}=\min\left(\frac{x^2(E(k))^2\beta_2+\rho(E^2(k))/E(k)}{2(1-\rho)}+\frac{x(\rho+E(k)-1)}{r(1-\rho)}\right)$$

$$(E(L))_\alpha^{U_4}=\max\left(\frac{x^2(E(k))^2\beta_2+\rho(E^2(k))/E(k)}{2(1-\rho)}+\frac{x(\rho+E(k)-1)}{r(1-\rho)}\right)$$

For case 5 are

$$(E(L))_\alpha^{U_5}=\max\left(\frac{x^2(E(k))^2\beta_2+\rho(E^2(k))/E(k)}{2(1-\rho)}+\frac{x(\rho+E(k)-1)}{r(1-\rho)}\right)$$

$$(E(L))_\alpha^{U_5}=\max\left(\frac{x^2(E(k))^2\beta_2+\rho(E^2(k))/E(k)}{2(1-\rho)}+\frac{x(\rho+E(k)-1)}{r(1-\rho)}\right)$$

For case 6 are

$$(E(L))_\alpha^{L_6}=\min\left(\frac{x^2(E(k))^2\beta_2+\rho(E^2(k))/E(k)}{2(1-\rho)}+\frac{x(\rho+E(k)-1)}{r(1-\rho)}\right)$$

$$(E(L))_\alpha^{U_6}=\max\left(\frac{x^2(E(k))^2\beta_2+\rho(E^2(k))/E(k)}{2(1-\rho)}+\frac{x(\rho+E(k)-1)}{r(1-\rho)}\right)$$

From the definitions of $\lambda(\alpha)$, $S_1(\alpha)$, $S_2(\alpha)$, $S_3(\alpha)$, $R(\alpha)$ and $K(\alpha)$ in (12) $x\in\lambda(\alpha)$, $u\in S_1(\alpha)$, $v\in S_2(\alpha)$, $w\in S_3(\alpha)$, $r\in R$, and $k\in K$ can be replaced by $x\in[x_\alpha^L, x_\alpha^U]$, $u\in[u_\alpha^L, u_\alpha^U]$, $v\in[v_\alpha^L, v_\alpha^U]$, $w\in[w_\alpha^L, w_\alpha^U]$, $r\in[r_\alpha^L, r_\alpha^U]$ and $k\in[k_\alpha^L, k_\alpha^U]$. The α-cuts form a nested structure with respect to α (Kaufman[5] and Zimmermann[11]); i.e. given $0<\alpha_2<\alpha_1\le 1$ we have $[x_{\alpha_1}^L, x_{\alpha_1}^U]\subseteq[x_{\alpha_2}^L, x_{\alpha_2}^U]$, $[u_{\alpha_1}^L, u_{\alpha_1}^U]\subseteq[u_{\alpha_2}^L, u_{\alpha_2}^U]$, $[v_{\alpha_1}^L, v_{\alpha_1}^U]\subseteq[v_{\alpha_2}^L, v_{\alpha_2}^U]$, $[w_{\alpha_1}^L, w_{\alpha_1}^U]\subseteq[w_{\alpha_2}^L, w_{\alpha_2}^U]$, $[r_{\alpha_1}^L, r_{\alpha_1}^U]\subseteq[r_{\alpha_2}^L, r_{\alpha_2}^U]$, $[k_{\alpha_1}^L, k_{\alpha_1}^U]\subseteq[k_{\alpha_2}^L, k_{\alpha_2}^U]$. Therefore, (12a), (12c), (12e), (12g), (12i), (12k) have the same smallest element and ( 12b), (12d), (12f), (12h), (12j), (12l) have the same largest element. To find the membership function $\phi_{E(\tilde{L})}(z)$, it suffices to find the left and right shape function of $[(E(L))_\alpha^L, (E(L))_\alpha^U]$, which is equivalent to finding the lower bound $(E(L))_\alpha^L$ and upper bound $(E(L))_\alpha^U$ of the α-cuts of $E(\tilde{L})$, which can be rewritten as:

$$(E(L))_\alpha^L=\min_{s.t. x_\alpha^L\le x\le x_\alpha^U, u_\alpha^L\le u\le u_\alpha^U, v_\alpha^L\le v\le v_\alpha^U, w_\alpha^L\le w\le w_\alpha^U, r_\alpha^L\le r\le r_\alpha^U, k_\alpha^L\le k\le k_\alpha^U}\left(\frac{x^2(E(k))^2\beta_2+\rho(E^2(k))/E(k)}{2(1-\rho)}+\frac{x(\rho+E(k)-1)}{r(1-\rho)}\right) \quad \ldots (13a)$$

$$(E(L))_\alpha^U=\max_{s.t. x_\alpha^L\le x\le x_\alpha^U, u_\alpha^L\le u\le u_\alpha^U, v_\alpha^L\le v\le v_\alpha^U, w_\alpha^L\le w\le w_\alpha^U, r_\alpha^L\le r\le r_\alpha^U, k_\alpha^L\le k\le k_\alpha^U}\left(\frac{x^2(E(k))^2\beta_2+\rho(E^2(k))/E(k)}{2(1-\rho)}+\frac{x(\rho+E(k)-1)}{r(1-\rho)}\right) \quad \ldots (13b)$$

