Available Online at http://www.recentscientific.com

CODEN: IJRSFP (USA)

International Journal of Recent Scientific Research
Vol. 9, Issue, 8(D), pp. 28544-28548, August, 2018

ISSN: 0976-3031

International Journal of Recent Scientific Research

DOI: 10.24327/IJRSR

Research Article

INTRODUCTION OF EXTENDED FINITE MELLIN TRANSFORM

Rangari, A.N[1]* and Sharma, V. D[2]

[1]Department of Mathematics, Adarsh College, Dhamangaon Rly. Dist-Amravati (M.S.), India
[2]Department of Mathematics, Arts, Commerce & Science College, Kiran NagarAmravati (M.S.), India

DOI: http://dx.doi.org/10.24327/ijrsr.2018.0908.2478

ARTICLE INFO

Article History:
Received 6th May, 2018
Received in revised form 10th June, 2018
Accepted 24th July, 2018
Published online 28th August, 2018

Key Words:

Finite Mellin Transform, Testing function space, Generalized function, Operators.

ABSTRACT

Integral transforms facilitate the conversion of complicated algebraic equations into simple and easily solvable expressions. In contrast to Fourier and Laplace transformations that were introduced to solve physical problems, Mellin transformation arose in a Mathematical, Physical and Engineering context. In the proposed work Extended Finite Mellin Transform is introduced.

INTRODUCTION

In mathematics, the Mellin transform is an integral transform that may be regarded as the multiplicative version of the two-sided Laplace transform. This integral transform is closely connected to the theory of Dirichlet series, and is often used in number theory, mathematical statistics, and the theory of asymptotic expansions; it is closely related to the Laplace transform, Fourier transform, and the theory of the gamma function and allied special functions.

The Mellin transform is widely used in computer science for the analysis of algorithms because of its scale invariance property. The Mellin transform method is applied to fractional differential equations (Klimek and Dziembowski, 2008) [1]. It is also used in statistics for finding means, variances, skewness, and fuzzy numbers and then apply to the random coefficient autoregressive (RCA) time series models (Appadoo, Thavaneswaran and Mandal, 2014) [2]. It is also useful for Geophysics. Integral transform facilitate the conversion of complicated algebraic equations into simple and easily solvable expressions. Their application is now gaining a lot of importance in geophysics, particularly in the area of signal processing and the quantitative interpretation of potential field data. Despite the popularity of integral transform in geophysical data analysis, the Mellin transform has remained virtually unexploited for geophysical applications (Mohan, Reddy and Ofoegbu, 1989) [3].

The main purpose of this paper is to generalized Finite Mellin transform in the distributional sense, Describing testing function spaces for Finite Mellin transform, Described some operators and properties for Finite Mellin transform. This paper is ordered as follows: Definitions are given in section 2, In section 3, Testing function spaces for Finite Mellin transform are given. The kernel of Finite Mellin transform is a member of testing function space is proved in section 4. In section 5, Definition of Distributional Generalized Finite Mellin transform is given. Some operators on testing function space are given in section 6. In section 7, Adjoint operators of Finite Mellin transform are given. Some properties of Generalized Finite Mellin transform are given in section 8. Lastly we conclude the paper.

The notations and terminology as per A. H. Zemanian [4], [5].

Definition: Finite Mellin Transform

The one Dimensional Finite Mellin transform with parameter s of $f(t)$ denoted by $M_f\{f(t)\} = F(s)$ performs a linear operation, given by the integral transform,

Corresponding author:* **Rangari, A.N
Department of Mathematics, Adarsh College, Dhamangaon Rly. Dist-Amravati (M.S.), India

$$M_f\{f(t)\}=F(s)=\int_0^a f(t)K(s)\,dt \qquad (2.1)$$

where the kernel $K(s)=\left(\frac{a^{2s+1}}{t^{s+1}}-t^s\right)$

Testing Function Spaces for Finite Mellin Transform

The Space $M_{f,b,c,\alpha}$:

Let I be the open set in $R_+\times R_+$ and E_+ denotes the class of infinitely differentiable function defined on I, the space $M_{f,b,c,\alpha}$ is given by,

$$M_{f,b,c,\alpha}=\left\{\phi:\phi\in E_+/\gamma_{b,c,q,}\left[\phi(t)\right]=\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}D_t^q\phi(t)\right|\le CA^q q^{q\alpha}\right\}$$

For each $q=0,1,2,3,\ldots\ldots\ldots\ldots$ and where the constants A and C depend on the testing function ϕ.

