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## **CODEN: IJRSFP (USA)**

International Journal of Recent Scientific Research Vol. 9, Issue, 8(D), pp. 28539-28543, August, 2018 International Journal of Recent Scientific Re*r*earch

DOI: 10.24327/IJRSR

# **Research Article**

## **PROPERTIES OF G\*-CLOSED SETS IN TOPOLOGICAL SPACES**

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DOI: http://dx.doi.org/10.24327/ijrsr.2018.0908.2477

#### **ARTICLE INFO**

## ABSTRACT

#### Article History:

Received 6<sup>th</sup>May, 2018 Received in revised form 10<sup>th</sup> June, 2018 Accepted 24<sup>th</sup>July, 2018 Published online 28<sup>th</sup> August, 2018

#### Key Words:

preopen sets, semipreclosed sets, gspclosed sets and gp-closed sets, gprclosed sets and g\*p-closed sets.

Mathematics Subject Classification: 54A05, 54B05, 54D10

The purpose of this paper is to define and study a new class of sets, called the class of G\*-closed

sets, which lies between the class of semipreclosed sets and the class of g-closed sets . Also, the

basic properties of these G\*-closed sets, G\*-open sets, G\*T 1/2-spaces, G\*- continuous functions,

G\*-irresolute functions, G\*-Hausdorff spaces and G\*-connected spaces are studied in this paper.

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## INTRODUCTION

Levine [16] generalized the closed set to generalized closed set (g-closed set) in topology for the first time. Since then it is noticed that many of the weaker forms of closed sets have been generalized. In 1982 and 1986, respectively, A.S.Mashhour et al [10] and D.Andrijevic [1] have defined and studied the concepts of semiopen sets, preopen sets and semiopen sets in topology. In 1993,1995, 1997, 1998, 2000, 2002, 2010, 2011 and 2014, respectively, Palaniappan et al. [23], Dontchev [11], Gnanambal [13], Noiri et al [22], M.K.R.S.Veera Kumar [26-28], M.Shyla Isac et al. [24], S.Bhattacharya [6] and K.Indrani et al [14], have defined and studied the concepts of rg-closed sets, gsp-closed sets, gp-closed sets, g\*-closed sets, g\*p-closed sets, Pre-semiclosed sets, rps-closed sets, grclosed sets and gr\*-closed sets in topological spaces. In this paper, we define and study a new class of sets, called the class of G\*-closed sets, which lies between the class of semipreclosed sets and the class of g-closed sets . Also,the basic properties of these G\*-closed sets, G\*-open sets, G\*T 1/2spaces, G\*- continuous functions, G\*-irresolute functions, G\*-Hausdorff spaces and G\*-connected spaces are studied in this paper.

### Preliminaries

Throughout this paper  $(X,\tau)$  and  $(Y,\sigma)$  (or simply X and Y) always means topological spaces on which no separation

axioms are assumed unless explicitly stated. Let A be a subset of space X. We denote the closure of A and the interior of A by Cl(A) and Int(A) respectively. A subset A of a space X is called regular open (in brief, r-open) if A = Int Cl(A) and regular closed (in brief, r-closed) if A = Cl Int(A).

The following definitions and results are useful in the sequel:

Definition: A subset A of a space X is said to be:

- 1. preopen [17] if  $A \subset Int Cl(A)$ .
- 2. semiopen [15] if  $A \subset ClInt(A)$ .
- 3. semipreopen [2] if  $A \subset Cl$  Int Cl(A).

The complement of a preopen (resp. semiopen, semipreopen) set of a space X is called preclosed [12] (resp. semiclosed [7], semipreclosed [2]).

**Definition [2]:** The union of all semipreopen sets contained in A is called the semipreinterior of A and is denoted by spInt(A). pInt(A)[11] and sInt(A)[5], rInt(A) [23] can be similarly defined.

**Definition** [2]: The intersection of all semipreclosed sets containing A is called the semipreclosure of A and is denoted by spCl(A). pCl(A)[7] and sCl(A)[4], rCl(A) [23] can be similarly defined.