Atleast one of x,u,v.w,r,k must hit the boundaries of their α-cuts to satisfy $\phi_{E(\tilde{L})}(z)$=α. This model is a set of mathematical programs with boundary constraints and lends itself to the systematic study of how the optimal solutions change with $x_\alpha^L$, $x_\alpha^U$, $u_\alpha^L$, $u_\alpha^U$, $v_\alpha^L$, $v_\alpha^U$, $w_\alpha^L$, $w_\alpha^U$, $r_\alpha^L$, $r_\alpha^U$, $k_\alpha^L$, $k_\alpha^U$ as α varies over (0,1]. The crisp interval $[(E(L))_\alpha^L, (E(L))_\alpha^U]$ obtained from (13) represents the α-cuts of $E(\tilde{L})$. Again by applying the results of Kaufmann [5] and Zimmermann [11], where $0<\alpha_2<\alpha_1\le 1$. In other words $(E(L))_\alpha^L$ increases and $(E(L))_\alpha^U$ decreases as α increases. Consequently, the membership function $\phi_{E(\tilde{L})}(z)$ can be found from (13). If both $(E(L))_\alpha^L$ and $(E(L))_\alpha^U$ in (13) are invertible with respect to α, then the left shape function L(z) = $[(E(L))_\alpha^L]^{-1}$ and the right shape function R(z) = $[(E(L))_\alpha^L]^{-1}$ can be derived, from which the membership function $\phi_{E(\tilde{L})}(z)$ is constructed:

$$\phi_{E(\tilde{L})}(z)=\begin{cases} L(z),(E(L))_{\alpha=0}^{L}\le z\le (E(L))_{\alpha=1}^{L}, \\ 1,(E(L))_{\alpha=1}^{L}\le z\le (E(L))_{\alpha=1}^{U}, \\ R(z),(E(L))_{\alpha=1}^{U}\le z\le (E(L))_{\alpha=0}^{U} \end{cases} \quad \ldots\ldots\ldots(14)$$

In most cases, the values of $(E(L))_\alpha^L$ and $(E(L))_\alpha^U$ cannot be solved analytically. Consequently, a closed form membership function for $\phi_{E(\tilde{L})}(z)$ cannot be obtained. However, the numerical solution for $(E(L))_\alpha^L$ and $(E(L))_\alpha^U$ at different possibility levels can be collected to approximate the shapes of L(z) and R(z) that is, the set of intervals $\{(E(L))_\alpha^L, (E(L))_\alpha^U / \alpha\in[0,1]\}$ shows the shape of $\phi_{E(\tilde{L})}(z)$, although the exact function is not known explicitly. Note that the membership functions for the expected waiting time of the customer in the orbit can be expressed in a similar manner.

**Numerical example.**

Consider an organisation's online recruitment policy in which candidates arrive in batches. Using α-cuts,the trapezoidal arrival size is trapezoidal fuzzy number [1,2,3,4] and the interval of confidence be represented by [1+α ,4-α]. Using (2) and (3), it is easy to find E(K) and $E(K^2)$. There are two stages of interview stage1 is technical interview and the stage2 is personal interview. The group calling rate for interview and the service times for two stages are trapezoidal fuzzy numbers represented by $\tilde{\lambda}$ = [0.2,0.3,0.4,0.5] $\tilde{S}_1$ = [0.25,0.26,0.27,0.28] , $\tilde{S}_2$ = [0.22,0.23,0.24,0.25]. As soon as the interview is completed, the interviewer may go for a vacation of random length $S_3$ with probability p = 0.5 or may continue to interview the next candidate if any with probability q = 0.5. $S_3$ represented by the trapezoidal fuzzy number $\tilde{S}_3$ = [0.21, 0.22, 0.23, 0.24]. The retrial rate is represented by the trapezoidal number = [0.1, 0.2, 0.3, 0.4]. The system manager wants to evaluate the performance measures of the system such as expected number of candidates in the orbit and expected waiting time of the candidate in the orbit.

Following (8),

$$E(L)=\frac{x^2(E(k))^2\beta_2+\rho(E^2(k))/E(k)}{2(1-\rho)}+\frac{x(\rho+E(k)-1)}{r(1-\rho)} \qquad E(W) = E(L) / \lambda E(X)$$

It is clear that in this example the steady state condition $\rho = \lambda E(X)(E(S_1) + E(S_2) + pE(S_3)) < 1$ is satisfied, thus the performance measure of interest can be constructed.