The Space $M_{f,b,c,\alpha,m}$:

This space is a subspace of (3.1) which is given by,

$$M_{f,b,c,\alpha,m}=\left\{\phi:\phi\in E_+/\gamma_{b,c,q,}\left[\phi(t)\right]=\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}D_t^q\phi(t)\right|\le C_\delta(m+\delta)^q q^{q\alpha}\right\}$$

for any $\delta>0$, where m is the constant depending on the function ϕ.

The space $M^{\upsilon}_{f,b,c,\alpha}$:

It is a negative space of (3.1) which is given by,

$$M^{\upsilon}_{f,b,c,\alpha}=\left\{\phi:\phi\in E_-/\lambda_{b,c,q,}\left[\phi(t)\right]=\sup_{I_2}\left|\lambda_{b,c}(-t)(-t)^{q+1}D_t^q\phi(t)\right|\le CA^q q^{q\alpha}\right\}$$

Here we also set

$$\lambda_{b,c}(t)=\begin{cases}t^{+b}, & 0<t<1\\ t^{+c}, & 1<t<a\end{cases}$$

Lemma

The function $\left(\frac{a^{2s+1}}{t^{s+1}}-t^s\right)$ is a member of $M_{f,b,c,\alpha}$ if $-b<\text{Re}s\le \text{b}$ for any real number c.

Proof: Let $\phi(t)=\frac{a^{2s+1}}{t^{s+1}}-t^s$

Consider, $\gamma_{b,c,q}\left[\phi(t)\right]=\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}D_t^q\phi(t)\right|$

$$=\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}D_t^q\left(\frac{a^{2s+1}}{t^{s+1}}-t^s\right)\right|$$

$$=\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}\left[P(-s-q)t^{-s-q-1}a^{2s+1}-P(s-q+1)t^{s-q}\right]\right|$$

where P is a polynomial in s and q.

$$=\sup_{I_1}\left|a^{2s+1}P(-s-q)t^{b-s}-P(s-q+1)t^{b+s+1}\right|\le C \qquad (4.1)$$

where

$$C=\sup_{I_1}\left\{a^{2s+1}P(-s-q)t^{b-s}-P(s-q+1)t^{b+s+1}\right\}$$

$\therefore \gamma_{b,c,q}\left[\phi(t)\right]\le C\frac{q^{q\alpha}}{q^{q\alpha}}$ by (4.1)

$=CA^q q^{q\alpha}$, where $A=\frac{1}{q^\alpha}$

Hence $\left(\frac{a^{2s+1}}{t^{s+1}}-t^s\right)\in M_{f,b,c,\alpha}$.

Here onwards for simplicity we say that $\phi(t)\in M_{f,b,c,\alpha}$ if $\left|\lambda_{b,c}(t)t^{q+1}D_t^q\phi(t)\right|<\infty$, for $q=0,1,2,3,\ldots..$. If as $t\to 0,\ b-s>0$ and $b+s>0$.

i.e. $b>\text{Re}s \quad s>-b$. i.e. $\text{Re}s<b \quad -b<\text{Res}$

i.e. if $-b<\text{Re}s\le b$ and for any real number c.

Thus $\phi(t)\in M_{f,b,c,\alpha}$ if $-b<\text{Re}s\le b$, for any real number c.