Definition: A subset A of a space X is called:

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- 1. generalized closed set ( in brief, g-closed) set [16] if  $Cl(A) \subset U$  whenever  $A \subset U$  and U is open in X.
- 2. generalized semiclosed (in brief, gs-closed) set [4] if  $sCl(A) \subset U$  whenever  $A \subset U$  and U is open in X.
- generalized semipreclosed (in brief, gsp-closed) set [11] if spCl(A)⊂U whenever A ⊂ U and U is open in X.
- generalized preclosed (in brief, gp-closed) set [22] if pCl(A)⊂U whenever A ⊂ U and
- 5. U is open in X.
- 6. gpr--closed set [13] if  $pCl(A) \subset U$  whenever  $A \subset U$  and U is r-open in X.
- 7. rg-closed set [23] if  $Cl(A) \subset U$  whenever  $A \subset U$  and U is r-open in X.
- 8. gr-closed set [6] if  $rCl(A) \subset U$  whenever  $A \subset U$  and U is open in X.
- 9. pre-semiclosed set [28] if  $spCl(A) \subset U$  whenever  $A \subset U$  and U is g-open in X.
- 10. g\*-closed set [26] if  $Cl(A) \subset U$  whenever  $A \subset U$  and U is g-open in X.
- g\*-preclosed (in brief, g\*p-closed) set [27] if pCl(A)⊂U whenever A ⊂ U and U is g-open in X.
- 12. g\*sp-closed set [20] if  $spCl(A) \subset U$  whenever  $A \subset U$  and U is g-open in X.
- 13. gr\*-closed set [14] if  $rCl(A) \subset U$  whenever  $A \subset U$  and U is g-open in X.

**Definition [21]:** The union of all gsp-open sets contained in A is called the gsp-interior of A and is denoted by gspInt(A).

**Definition** [21]: The intersection of all gsp-closed sets containing A is called the gsp-closure of A and is denoted by gspCl(A).

*Lemma [21]:* For every subset  $A \subset X$ , the following hold.

- 1. gspCl(X-A) = X-gspInt(A).
- 2. gspInt(X-A) = X-gspCl(A).

**Definition:** A function  $f: X \rightarrow Y$  is called:

- 1. precontinuous [17] if the inverse image of each open set of Y is preopen in X.
- 2. semiprecontinuous [19] if the inverse image of each open set of Y is semipreopen in X.

**Definition:** A function f:  $X \rightarrow Y$  is called:

- 1. g-continuous [5] if the inverse image of each closed set of Y is g-closed in X.
- 2. rg-continuous [3] if the inverse image of each closed set of Y is rg-closed in X.
- 3. gsp-continuous [11] if the inverse image of each closed set of Y is gsp-closed in X.
- 4. g\*-continuous [26] if the inverse image of each closed set of Y is g\*-closed in X.
- 5. g\*sp-continuous [20] if the inverse image of each closed set of Y is g\*sp-closed in X.
- 6. gr\*-continuous [14] if the inverse image of each closed set of Y is gr\*-closed in X.
- 7. rps-continuous [25] if the inverse image of each closed set of Y is rps-closed in X.

**Definition [5]:** A space X is said to be g-connected if X cannot be written as the disjoint union of two nonempty g-open sets in X.

**Definition** [1]: A space X is said to be rg-connected if X cannot be written as the disjoint union of two nonempty rg-open sets in X.

**Definition [8]:** A space X is said to be g-Hausdroff if whenever x and y are distinct points of X there exist disjoint g-open sets U and V such that  $x \in U$  and  $y \in V$ .

**Definition** [8]: A space X is said to be rg-Hausdroff if whenever x and y are distinct points of X there exist disjoint  $g^*p$ -open sets U and V such that  $x \in U$  and  $y \in V$ .

## Properties of G\*-closed sets

We, define the following.

**Definition:** A subset A of a space X is said to be G\*-closed set if  $gspCl(A) \subset U$  whenever  $A \subset U$  and U is g-open in X.

The complement of a G\*-closed set is called G\*-open set.

Lemma: Let X be a space. Then,

- 1. Every closed (and hence preclosed, semipreclosed, rclosed) set is G\*-closed set.
- 2. Every g\*-closed (and hence g\*p-closed, g\*sp-closed, gr\*-closed, rps-closed) set is G\*-closed set.
- Every G\*-closed set is g-closed (and hence gs-closed, gsp-closed, gp-closed, gpr-closed, rg-closed, grclosed, pre-semiclosed) set.

We, prove the following.

