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Research Article

RELATIONAL STRUCTURE OF S-FUZZY SOFT SUBGROUPS

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ABSTRACT

In this paper, soft subgroup structure under fuzzy techniques over s-norm has been discussed. By using s-norm of S, we characterize some basic properties of S-fuzzy soft subgroups and Cartesian product of normal subgroups. Also, we define the relational concept of S-fuzzy soft subgroups and prove some basic properties.

INTRODUCTION

Soft set is a parameterized general mathematical tool which deals with a collection of approximate descriptions of objects. Each approximate description has two parts, a predicate and an approximate value set. In classical mathematics, a mathematical model of an object is constructed and defines the notion of exact solution of this model. The origin of soft set theory could be traced from the work of Pawlak [7] in 1993 entitled hard and soft set in Proceeding of the international E Work shop on rough sets and discovery at Banff. His notion of soft sets is a unified view of classical, rough and fuzzy Sets. In order to solidify the theory of soft set, P.K. Maji et al., [5] in 2002, defined some basic terms of the theory such as equality of two soft sets, subsets and super set of a soft set, complement of a soft set, null soft set, and absolute soft set with examples. This motivated D. Molodtsov's work [6] in 1999 titled soft set Theory first results. The notion of a fuzzy subset of a set is due to LotfiZadeh [8]. At present this concept has been applied to many mathematical branches, such as group, functional analysis, probability theory, topology, and so on. The notion of fuzzy subgroup was introduced by A. Rosenfeld et.al [3], [5] in his pioneering paper. In fact many basic properties in group theory are found to be carried over to fuzzy groups. In 1979 Anthony and Sherwood [2] redefined a fuzzy subgroup of a group using the concept of triangular norm (t-norm, for short). In this paper, S-fuzzy soft subgroup structure has been analyzed and discussed some related properties.

Section-2 Preliminaries

Definition 2.1:Let G_1 and G_2 be two arbitrary groups with a multiplication binary operations and identities e_1, e_2 respectively. A fuzzy subset of G_1 x G_2 , we mean function from G_1 x G_2 into [0,1]. The set of all fuzzy subsets of G_1 x G_2 is called the [0, 1] –power set of G_1 x G_2 and is denoted by $[0,1]^{G_1 x G_2}$.

Definition 2.2: By an s-norm S, we mean a function S: [0,1] x [0,1] → [0,1] satisfying the following conditions;

(S1) S(x, 0) = x
(S2) S(x ,x) ≤ S(y,z) if y ≤ z.
(S3) S(x,y) = S(y,x)
(S4) S (x, S(y,z)) = S (S(x,y), z), for all x,y,z ε [0,1].

Proposition 2.3: For an s-norm S, then the following statements holds S(x,y) ≥ sup {x ,y}, for all x,yε [0,1].we say that S is idempotent if for all x ε [0,1], S(x,x) = x.

Example 2.4:The basic s-norms are

Sm(x,y) = sup {x,y}
Sb(x,y) = inf {0, x+y -1} and
Sp(x,y) = xy, which are called standard union, bounded sum, and algebraic product respectively.

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Definition 2.5: A soft set f_A over U is defined as f_A: E→ P(U) such that $f_A(x)=\emptyset$ if $x \notin$A.

Clearly, a soft set is not a set. For illustration, Molodtsov consider several examples in [6].

Definition 2.6: Let A be a fuzzy soft subset of a group G_1xG_2.Then A is called a fuzzy soft subgroup of G_1xG_2 under a s-norm S (S-fuzzy soft subgroup) iff for all (a,b), (c,d) ε G_1xG_2

(SFSG1) A((a,b) (c,d)) ≤ S(A(a,b), A(c,d)),
(SFSG2) $A(a,b)^{-1} \le$ A(a,b).

Denote by SFS (G_1xG_2), the set of all S-fuzzy soft subgroups of G_1xG_2.

Example 2.7: Let Z_2 = {e,a}, Z_3 = {e, a, b} be two additive groups. Then

Z_1xZ_2 = { (e,e), (e,a), (e,b),(a,e), (a,a), (a,b)}. Define fuzzy soft set A in Z_2 x Z_3 by A(e,e) = 0.4, A(a,e) = 0.6, A(e,b) = A(e,a) = 0.7, A(e,e) = A(a,b) = 0.8. If S(x,y) = S_b(x,y) = inf {0, x +y-1 }, for all (x,y) ε Z_2xZ_3 , then A ε SFS (Z_1x Z_2).

Definition 2.8: Let A_1,A_2ε SFS(G_1xG_2) and (a,b) ε G_1xG_2. we define

1. $A_1 \sqsubseteq A_2$ iff $A_1(a,b) \ge A_2$ (a,b),
2. $A_1 = A_2$ iff $A_1(a,b) = A_2(a,b)$,
3. ($A_1 \sqcup A_2$) (a ,b) = S { A_1 (a,b) , A_2 (a,b)}.

