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Research Article

DOMINATION PARAMETERS OF SPLIT GRAPHS

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ABSTRACT

A set D of vertices in a graph G is a dominating set if every vertex not in D is adjacent to atleast one vertex in D. The domination number γ(G) is the minimum cardinality of a dominating set of G. In this paper we investigate certain domination parameters of cyclic split graph, uniform n-fan split graph, uniform n-wheel split graph and uniform n-star split graph.

INTRODUCTION

A subset D of the vertex set V in an undirected graph G is a dominating set if every vertex not in D is adjacent to atleast one vertex in D. The theory of dominating sets was introduced formally by Ore and Berge and now it is one of the vast areas of research. This concept evolves in finding a team of representatives. For example in the problem of locating radar stations, a number of strategic locations are to be kept under observation. These form the dominating sets. Finding the minimum number of strategic locations form the minimum dominating set. We call this as domination number denoted as $\gamma(G)$. Domination number is the minimum cardinality of a dominating set of G.

The concept of domination in graphs began in the mid of 1860s with the game of chess. The goal of the problem was to use certain chess pieces to dominate the squares of the chess board. Knowing that a queen can move horizontally, vertically or diagonally, de Jaenish in 1862 considered the problem of finding the minimum number of queens that can be placed on a chess board such that every square is either occupied by a queen in a single move. This is referred as the n queens problem [3,18].

Total domination in graphs was introduced by Cockayne, Dawes and Hedetniemi. A dominating set D of a graph G is a *total dominating set*, if its induced sub graph has no isolated vertices. Alternately we define a set D of vertices of an undirected graph G as a total dominating set, if each vertex in V is adjacent to some vertex of D. The minimum cardinality of a total dominating set is the *total domination number* denoted by $\gamma_t(G)$ [11].

A dominating set D of a graph G is a *connected dominating set*, if its induced sub graph is connected. The minimum cardinality of a connected dominating set is the *connected domination number* denoted by $\gamma_c(G)$[18]. The study of connected domination has extensive application in routing problems.

A dominating set $D$of a graph G is an *independent dominating set*, if its induced subgraph has no edges. Alternately, we define a set D of vertices of an undirected graph G is an independent dominating set, if D is an independent set and every vertex not in D is adjacent to a vertex in D.The minimum cardinality of an independent dominating set is the *independent domination number* denoted by $\gamma_i(G)$ [2].

A wheel W_n is a graph obtained from the cycle $C_n \geq 3$ by adding a new vertex. A *cyclic split graph* $C_nK_r^k$ has a complete graph K_r with vertices v_1,v_2,…v_r and kr wheels $W_{i,j}$ attached at each vertex v_i in K_r, such that $W_{i,j}$= v_i+$C_{i,j}$, $1 \leq i \leq$

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r and $1 \leq j \leq k$. The deletion of the spokes of the wheel results in the disjoint union of the complete graph K_r and kr independent cycles $C_{i,j}$, $1 \leq i \leq r$ and $1 \leq j \leq k$ where each cycle has n vertices which are labeled as $a_{n,i,j}$ [6].

A fan graph $F_{m,n}$ is defined as graph join $\overline{K_m} + P_n$, where $\overline{K_m}$ is an empty graph on m nodes and P_n is the path graph on n nodes.A *uniform n-fan split graph* SF_n^r contains a star S_{n+1} with hub at x such that the deletion of n edges of S_{n+1} partitions the graph into n independent fans $F_r^i = P_r^i + K_1$, $1 \leq i \leq n$ and an isolated vertex [14].

Let u_i, $1 \leq i \leq n$ be thevertices of the complete graph K_n. Let $W_{r+1}^i = C_r^i + K_1$ be the wheels with hubs w_i, $1 \leq i \leq n$ respectively. Let $u_i w_i$, $1 \leq i \leq n$ be an edge. The graph constructed is called uniform *n-wheel split graph* and denoted $K_n W_r$ [5,14].

A star graph S_n of order n is a tree on n nodes with one node having vertex of degree $n-1$ and the other $n-1$ vertices having degree one. A *uniform n-star split graph* ST_n^r contains a clique K_n such that the deletion of nC_2 edges of K_n partitions the graph inton-independent star graphs S_{r+1}. The number of vertices are $n(r+1)$[5].