Distributional Generalized Finite Mellin Transform (M_fT)

For $f(t)\in F^*_{f,b,c,\alpha}$, where $F^*_{f,b,c,\alpha}$ is the dual space of $M_{f,b,c,\alpha}$ and $-b<\text{Re}s<b$. The distributional Finite Mellin transform is defined as,

$$M_f\{f(t)\}=F(s)=\langle f(t),\phi(t,s)\rangle, \qquad (5.1)$$

where $\phi(t,s)=\left(\frac{a^{2s+1}}{t^{s+1}}-t^s\right)$ and for each fixed t $(0<t<a)$. The right hand side of (5.1) is meaningful because according to lemma 4, $\phi(t,s)\in M_{f,b,c,\alpha}$ and $f(t)\in M^*_{f,b,c,\alpha}$.

Operators on the Space $M_{f,b,c,\alpha}$

Proposition: If $\phi(t)\in M_{f,b,c,\alpha}$ and σ is any fixed real number then $\phi(t+\sigma)\in M_{f,b,c,\alpha}$, $t+\sigma>0$ and $\phi(t+\sigma)\in M^{\upsilon}_{f,b,c,\alpha}$, $t+\sigma<0$.

Proof Consider,

$$\gamma_{b,c,q}\phi(t+\sigma)=\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}D_t^q\phi(t+\sigma)\right|$$

$$=\sup_{I_1}\left|\lambda_{b,c}(t'-\sigma)(t'-\sigma)^{q+1}D_t^q\phi(t')\right|$$

Where $t'=t+\sigma \quad \therefore t=t'-\sigma$

$$\le CA^q q^{q\alpha}$$

Thus, $\phi(t+\sigma)\in M_{f,b,c,\alpha}$ for $t+\sigma>0$.

Similarly it can shown that $\phi(t+\sigma)\in M^{\upsilon}_{f,b,c,\alpha}$, for $t+\sigma<0$.

Proposition: The translation (Shifting) operator $\sigma:\phi(t)\to\phi(t+\sigma)$ is a topological automorphism on $M_{f,b,c,\alpha}$, for $t+\sigma>0$, and it is a topological isomorphism from $M_{f,b,c,\alpha}$ onto $M^{\upsilon}_{f,b,c,\alpha}$, for $t+\sigma<0$.

Proposition: If $\phi(t)\in M_{f,b,c,\alpha}$ and $r>0$, strictly positive number, then $\phi(rt)\in M_{f,b,c,\alpha}$.

Proof: Consider, $\gamma_{b,c,q}\phi(rt)=\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}D_t^q\phi(rt)\right|$

$$=\sup_{I_1}\left|\lambda_{b,c}\left(\frac{T}{r}\right)\left(\frac{T}{r}\right)^{q+1}D_T^q\phi(T)\right|,$$

Where, $rt=T \quad \therefore t=\dfrac{T}{r}$

$$=M\sup_{I_1}\left|\lambda_{b,c}(T)(T)^{q+1}D_T^q\phi(T)\right|,$$

Where, M is a constant depending on r.

$$\le MCA^q q^{q\alpha}\le C'A^q q^{q\alpha}$$

where $C'=MC$.

Thus, $\phi(rt)\in M_{f,b,c,\alpha}$, for $r>0$.

Proposition: If $r>0$ and $\phi(t)\in M_{f,b,c,\alpha}$ then the scaling operator $R:M_{f,b,c,\alpha}\to M_{f,b,c,\alpha}$ defined by $R\phi=\psi$ where $\psi(t)=\phi(rt)$ is a topological automorphism.

Proposition: The operator $\phi(t)\to D_t\phi(t)$ is defined on the space $M_{f,b,c,\alpha}$ and transforms this space into itself.

Proof: Let $\phi(t)\in M_{f,b,c,\alpha}$. If $D_t\phi(t)=\psi(t)$, we have

$$\gamma_{b,c,q}\psi(t)=\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}D_t^q\psi(t)\right|$$

$$=\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}D_t^qD_t\phi(t)\right|$$

$$=\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}D_t^{q+1}\phi(t)\right|$$

$$\le CA^{q+1}(q+1)^{(q+1)\alpha},$$

$q=0,1,2,3,.....$

Therefore, $\psi(t)\in M_{f,b,c,\alpha}$ i.e. $D_t\phi(t)\in M_{f,b,c,\alpha}$.

i.e. $\gamma_{b,c,q}\left(D_t\phi(t)\right)=\gamma_{b,c,(q+1)}\left(\phi(t)\right)$.