Lemma: Let A be a G\*-closed set of space X. Then,

- 1. gspCl(A)-A does not contain any nonempty g-closed set.
- If A⊂ B ⊂ gspCl(A), then B is also a G\*-closed set of X.

**Proof.** (i) Let F be a g-closed set contained in gspCl(A) -A, then gspCl(A)  $\subset X - F$  since X - F is g-open with  $A \subset X$ -F and A is G\*-closed set. Then,  $F \subset (X$ -gspCl(A))  $\cap$  (gspCl(A)-A)  $\subset$  (X-gspCl(A))  $\cap$  gspCl(A) =  $\emptyset$ . Thus,  $F = \emptyset$ .

(ii)Let G be a g-open set of X such that  $B \subset G$ . Then,  $A \subset G$ . Since  $A \subset G$  and A is G\*-closed set, then  $gspCl(A) \subset G$ . Then,  $gspCl(B) \subset gspCl(gspCl(A)) = gspCl(A)$  since  $B \subset gspCl(A)$ . Thus,  $gspCl(B) \subset gspCl(A) \subset G$ . Hence, B is also a G\*-closed set of X.

*Lemma:* A subset A of a space X is called G\*-open iff  $F \subset$  gspInt(A) whenever F is g-closed and  $F \subset A$ . Proof follows from Lemmas-2.7 and 3.3.

*Lemma:* If  $gspInt(A) \subset B \subset A$  and A is G\*-open set, then B is also G\*-open set.

### Proof: Obvious.

*Lemma:* For any  $A \subset X$ , then gspInt(gspCl (A) – A) =  $\emptyset$ . Proof : Obvious.

*Lemma:* If  $A \subset X$  is G\*-closed, then gspCl(A) –A is G\*-open. Proof follows from Lemma-3.6.

Next, we introduce the following.

Definition: A space X is said to be a:

- 1. G\*C-space if every G\*- closed set in it is closed.
- 2. G\*PC-space if every G\*-closed set in it is preclosed.

- 3. G\*SP- space if every G\*--closed set in it is semipreclosed.
- 4.  $_{1/2}T_{g^*}$ -space if every G\*-closed set in it is g\*-closed.
- 5.  $_{1/2}T_{g^*sp}$ -space if every G\*-closed set in it is g\*sp-closed.
- 6.  $_{1/2}T_{gr}$ -space if every G\*-closed set in it is gr\*-closed.
- 7.  ${}_{g}T_{G^*}$ -space if every g-closed set in it is G\*-closed.
- 8.  $_{gsp}T_{G^*}$ -space if every gsp-closed set in it is G\*-closed.
- 9.  $_{rg}T_{G^*}$ -space if every rg-closed set in it is G\*-closed.

We, define the following

**Definition:** The union of all  $G^*$ -open sets which contained in A is called the  $G^*$ -interior of A and is denoted by  $G^*Int(A)$ .

**Definition:** The intersection of all G\*-closed sets containing set A is called the G\*-closure of A and is denoted G\*Cl(A). **Lemma:** Let  $x \in X$ , then  $x \in G*Cl(A)$  if and only if  $V \cap A \neq \emptyset$  for every G\*-open set V containing x.

Proof is easy and hence omitted.

**Remark:** The G\*-closure of a set A is not always G\*-closed set.

We, state the following.

Lemma: Let A and B be subsets of X. Then

- 1.  $G^*Cl(\emptyset) = \emptyset$  and  $G^*Cl(X) = X$ .
- 2. If  $A \subset B$ ,  $G^*Cl(A) \subset G^*Cl(B)$ .
- 3.  $A \subset G^*Cl(A)$ .
- $4. \quad G^*Cl(A)=G^*Cl(G^*Cl(A)).$
- 5.  $G^*Cl(A) \cup G^*Cl(B) \subset G^*Cl(A \cup B).$
- 6.  $G^*Cl(A \cap B) \subset G^*Cl(A) \cap G^*Cl(B)$

We, define the following.

**Definition:** A function  $f: X \rightarrow Y$  is called G\*-continuous if  $f^1$  (V) is G\*-closed in X for every closed subset V of Y. **Definition:** A function f:  $X \rightarrow Y$  is called G\*-irresolute if  $f^1$  (V) is G\*-closed in X for every G\*- closed subset V of Y.