It is well known and generally accepted that the problem of determining the domination number of an arbitrary graph is a difficult one. Because of this, researchers have turned their attention to the study of classes of graphs for which the domination problem can be solved in polynomial time.

In this paper we investigate total, connected and independent domination numbers of cyclic split, uniform n-wheel split, uniform n-fan split and uniform n-star split graphs.

LITERATURE SURVEY

Domination in graph theory is studied by Cockayne, Hedetneimi and Dawes [1, 3]. Jasintha Quadras *et a*l. initiated the quest of finding the total bondage number and total domination number of windmill graph, friendship graph, circular necklace graph, helm graph, wheel graph [4]. A transversal in a hyper graph is a subset of vertices which intersects every edge. These edges are the neighbourhoods of the vertices of G. [16] reveal a traversal is a total dominating set.

The domination number of G is almost the point-point covering of G is explained in [15]. Results on complement of connected domination number is explained in [10]. W.J.Desormeaux *et a*l. have defined when $diam(G) = k$ and the diameter of a disconnected graph is empty then diameter 2-graph is a connected graph [11].

The independent domination number of path, cycle and complete bipartite graph is found in [2]. The independent domination number of helm graph, web graph and wheel graph is determined in [17]. The domination number of G equals the independent domination number if G doesnot have an induced subgraph isomorphic to $K_{3,3}$. This interpretation is given by R.B.Allan *et a*l. [13].

The local metric dimension of cyclic split graph has been investigated in [6]. The inverse domination number for cyclic split graph is determined by Jude Annie Cynthia *et a*l. [8]. The cordial labelling and antimagic labelling of cyclic split graph is investigated in [7, 9]. Pinnar Heggernes *et al.* have proved that disjoint paths remain NP-complete on split graphs [12]. S.Roy found the packing chromatic number of uniform n-fan and n-wheel split graphs [14]. The average distance of uniform n-star split graph and uniform n-wheel split graph is determined in [5].

Domination Parameters of Cyclic Split Graphs

Lemma3.1: $\gamma_t(G) \leq r$

Proof: Consider $C_n K_r^k$. Let v_j denote the vertices of complete graph and each v_j is adjacent to n vertices in the wheel which are labeled as $a_{i,j,k}$, $i = 1,2,..n$. Each vertex v_j dominates $a_{i,j,k}$, $1 \leq i \leq n$,$1 \leq j \leq r$, $1 \leq k \leq m$. Also each v_j dominates v_{j+1}, where $1 \leq j \leq r$. Thus the set of vertices v_j where the indices are taken modulo n forms a total dominating set. Hence the total domination number, $\gamma_t(G) \leq r$.

Lemma 3.2: $\gamma_t(G) > r-1$

Proof: Without loss of generality choose $v_1, v_2 \ldots v_{r-1}$ to be the members of the total dominating set of $C_n K_r^k$. Then each v_j dominates $a_{i,j,k}$, $1 \leq i \leq n$,$1 \leq j \leq r-1$,$1 \leq k \leq m$. The vertex v_r is an isolated vertex in $C_n K_r^k$. This contradicts the statement a total dominating set has no isolated vertices. Hence there cannot be $r-1$ vertices comprising the total dominating set(i.e) $\gamma_t(G) = r$ which means $\gamma_t(G) > r-1$.

Theorem 3.1: *The total domination number of the cyclic split graph* $C_n K_r^k$ *is* $\gamma_t(G) = r$.

Proof: The proof follows from above two lemmas.

Theorem 3.2: *The connected domination number of the cyclic split graph* $C_n K_r^k$ *is* $\gamma_c(G) \leq r$.

Proof: Let v_j denote the vertices in the complete graph of $C_n K_r^k$.The vertices of complete graph $v_j, v_{j+1}, v_{j+2}, \ldots v_r$ forms the dominating set of G. The dominating set formed is itself connected. Hence we can say every dominating set of the cyclic split graph $C_n K_r^k$ is a connected dominating set.

Remark: The induced sub graph of the vertices of the total dominating set of $C_n K_r^k$ is connected.