Also $A_1 \sqcup A_2 = A_2 \sqcup A_1$ and $A_1 \sqcup (A_2 \sqcup A_3) = (A_1 \sqcup A_2) \sqcup A_3 = A_1 \sqcup (A_2 \sqcup A_3)$ (property(S3 and S4)).

Lemma-2.9: Let S be a s-norm. Then S (S(a,b), S(w,c)) = S(S(a,w), S(b,c)) for all a,b,w,c ε [0,1].

Section: Properties of S-Fuzzy Soft Subgroup Strctures

Proposition 3.1: Let A_1, A_2ε SFS (G_1 xG_2). Then $A_1 \sqcup A_2$ε SFS (G_1 xG_2).

Proof: Let (a, b), (c ,d) ε G_1 x G_2.

($A_1 \sqcup A_2$)((a,b) (c,d)) = S (A_1((a,b) (c,d)), A_2((a,b)(c,d)))
≤ S (S (A_1(a,b), A_1(c,d), S(A_2(a,b), A_2(c,d)))
= S(S (A_1(a,b), A_2(a,b), S(A_1(c,d), A_2(c,d)).Also
($A_1 \sqcup A_2$)($(a,b)^{-1}$ = S($A_1(a ,b)^{-1}$, $A_2(a,b)^{-1}$)
≤ S(A_1(a,b), A_2(a,b)) = ($A_1 \sqcup A_2$)((a,b).

Corollary : Let J_n = {1,2,3 …… n }. If {A_i / iεJ_n} ⊑ SFS (G_1xG_2). Then A = $\bigcup_{i\varepsilon Jn} Ai$ε SFS ($G_1xG_2$).

Example 3.2: Let Z_3 = {0,1,2} be an additive group. Then

Z_3 x Z_3 = { (0,0), (0,1), (0,2), (1,0), (1,1), (1,2), (2,0), (2,1), }. Define fuzzy soft sets A_1,A_2 in Z_3x Z_3 by

A_1(0,0) = 0.5	A_2(0,0) = 0.3
A_1(0,1) = 0.5	A_2(0,1) = 0.3
A_1(0,2) = 0.6	A_2(0,2) = 0.4
A_1(1,0) = 0.6	A_2(1,0) = 0.4
A_1(2,0) = 0.7	A_2(2,0) = 0.5
A_1(1,1) = 0.7	A_2(1,1) = 0.5
A_1(2,2) = 0.8	A_2(2,2) = 0.6
A_1(2,1) = 0.8	A_2(2,1) = 0.6

respectively. If S(a,b) = S_b(a,b) = inf { 0, a +b -1}, for all (a, b) ε Z_3 xZ_3, then A_1, A_2, $A_1 \sqcup A_2$ε SFS(Z_3xZ_3).

Lemma 3.3: Let A be a fuzzy soft subset of a finite group G_1 x G_2 and S be idempotent. If A satisfies condition (SFSG1) of Definition -2.1, then A ε SFS (G_1 xG_2).

Proof: Let (a,b) ε G_1xG_2. (a ,b) ≠ (e_1 , e_2).

Since G_1 xG_2 is finite, (a, b) has finite order, say n > 1.
So $(a, b)^n = (e_1, e_2)$ and $(a, b)^{-1} = (a, b)^{n-1}$.
Now by using (SFSG1) repeatedly, we have that
$A((a, b)^{-1}) = A((a, b)^{n-1})= A((a, b)^{n-2} (a, b))\le S(A(a, b)^{n-1} , A(a, b))$
≤ S(A(a, b), A(a, b),……… A(a, b)) (n-times) = A(a, b).

Lemma 3.4: Let A εSFS(G_1 xG_2).If S be an idempotent.Then for all (x,y) ε G $_1$X G_2 ,and n ≥ 1,

1. $A(e_1, e_2)^n \le$ A(a, b);
2. $A(a, b)^n \le$ A(a, b);
3. A(a, b) = $A(a, b)^{-1}$.

Proof: Let (a,b) ε G_1 xG_2 and n ≥ 1.

1. $A(e_1, e_2)^n = A((a, b) (a, b)^{-1})^n \le S(A(a, b), A(a, b)^{-1})^n$
   a. ≤ S(A(a, b), A(a, b), ……….. A(a, b)) (n-times)= A(a, b).
2. $A(a, b)^n$ = A((a, b), (a, b) ……… (a, b))
   a. ≤ S (A(a, b), A(a ,b) ……….. A(a, b)) (n-times) = A(a, b)
3. A(a,b) = $A((a,b)^{-1}) \le A(a,b)^{-1} \le$ A(a,b).So A(a,b) = $A(a,b)^{-1}$.

Proposition 3.5: Let A ε SFS (G_1 xG_2) and (a, b) ε G_1xG_2. If S be idempotent, then A ((a, b)(c, d)) = A(c, d) for all (c, d) ε G_1xG_2 if and only if A(a, b) = A(e_1, e_2).