**Definition:** A function f:  $X \rightarrow Y$  is called strongly G\*continuous if the inverse image of each G\*-open set of Y is open in X.

In view of Lemma-3.2 above, we have the following.

**Lemma 3.17:** Let  $f : X \rightarrow Y$  be a function. Then,

- 1. If f is continuous ( and hence precontinuous, semiprecontinuous) function, then it is G\*-continuous.
- 2. If f is G\*-continuous function, then it is g-continuous.
- 3. If f is G\*-continuous function, then it is gsp-continuous.
- 4. If f is G\*-continuous function, then it is rg-continuous.
- 5. If f is g\*-continuous function, then it is G\*-continuous.
- 6. If f is g\*sp-continuous function, then it is G\*-continuous.
- 7. If f is rps-continuous function, then it is G\*-continuous.

**Lemma 3.18:** Every G\*-irresolute function is G\*-continuous **Proof:** Suppose  $f: X \rightarrow Y$  is G\*-irresolute. Let V be any closed subset of Y. Then V is G\*-closed set in Y, lemma -3.2. Since f is g\*sp-irresolute,  $f^{-1}(V)$  is g\*sp-closed in X. This proves the lemma.

**Theorem:** Let  $f:X \rightarrow Y$  and  $g:Y \rightarrow Z$  be two G\*-irresolute functions, then g of is also G\*-irresolute function.

**Theorem:** Let  $f:X \rightarrow Y$  be a function. Then the following are equivalent.

- a. f is G\*-continuous
- b. The inverse image of each open set Y is G\*-open in X
- c. The inverse image of each closed set in Y is G\*-closed in X

**Proof:** (i) $\Rightarrow$ (ii): Let U be open in Y. Then Y-U is closed in Y. By (i)  $f^{1}(Y-U)$  is G\*-closed in X. But  $f^{1}(Y-U) = X-f^{1}(U)$  which is G\*-closed in X. Therefore  $f^{1}(U)$  is G\*-open in X. (ii) $\Rightarrow$ (iii) and (iii) $\Rightarrow$ (i) follow easily.

In view of Definition -3.10 and Lemma-3.11, we prove the following.

**Theorem:** If a function  $f:X \rightarrow Y$  is G\*-continuous then  $f(G^*Cl(A)) \subseteq Cl(f(A))$  for every subset A of X.

**Proof:** Let  $f:X \rightarrow Y$  be G\*-continuous. Let  $A \subset X$ . Then Cl(f(A)) is closed in Y. Since f is G\*-continuous,  $f^1(Cl(A))$  is G\*-closed in X. Suppose  $y \in f(x)$ ,  $x \in G*Cl(A)$ . Let U be an open set containing  $y \in f(x)$ . Since f is G\*-continuous. by Theorem 3.22,  $f^1(U)$  is G\*-open set containing x so that  $f^1(U) \cap A \neq \emptyset$  by Lemma 3.11. Therefore  $f^1(f^1(U) \cap A) \neq \emptyset$ , which implies  $f(f^1(U) \cap f(A)) \neq \phi$ .Since  $f(f^1(U)) \subseteq A$ ,  $U \cap f(A) \neq \emptyset$ . This proves that  $y \in Cl(f(A))$  that implies  $f(G*Cl(A)) \subseteq Cl(f(A))$ .

**Theorem:** If a function  $f:X \rightarrow Y$  is G\*-irresolute then  $f(G*Cl(A)) \subseteq G*Cl(f(A))$  for every subset A of X. Proof: Similar to Th.3.23.

*Theore:* Let  $f:X \rightarrow Y$  be G\*-continuous and  $g: Y \rightarrow Z$  be continuous, then gof:  $X \rightarrow Z$  be G\*-continuous.

**Proof:** Let V be any open subset of Z. Then  $g^{-1}(V)$  is open in Y,since g is continuous function. Again, f is G\*- continuous and  $g^{-1}(V)$  is open set in Y then  $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$  is G\* - open in X. This shows that gof is G\*-continuous.

*Theore:* Let  $f:X \rightarrow Y$  be G\*-continuous and  $g: Y \rightarrow Z$  be strongly G\*-continuous, then gof:  $X \rightarrow Z$  be G\*-irresolute.