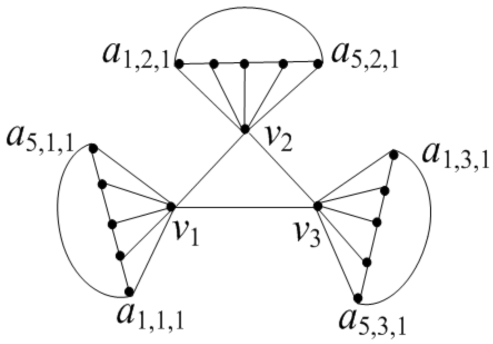

Fig 1 cyclic split graph $C_5 K_3^1$

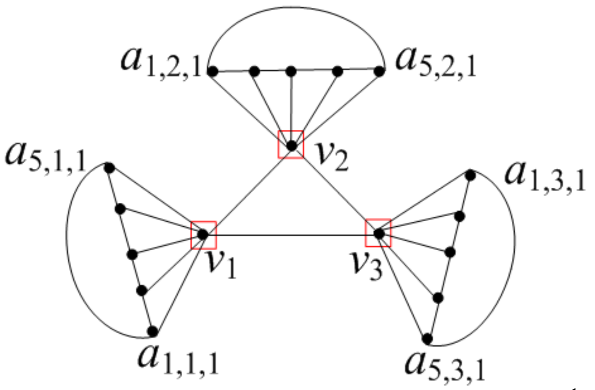

Fig 2 connected and total dominating set of cyclic split graph $C_5K_3^1$

Observation

1. The total dominating set and connected dominating set of $C_nK_r^k$ are the same.
2. The cyclic split graph $C_n\ K_r^k$ has $\gamma(G)$= $\gamma_t(G)$ = $\gamma_c(G)$ = *r, for* $1 \le r \le n$.
3. Every connected dominating set is a total dominating set.
4. The total dominating set of $C_nK_r^k$is a connected dominating set.

Theorem3.3: *The independent domination number of the cyclic split graph* $C_nK_r^k$ *is*

$$\gamma_i(G) = \left\{(r-1)k\left\lceil\frac{n}{3}\right\rceil\right\} + 1, \textit{for } n \ge 5 \textit{ and kvarying.}$$

Proof: Consider the cyclic split graph $C_nK_r^k$. Choose v_1 to be a member of the dominating set *D*. Then v_1 dominates all the vertices of the cycle $a_{n,i,j}$.Consider the wheels attached to a vertex v_{i}, $2 \le i \le r$. Let $a_{m,i,j}$, $1 \le m \le n$, $2 \le i \le r, 1 \le j \le k$ be the set of vertices of the wheel $W_{i,j}$, $2 \le i \le r, 1 \le j \le k$ attached at v_i. Then the vertices $a_{m,i,j}$ dominates $a_{m-1,i,j}$ and $a_{m+1,i,j}$, where $m = 2,5, \ldots n - (n \bmod 3) - 1$. If $n \bmod 3 = 1$ *or* 2, then the vertex also belongs to the dominating set *D*. Hence *D* forms an independent dominating set and $\gamma_i(G) = \left\{(r-1)k\left\lceil\frac{n}{3}\right\rceil\right\} + 1$.

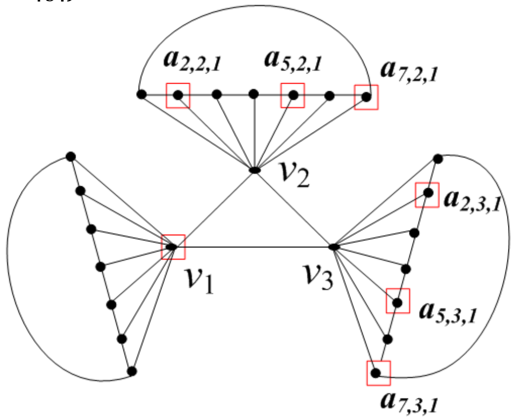

Fig 3 independent dominating set of cyclic split graph $C_7K_3^1$

Domination Parameters of Fan Split Graph, Wheel Split Graph and Star Split Graph

Let v_i denote the vertices in a star S_{r+1} and each v_i, $1 \le i \le n$ is adjacent to m vertices in the fan labelled as $a_{m,i}$, $1 \le i \le n, 1 \le m \le r$.

Theorem4.1: *The connected and total domination number of a fan split graph is* $\gamma_t = \gamma_c = n + 1$.