Proof: Suppose that A((a, b) (c, d)) = A(c, d), for all (c, d) ε G_1xG_2.

Then by letting (c, d) = (e_1,e_2), we get that A(a, b) = A(e_1, e_2).
Conversly, suppose that A (a, b) = A(e_1,e_2). By lemma-3.5, we have

A(a, b) ≤ A((a, b)(c, d)), A(c, d)). Now
A((a, b)(c, d)) ≤ S (A(a, b), A(c, d)) ≤ S(A(c, d), A(c, d)) = A(c, d).Also
A(c, d) = $A((a, b)^{-1}$(a, b)(c, d))) ≤ S(A(a, b), A(a, b),(c, d))) ≤ S (A(a, b)(c, d), A((a, b)(c, d))
= A((a, b)(c, d)).

Example 3.6: Let Z_2 = {e, a} and A be a fuzzy soft set in Z_1 xZ_2 as
A(e, e) = 0.2, A(a, e) = 0.4, A(e, a) = 0.3, A(a,a) = 0.5.

If S(a, b) = S_m(a, b) = max {a,b} ,for all (a, b) ε Z_1xZ_2, then A((a, b)(c, d)) = A(c, d) for all (c, d) ε Z_1xZ_2 if and only if A(a, b) = A(0,0).

Definition 3.7: Let ϕ be a mapping from G_1xG_2 into H_1xH_2, A ε $[0,1]^{G1xG1}$ and α ε $[0,1]^{H1xH2}$. By (D.S Malik) ϕ(A) ε $[0,1]^{H1xH2}$ and $\phi^{-1}(\alpha)$ ε $[0,1]^{G1xG2}$, defined by for all (c,d) ε H_1x H_2, ϕ(A) (c, d) = inf { A(a, b) / (a, b) ε G_1 xG_2, f(a, b) = (c, d)} if ϕ^{-1}(c, d) is empty. Also for all (a, b) ε G_1x G_2, $\phi^{-1}(\alpha)$ (a, b) = α (ϕ(a, b)).

Lemma 3.8: Let A ε SFS(G_1xG_2) and H_1xH_2 be a group. Suppose that ϕ is aepimorphiam of G_1xG_2 into H_1xH_2. Then ϕ(A) ε SFS(H_1xH_2).

Proof: Let (α_1, α_2), (β_1,β_2) ε H_1xH_2 and (a,b),(c,d) ε $G_1x\ G_2$.If (α_1, α_2) ε $\phi(G_1xG_2)$ or (β_1, β_2) ε $\phi(G_1xG_2)$, then$\phi(A)(\alpha_1, \alpha_2) = \phi(A)(\beta_1, \beta_2) = 0 \leq \phi(A)((\alpha_1, \alpha_2)(\beta_1, \beta_2))$.

Suppose $(\alpha_1, \alpha_2) = \phi$ (a, b) and $(\beta_1, \beta_2) = \phi$ (c, d), then

$\phi(A)((\alpha_1,\alpha_2)(\beta_1, \beta_2))$ = inf { (A(a, b) (c, d) / $(\alpha_1, \alpha_2) = \phi$ (a, b)and $(\beta_1, \beta_2) = \phi$ (c, d)}

≤ inf { S(A(a, b), A(c, d) / $(\alpha_1, \alpha_2) = \phi$ (a, b)and $(\beta_1, \beta_2) = \phi$ (c, d)}

= S (inf { (A(a, b) / $(\alpha_1, \alpha_2) = \phi$ (a, b), inf {A(c, d) / $(\beta_1, \beta_2) = \phi$ (c, d)}

= $S(\phi(A)(\alpha_1, \alpha_2), \phi(A)(\beta_1, \beta_2))$.Also since A ε $SFS(G_1xG_2)$, we have

$\phi(A)(\alpha_1, \alpha_2)^{-1}) = \phi(A)(\alpha_1, \alpha_2)$.

Lemma 3.9: Let H_1 x H_2 be a group and α ε$SFS(H_1xH_2)$. If ϕ be a epimorphism of G_1xG_2 into H_1xH_2, then $\phi^{-1}(\alpha)$ ε$SFS(G^1xG_2)$.

Proof: Let (a,b), (c,d) ε G_1xG_2.Then $\phi^{-1}(\alpha)$ ((a,b)(c,d)) = α (ϕ(a,b)(c,d))= α (ϕ(a, b) ϕ(c,d))

≤ S(α(ϕ(a,b), α(ϕ(c,d))= $S(\phi^{-1}(\alpha)(a,b)\ \phi^{-1}(\alpha)(c,d))$.

The proof is completed.

CONCLUSION

Usually the mathematical model is too complicated and the exact solution is not easily obtained. So, the notion of approximate solution is introduced and the solution is calculated. By using s-norm of S, we characterize some basic properties of S-fuzzy soft subgroups and cartesian product of normal subgroups. Also, we define the relational concept of S-fuzzy soft subgroups and proved some elementary aspects.

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