**Proof:** Let V be any G\*-open subset of Z. Then  $g^{-1}(V)$  is open in Y, since g is strongly G\*-continuous function. Again, f is G\*-continuous and  $g^{-1}(V)$  is open set in Y then  $f^{-1}(g^{-1}(V)) =$ (gof)<sup>-1</sup>(V) is G\*-open in X. This shows that gof is G\*irresolute.

We, define the following

**Definition:** A function  $f:X \rightarrow Y$  is called contra  $G^*$  - continuous if  $f^1(V)$  is  $G^*$ -closed in X for each open set V in Y

**Theorem:** Let f:  $X \rightarrow Y$  be G\*-continuous and g :  $Y \rightarrow Z$  be contra-continuous, then gof :  $X \rightarrow Z$  be contra G\*-continuous. **Proof:** Obvious.

We, define the following.

**Definition:** A function  $f: X \rightarrow Y$  is called  $(g, G^*)$ -continuous if the inverse image of each g-open set of Y is  $G^*$ -open in X.

Clearly, every (g, G\*)-continuous function is G\*-continuous function, since every open set is g-open set..

**Theorem:** Let f:  $X \rightarrow Y$  be (g, G\*)-continuous function and g :  $Y \rightarrow Z$  be g-continuous then gof :  $X \rightarrow Z$  is G\*-continuous function.

#### Proof: Obvious.

**Theorem:** Let  $f: X \rightarrow Y$  be  $(g, G^*)$ -continuous function and g  $:Y \rightarrow Z$  be gc-irresolute then gof  $:X \rightarrow Z$  is  $(g, G^*)$ -continuous function.

#### Proof: Obvious.

We, define the following.

**Definition:** A function  $f: X \rightarrow Y$  is called (rg,  $G^*$ )-continuous if the inverse image of each rg-open set of Y is  $G^*$ -open in X.

Clearly, every  $(rg,G^*)$ -continuous function is  $G^*$ -irresolute. The routine proofs of the following are omitted.

**Theorem:** Let f:  $X \rightarrow Y$  be (rg, G\*)-continuous function and g :  $Y \rightarrow Z$  be rg-continuous then gof :  $X \rightarrow Z$  is G\*-continuous function.

**Theorem:** Let  $f: X \rightarrow Y$  be (rg,  $G^*$ )-continuous function and g :  $Y \rightarrow Z$  be rg-irresolute then gof :  $X \rightarrow Z$  is (rg,  $G^*$ ) -continuous function.

In view of Definition-3.8, we have the following.

*Lemma:* Let  $f: X \rightarrow Y$  be a G\*-continuous function. Then,

- 1. If X is a G\*SP- space, then f is semiprecontinuous function.
- 2. If X is a  $_{1/2}T_{g^*}$ -space, then f is g\*-continuous function.
- 3. If X is a  $_{1/2}T_{gr}^{*}$ -space, then f is gr\*-continuous function.

*Lemma:* Let  $f: X \rightarrow Y$  be a gsp-continuous function with X is  ${}_{gsp}T_{G^*}$ -space,then f is

G\*-continuous function.

**Lemma:** Let  $f: X \rightarrow Y$  be a rg-continuous function with  ${}_{rg}T_{G^*}$ -space, then f is

G\*-continuous function.

We, define the following.

**Definition:** A space X is said to be G\*-connected if X cannot be written as the

disjoint union of two nonempty G\*-open sets in X.

We, give the following.

*Lemma:* For a space X, the following are equivalent : (i)X is G\*-connected.

(ii) X and  $\varnothing$  are the only subsets of X which are both G\*-open and G\*-closed.

(iii)Each G\*-continuous function of X into some discrete space Y with at least two points is a constant function. Proof : Obvious.

#### **Theorem:** Let $f: X \rightarrow Y$ be a function

- a. If X is G\*-connected and if f is G\*-continuous, surjective, then Y is connected
- b. If X is G\*-connected and if f is G\*-irresolute, surjective, then Y is G\*-connected.

**Proof**: (i) Let X be G\*-connected and f be G\*-continuous, surjective. Suppose Y is disconnected. Then  $Y = A \cup B$ , where A and B are disjoint nonempty open subset of Y. Since f is G\*-continuous surjective then,  $X = f^{-1}(A) \cup f^{-1}(B)$  where  $f^{-1}(A)$  and  $f^{-1}(B)$  are disjoint nonempty G\*-open subsets of X. This

contradicts the fact that X is  $G^*$ -connected. Therefore, Y is connected. This proves (i).