Proof: Consider the fan split graph SF_n^r. Each vertex v_i dominates $a_{m,i}$, $1 \le i \le n, 1 \le m \le r$. Thus the collection v_n forms a dominating set. Deleting n edges of the star S_{r+1} partitions SF_n^r into n-independent fans, F_r^i , $1 \le i \le n$ and the isolated vertex x. The collection v_n and the isolated vertex x form a total dominating set which is also connected. Hence $\gamma_t = \gamma_c = n + 1$.

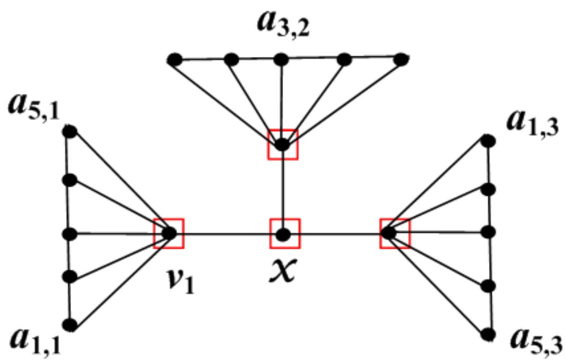

Fig 4 connected and total domination number of SF_n^r

Theorem 4.2 *The independent domination number of a fan split graph* SF_n^r *is* $\gamma_i(G) = n$, *where* $1 \le i \le n$.

Proof: Consider the collection $\{v_i, 1 \le i \le n\}$.We can see that $d(v_i, v_j) = 2$, for $i \ne j, 1 \le i \le n$. Therefore $\{v_i, 1 \le i \le n\}$ forms an independent dominating set. Hence $\gamma_i(G) = n$.

Remark

1. $\gamma(G)$= $\gamma_t(G) = \gamma_c(G) > \gamma_i(G)$ for SF_n^r , *n* ≥ *3, r*≥ *3.*
2. $\gamma_i(G) = n$ for SF_n^r.

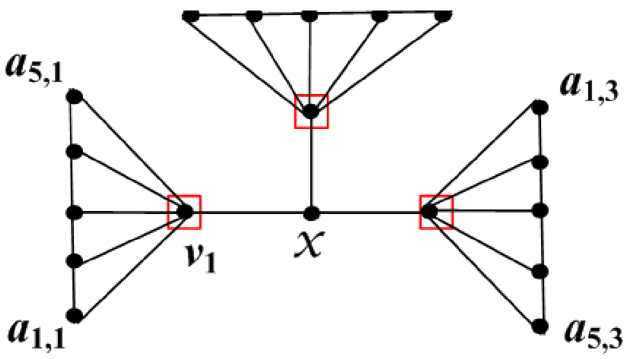

Fig 5 independent domination number of fan split graph SF_n^r

Theorem4.3: *The connected and total domination number of* K_nW_r *is* $\gamma_t(G) = \gamma_c(G) = 2n$. *The independent domination number of a wheel split graph* K_nW_r *is* $\gamma_i(G) = n$, *where* $1 \le i \le n$.

Proof: Consider K_nW_r. Let u_i,$1 \le i \le n$ denote the vertices of the complete graph and each $u_i$is adjacent to w_i, $1 \le i \le n$. Each w_i is adjacent to k vertices $a_{k,i}$. We can see that $d\ (w_i, w_j) = 3$, for $i \ne j$, $1 \le i \le n$. Thus the collection w_i forms the independent dominating set. (i.e.) $\gamma_i(G) = n$.We observe each u_i dominate themselves being a complete graph. Further each u_i dominates the hubs of the wheels w_i, $1 \le i \le$

n. The collection u_i *and* w_i forms a total dominating set which is also connected. Hence $\gamma_t = \gamma_c = 2n$.

Remark:

1. $\gamma(G) = \gamma_t(G) = \gamma_c(G) > \gamma_i(G)$ for K_nW_r , $n \geq 3$, $r \geq 3$
2. $\gamma_i(G) = n$ for K_nW_r.