Proposition: The differential operator of M_f-type $M_f:\phi(t)\to D_t\phi(t)$ is a topological autopmorphism on $M_{f,b,c,\alpha}$.

Proposition: For $m=(m_1,m_2)$, where $m_1,m_2=0,1,2,.........$, if $\phi(t)\in M_{f,b,c,\alpha}$. Then $\psi(t)\in M_{f,b,c,\alpha}$ where $\psi(t)=D^m\phi(t)$. Further the mapping $\Delta=D^m\phi:\phi\to\psi$ is one-one, linear and continuous.

Proof: For $\psi\in M_{f,b,c,\alpha}$,

$$\gamma_{b,c,q}\psi(t)=\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}D_t^q\psi(t)\right|$$

$$=\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}D_t^qD^m\phi(t)\right|$$

$$=\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}D_t^{q+1}\phi(t)\right|\le CA^q q^{q\alpha}$$

(6.7.1)

Thus $\psi(t)\in M_{f,b,c,\alpha}$ if $\phi(t)\in M_{f,b,c,\alpha}$.

It is obviously linear. It is injective for, if $D^m\phi=0$ then $\phi=c$, c is a constant. If $c=0$ then $\phi=0$ and D is injective. But if $c\neq0$ then,

$\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}D_t^qc\right|=\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}c\right|$, for $q=0$.

As the right hand side is not bounded we conclude that $\phi\notin M_{f,b,c,\alpha}$, which is a contradiction. Hence 'c' must be zero and therefore $\phi=0$. For continuity we observe from equation (6.7.1) that,

$$\gamma_{b,c,q}\left(D^m\phi\right)\le M\gamma_{b,c,q}(\phi),$$

Where M is some constant. Thus the theorem is proved.

Proposition: For $\tau\in R$ and $\phi(t)\in M_{f,b-\tau,c,\alpha}$, $E(\phi)=\psi(t)=e^{-\tau t}\phi(t)\in M_{f,b,c,\alpha}$.

Proof: Let $\phi(t)\in M_{f,b-\tau,c,\alpha}$,

Consider, $\gamma_{b,c,q}\psi(t)=\sup_{I_1}\left|\lambda_{b,c}(t)t^{q+1}D_t^qe^{-\tau t}\phi(t)\right|$

$$=\sup_{I_1}\left|\sum_{i=0}^{1}b_ie^{-\tau t}\lambda_{b,c}(t)t^{q+1}D_t^q\phi(t)\right|$$

$$=\sup_{I_1}\left|\sum_{i=0}^{1}b_ie^{-\tau t}t^{b+q+1}D_t^q\phi(t)\right|$$

$$\le CA^q q^{q\alpha}.$$

Thus $\psi(t)\in M_{f,b,c,\alpha}$ if $\phi(t)\in M_{f,b-\tau,c,\alpha}$.

Adjoint Operators of Fourier-Finite Mellin Transform

Proposition: The adjoint shifting operator is a continuous function from $M^{*}_{f,b,c,\alpha}$ to $M^{*}_{f,b,c,\alpha}$. The adjoint operator $f(t)\rightarrow f(t-\sigma)$ leads to the operation transform formula

$M_f\{f(t-\sigma)\}=RM_f\{f(t)\}$.