First it is easy to find

$[x_\alpha^L, x_\alpha^U] = [0.2+0.1\alpha, 0.5-0.1\alpha]$

$[u_\alpha^L, u_\alpha^U] = [0.25+0.01\alpha, 0.28-0.01\alpha]$

$[v_\alpha^L, v_\alpha^U] = [0.22+0.01\alpha, 0.25-0.01\alpha]$

$[w_\alpha^L, w_\alpha^U] = [0.21+0.01\alpha, 0.5=24-0.01\alpha]$

$[r_\alpha^L, r_\alpha^U] = [0.1+0.1\alpha, 0.4-0.1\alpha]$

Next it is obvious that when x = $x_\alpha^L$ and r = $r_\alpha^U$ , the expected number of candidates in orbit attains its minimum value and when x = $x_\alpha^U$ and r = $r_\alpha^L$ the expected number of candidates attains its maximum value.

**Table 1** The α-cuts of the performance measures of 11 α values

| α | $(E(L))_\alpha^L$ | $(E(L))_\alpha^U$ | $(E(W))_\alpha^L$ | $(E(W))_\alpha^U$ |
|---|---|---|---|---|
| 0.0 | 1.9686 | 53.8815 | 3.9372 | 43.1052 |
| 0.1 | 2.1547 | 45.9402 | 4.1042 | 37.5022 |
| 0.2 | 2.3573 | 38.2473 | 4.286 | 31.8728 |
| 0.3 | 2.5778 | 32.7816 | 4.4831 | 27.8992 |
| 0.4 | 2.8182 | 28.2658 | 4.697 | 24.5790 |
| 0.5 | 3.081 | 24.3981 | 4.9296 | 21.6872 |
| 0.6 | 3.3691 | 21.5161 | 5.1832 | 19.5601 |
| 0.7 | 3.6842 | 18.9219 | 5.4581 | 17.6018 |
| 0.8 | 4.0328 | 16.7474 | 5.7611 | 15.9499 |
| 0.9 | 4.4172 | 14.8687 | 6.0927 | 14.5060 |
| 1.0 | 4.8403 | 13.2529 | 6.4537 | 13.2529 |

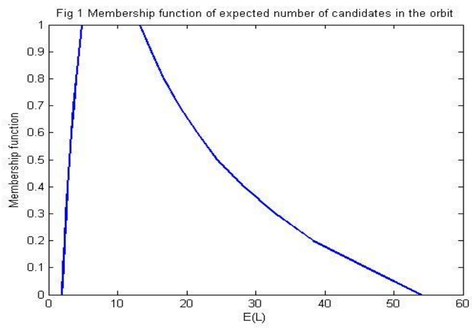


**Fig 1** Membership function of expected number of candidates in the orbit

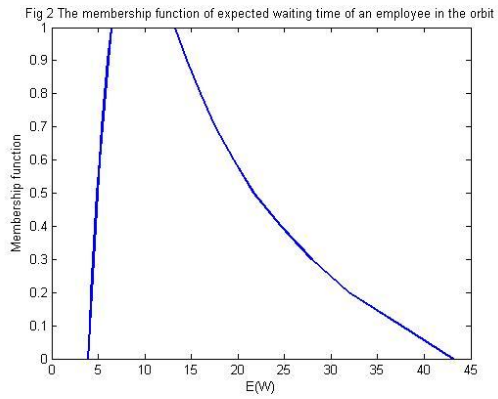


Fig 2 Membership function of expected waiting time of an employee in the orbit

With the help of MATLAB 7.04, we perform α-cuts of fuzzy expected number of candidates in the orbit and expected waiting time of a candidate in the orbit at eleven distinct α levels 0,0.1,0.2,…..1.0. Crisp intervals for fuzzy expected number of candidates in orbit and expected waiting time of a candidate in the orbit are presented in table 1. Fig 1 depicts the rough shape of $\phi_{E(\tilde{L})}$. The rough shape turns out rather fine and looks like a continuous function. The α-cut represent the probability that these two performance measure will lie in the associated range. Specially, α = 0 the range, the performance measures could appear and for α = 1 the range, the performance measure are likely to be. For example, while these two performance measures are fuzzy, the most likely value of the expected length of the orbit falls between 4.8403 and 13.2529 , and its value is impossible to fall outside the range of 1.9686 and 53.8815; it is definitely possible that the expected waiting time in the orbit falls between 6.4537 and 13.2529 time units approximately, and it will never fall below 3.9372 and above 43.1052 time units.

## CONCLUSION

Single server batch arrival retrial queuing models with heterogeneous service and generalised vacation have wider applications in communication system to evaluate system performance. This paper applies the concept of α-cuts and zadeh's extension principle and constructs the membership functions of the expected number of customers in the orbit and expected waiting time of the customer in the orbit using the paired NLP models. Following the proposed approach, the α-cuts of the membership functions are found to attain explicit closed form expression for the system characteristics. Since the performance measure is expressed by the membership function rather than by a crisp value, it maintains the fuzziness of input information and the results can be used to represent the fuzzy system more accurately.

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