(ii) Let X be G\*-connected and f be G\*-irresolute surjective. Suppose Y is not G\*-connected. Then Y = AUB, where A and B are disjoint nonempty G\*-open subsets of Y. Since f is G\*-irresolute surjective, then  $X = f^{-1}(A) \cup f^{-1}(B)$  where  $f^{-1}(A)$  and  $f^{-1}(B)$  are disjoint nonempty G\*-open subsets of X. This implies X is not G\*-connected, which is contradiction. Therefore Y is G\*-connected. This prove (ii).

#### **Theorem:** Let $f: X \rightarrow Y$ be a function

- a. If X is  $G^*$ -connected and if f is (g,  $G^*$ )-continuous, surjective, then Y is g-connected.
- b. If X is G\*-connected and if f is (rg, G\*)-continuous, surjective, then Y is rg-connected.

Proof: Obvious.

We, define the following.

**Definition:** A space X is said to be G\*-Hausdroff if whenever x and y are distinct points of X, there exist disjoint G\*-open sets U and V such that  $x \in U$  and  $y \in V$ .

Next, we prove the following.

**Theorem 3.41:** Let X be a space and Y be Hausdorff. If  $f:X \rightarrow Y$  be G\*-continuous injective, then X is G\*-Hausdorff

**Proof:** Let x and y be any two distinct points of X . Then f(x) and f(y) are distinct points of Y, because f is injective. Since Y is Hausdorff , there exist two disjoint open sets U and V such that

 $f(x) \in U$  and  $f(y) \in V$ . Since f is G\*-continuous and  $U \cap V = \emptyset$ , we have  $f^{1}(U)$  and  $f^{1}(V)$  are disjoint G\*-open sets in X such that  $x \in f^{1}(U)$  and  $y \in f^{1}(V)$ . Hence X is G\*-Hausdorff space.

**Theorem:** Let X be a space and Y be  $G^*$  -Hausdorff. If  $f:X \rightarrow Y$  be  $G^*$ -irresolute injective, then X is  $G^*$ -Hausdorff.

**Proof**: Let x and y be any two distinct points of X. Then f(x) and f(y) are distinct points of Y,

because f is injective. Since Y is G\*-Hausdorff, there exist two disjoint G\*-open sets U and V such that  $f(x) \in U$  and  $f(y) \in V$ . Since f is G\*-irresolute and  $U \cap V = \emptyset$ , we have  $f^1(U)$  and  $f^1(V)$  are disjoint G\*-open sets in X such that  $x \in f^1(U)$  and  $y \in f^1(V)$ . Hence X is G\*-Hausdorff space.

**Theorem:** Let X be a space and Y be g-Hausdorff. If  $f:X \rightarrow Y$  be  $(g, G^*)$ -continuous injective, then X is  $G^*$ -Hausdorff.

**Proof**: Let x and y be any two distinct points of X. Then f(x) and f(y) are distinct points of Y, because f is injective. Since Y is g -Hausdorff, there exist two disjoint g-open sets U and V such that  $f(x) \in U$  and  $f(y) \in V$ . Since f is  $(g,G^*)$ -continuous and  $U \cap V = \emptyset$ , we have  $f^1(U)$  and  $f^1(V)$  are disjoint G\*-open sets in X such that  $x \in f^1(U)$  and  $y \in f^1(V)$ . Hence X is G\*-Hausdorff space.

**Theorem:** Let X be a space and Y be rg-Hausdorff. If  $f:X \rightarrow Y$  be (rg, G\*) -continuous injective, then X is G\*-Hausdorff.

Proof: Similar to Th. 3.43.

**Theorem:** Let X be a space and Y be  $G^*$ -Hausdorff. If  $f:X \rightarrow Y$  be strongly  $G^*$ -continuous injective, then X is Hausdorff.

**Proof:** Similar to Th.3.43.

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## How to cite this article:

Govindappa Navalagi., 2018, Properties of G\*-Closed Sets in Topological Spaces. *Int J Recent Sci Res.* 9(8), pp.28539-28543. DOI: http://dx.doi.org/10.24327/ijrsr.2018.0908.2477

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