Proof: Consider,

$$M_f\{f(t-\sigma)\}=\left\langle f(t-\sigma),\left(\frac{a^{2s+1}}{t^{s+1}}-t^s\right)\right\rangle$$

$$=\left\langle f(t),\left(\frac{a^{2s+1}}{(t+\sigma)^{s+1}}-(t+\sigma)^s\right)\right\rangle$$

$$=\left\langle f(T-\sigma),\left(\frac{a^{2s+1}}{T^{s+1}}-T^s\right)\right\rangle \quad \text{where} \quad t+\sigma=T, \quad t=T-\sigma$$

$$=R\left\langle f(t),\left(\frac{a^{2s+1}}{t^{s+1}}-t^s\right)\right\rangle,$$

where, R is constant depending on σ.

$$=RM_f\{f(t)\}$$

$$\therefore M_f\{f(t-\sigma)\}=RM_f\{f(t)\}.$$

Proposition: The adjoint scaling operator is a continuous function from $M^{*}_{f,b,c,\alpha}$ to $M^{*}_{f,b,c,\alpha}$. The adjoint operator $f(t)\rightarrow\frac{1}{q}f\left(\frac{t}{q}\right)$ corresponding transform formula is

$M_f\left\{\frac{1}{q}f\left(\frac{t}{q}\right)\right\}=QM_f\{f(t)\}$.

Proof: Consider,

$$M_f\left\{\frac{1}{q}f\left(\frac{t}{q}\right)\right\}=\left\langle\frac{1}{q}f\left(\frac{t}{q}\right),\left(\frac{a^{2s+1}}{t^{s+1}}-t^s\right)\right\rangle$$

$$=\left\langle f(t),\left(\frac{a^{2s+1}}{(tq)^{s+1}}-(qt)^s\right)\right\rangle$$

$$=\left\langle f\left(\frac{T}{q}\right),\left(\frac{a^{2s+1}}{T^{s+1}}-T^s\right)\right\rangle \quad \text{where} \quad qt=T,\ t=\frac{T}{q}$$

$$=Q\left\langle f(t),\left(\frac{a^{2s+1}}{t^{s+1}}-t^s\right)\right\rangle$$

where, Q is a constant depending on q.

$$=QM_f\{f(t)\}.$$

$$\therefore M_f\left\{\frac{1}{q}f\left(\frac{t}{q}\right)\right\}=QM_f\{f(t)\}$$

Properties of Generalized Finite Mellin Transform

Linearity Property: If $M_f\{f(t)\}$ is generalized finite mellin transform of $f(t)$ and $M_f\{g(t)\}$ is generalized finite mellin transform of $g(t)$ then

$$M_f\{C_1f(t)+C_2g(t)\}(s)=C_1M_f\{f(t)\}(s)+C_2M_f\{g(t)\}(s)$$

Proof: Consider,

$$M_f\{C_1f(t)+C_2g(t)\}(s)=\int_0^a\left[C_1f(t)+C_2g(t)\right]\left(\frac{a^{2s+1}}{t^{s+1}}-t^s\right)dt$$

$$=C_1\int_0^a\left(\frac{a^{2s+1}}{t^{s+1}}-t^s\right)f(t)\,dt+C_2\int_0^a\left(\frac{a^{2s+1}}{t^{s+1}}-t^s\right)g(t)\,dt$$

$$=C_1M_f\{f(t)\}(s)+C_2M_f\{g(t)\}(s)$$

Result: $M_f\{f(\alpha t)\}(s)=\alpha^{-s-1}M_{f,a\alpha}\{f(t)\}(s)$

Proof: $M_f\{f(\alpha t)\}(s)=\int_0^a\left(\frac{a^{2s+1}}{t^{s+1}}-t^s\right)f(\alpha t)\,dt$

$$=\int_0^a\left(a^{2s+1}.t^{-s-1}-t^s\right)f(\alpha t)\,dt$$

Put $\alpha t=T \quad \therefore t=\frac{T}{\alpha} \quad \Rightarrow \alpha dt=dT \quad \therefore dt=\frac{dT}{\alpha}$

$$=\int_0^a\left(a^{2s+1}.\left(\frac{T}{\alpha}\right)^{-s-1}-\left(\frac{T}{\alpha}\right)^s\right)f(T)\frac{dT}{\alpha}$$