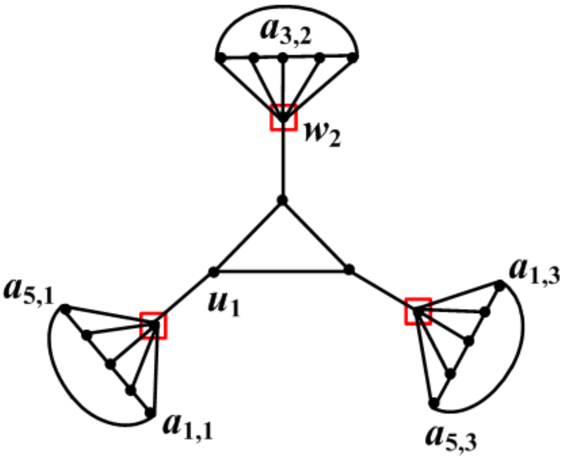

Fig 6 independent domination number of wheel split graph K_nW_r

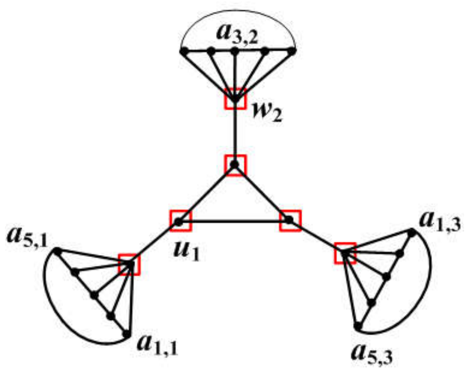

Fig 7 connected and total domination number of wheel split graph K_nW_r

Theorem4.4:*The connected and total domination number of a star split graph ST_n^r is $\gamma_t(G) = \gamma_c(G) = n$, for all n .*

Proof: Consider ST_n^r. Let v_i, $1 \leq i n$ denote the vertices of the complete graph K_n. Each v_i is adjacent to $a_{t,i}$,$1 \leq t \leq r$, $1 \leq i \leq$ n. Deleting all edges of K_n results into n-independent star graphs S_{r+1}. Hence we observe that the n-vertices of clique K_n are the total dominating set. This is also a connected dominating set. Hence $\gamma_t(G) = \gamma_c(G) = n$.

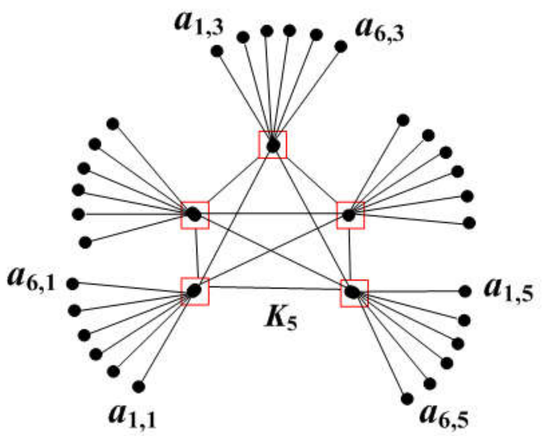

Fig.8 connected and total domination number of star split graph ST_n^r

Theorem4.5: *The independent domination number of a star split graph ST_n^r is $\gamma_i(G) = r(n-1) + 1$*

Proof: Consider the star split graph ST_n^r. Choosing v_1 to be a member of the dominating set, it dominates all the vertices of K_n.Consider the star attached to v_i, $2 \leq i \leq n$. Let $a_{t,i}$,$1 \leq t \leq$ r, $2 \leq i \leq n$be the set of vertices of the star S_{r+1} attached at v_i, $2 \leq i \leq n$. Then each $a_{t,i}$, $2 \leq i \leq n$ individually dominate. Hence each $a_{t,i}$ also belongs to the dominating set D and D forms an independent dominating set. Thus$\gamma_i(G) = r(n-1) + 1$.

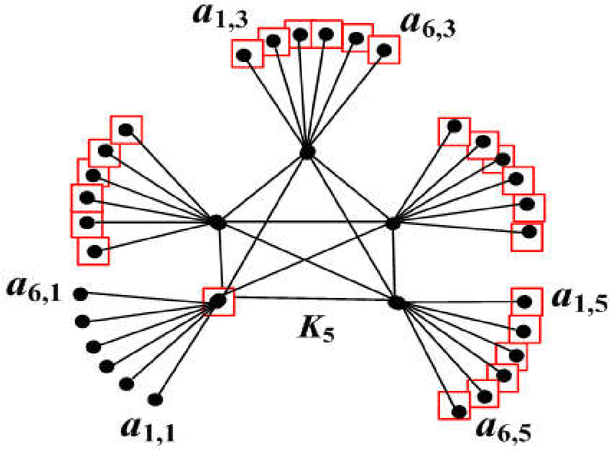

Fig 9 independent domination number of star split graph ST_n^r

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