$$=\frac{1}{\alpha}\int_0^a\left(a^{2s+1}.\frac{T^{-s-1}}{\alpha^{-s-1}}-\frac{T^s}{\alpha^s}\right)f(T)\,dT$$

$$=\frac{1}{\alpha}\int_0^a\frac{1}{\alpha^s}\left(a^{2s+1}.\alpha^s\frac{T^{-s-1}}{\alpha^{-s-1}}-\alpha^s\frac{T^s}{\alpha^s}\right)f(T)\,dT$$

$$=\frac{1}{\alpha^{s+1}}\int_0^a\left(a^{2s+1}.\alpha^{s+s+1}T^{-s-1}-T^s\right)f(T)\,dT$$

$$=\alpha^{-s-1}\int_0^a\left(a^{2s+1}.\alpha^{2s+1}T^{-s-1}-T^s\right)f(T)\,dT$$

$$= \alpha^{-s-1}\int_0^a \left((a\alpha)^{2s+1}.T^{-s-1} - T^s \right) f(T)\,dT$$

$$= \alpha^{-s-1}\int_0^a \left(\frac{(a\alpha)^{2s+1}}{T^{s+1}}.-T^s \right) f(T)\,dT$$

$$= \alpha^{-s-1} M_{f,a\alpha}\{f(t)\}(s).$$

$$\therefore M_f\{f(\alpha t)\}(s) = \alpha^{-s-1} M_{f,a\alpha}\{f(t)\}(s)$$

CONCLUSIONS

In the present work generalized Finite Mellin transform is introduced in the distributional sense. Some operators and Adjoint operators for Finite Mellin transform is obtained which will be useful for solving Partial differential equations.

References

1. Klimek, Malgorzata,and Dziembowski, Daniel, "On Mellin transform application, to solution of fractional differential equations", Scientific Research of the Institute of Mathematics and computer science, Volume 7, Issue 2, Pages 31—42, 2008,
2. Appadoo, S.S., Thavaneswaran, A., and Mandal, S., "Mellins Transform and Application to some Time Series Models", *International Scholarly Research Notices, (ISRN Applied Mathematics),* Volume 2014, Article ID 976023,18 Feb. 2014.
3. Mohan, N.L., Reddy, K.G., Ofoegbu, C.O., "The Mellin Transform, Its Properties and Applications in Geophysics", Proceedings of the 7th International Mathematical Geophysics Seminar held at the Free University of Berlin, pp. 8-11, Feb. 1989.
4. Zemanian A.H., "Distribution theory and transform analysis", McGraw Hill, New York, 1965.
5. Zemanian, A. H., "Generalized integral transform", Inter science publisher, New York, 1968.
6. Lokenath Debnath, and Bhatta Dambaru, Integral Transforms and their Applications, Chapman and Hall/CRC Taylor and Francis Group Boca Raton London, New York, 2007.
7. Sharma, V. D., and Rangari, A. N., "Generalized Fourier-Finite Mellin Transform", *International Journal of Mathematical Archive,* Volume 3, pp. 1257-1260, March 2012.
8. Sharma, V. D., and Rangari, A. N., "Operation Transform Formulae of Fourier-Laplace Transform, *International Journal of Pure and Applied Sciences and Technology,* 15(2), pp. 62-67, 2013.
9. Sharma, V. D., and Rangari, A. N., "Operational Calculus on Fourier-Finite Mellin Transform, *International Journal of Engineering and Innovative Technology (IJEIT),* Volume 3, Issue 4, pp. 161-164, October 2013.
10. Sharma, V. D., and Rangari, A. N., "Adjoint Operators of Fourier-Finite Mellin Transform, *International Journal of Emerging Trends & Technology in Computer Science (IJETTCS),* Volume 4, Issue 6, pp. 10-13, Nov.-Dec. 2015.

How to cite this article:

Rangari, A.N and Sharma, V. D., 2018, Introduction of Extended Finite Mellin Transform. *Int J Recent Sci Res.* 9(8), pp.28544-28548.DOI: http://dx.doi.org/10.24327/ijrsr.2018.0908.2478

****