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RESEARCH ARTICLE

NATURAL FREQUENCIES OF AN IMMERSED RAYLEIGH BEAM CARRYING AN ECCENTRIC TIP MASS WITH MASS MOMENT OF INERTIA

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ABSTRACT

The present paper deals with a method for studying the dynamic analysis of an offshore structure having the form of a column partially immersed in a fluid. The non-uniform column carries a concentrated mass with eccentricity "*e*" and rotary inertia J_M, at its free end. The column is idealized as a Rayleigh beam supported by translational and rotational springs, at the bottom.

For investigating the dynamic behaviour of the structure under consideration, the effect of the rotary inertia of the concentrated mass and its eccentricity are all taken into account. Applying the Hamilton principle, the equation of motion is derived by means ofa set the orthogonal polynomials, which satisfy the boundary conditions. Taking into account the effects of the kinematic and inertial parameters of the structure, the roots of the transcendental equation are obtained by employing a symbolic numerical code. For analysing the influence of the several non dimensional parameters on natural frequencies values and shape modes, a lot of numerical examples are presented and the results are validated by making comparisons with the results in literature and reported in bibliography.

INTRODUCTION

Since one can predict the dynamic behaviours of some structural systems, such as piles, water towers, fixed-type platforms, robot arms and tall buildings, from tapered beams and columns with reasonable accuracy, several researchers devoted themselves to the free vibration analysis of such structural models. In offshore and ocean engineering, such structure are usually simplified in analysis as tapered beam or column partially immersed in a liquid and because of the beam liquid interaction problem is of significance in these branches of engineering, many authors have been dealt with the dynamic behaviour of these structural systems. In particular, the analysis of the dynamic behaviour of the structures subjected to seismic actions is fundamental for evaluating their safety and performance with regard to the earthquakes, wind loads and vibrations. On the other hand, in the modern engineering, it is of utmost interest to ensure the performance and reliability of the structures with particular attention to the earthquake actions whose effects can give rise to critical conditions on their structural response. Therefore, the knowledge of the dynamic behaviour of the structural systems, such as towers, piles, tall buildings, offshore platforms and onshore structure, and the ability to predict dynamic response from the modal data is of utmost interest in mechanics, ocean and coastal engineering. Because of the wind and waves loads and the earthquakes actions are the prominent sources of excitation, the calculation of natural frequencies and associated mode shapes represents a significant preliminary study to evaluate the dynamic response of offshore and onshore structures. For dealing with analysis of the dynamic behaviour of these structures and finding the free frequencies values, the simplest structural system employed is a beam/column, having a variable cross-section. The literature regarding the free vibration analysis of beams/columns is relatively rich and in the majority of the papers, Euler-Bernoulli and Timoshenko uniform and tapered beams were considered.

Assuming the Euler- Bernoulli hypotheses, in some papers the governing equations have been solved analytically in closed-form, subjected to the geometrical conditions depending on specific tapering ratios and in which the frequencies are derived in terms of Bessel functions. For non-uniform beam (particularly the linearly tapered) with tip mass, the reports of Mabie *et al*. [1], Goel [2], Abrate [4], Craver *et al*. [5], Auciello [6], Auciello *et al*. [7], Auciello [8], Ece *et al*. [9] and Firouz-Abadi *et al*. [10] are the most concerned. In other works, a particular attention has been given to tapered beams with variable characteristics of geometry and elasticity. For example, Aucielloet *al*. [11], and Auciello *et al*.[12] have analysed the free vibration frequencies of a beam composed of two tapered beam sections, with different physical characteristics and with a mass at its end. The results achieved, by using

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orthogonal polynomials as trial functions, have permitted interventions on geometric parameters so allowing a structural optimization of the beam with respect to its dynamic behaviour. In offshore engineering, since the dynamic behaviours of the structures such as piles and towers, surrounded by water, can be predicted from a cantilever beam carrying a tip mass with reasonable accuracy, the literature concerned is plenty. For example, Uscilowska and Kolodziej [13] have presented closed form, exact frequency equations and mode shape functions for a partially immersed column with eccentrically located tip mass. The column under consideration has been modelled as a distributed parameter cantilever with the lumped mass at the top and in the analysis the rotary inertia of the lumped mass has been taken into account; Wu *et al*. [14] have calculated the values of the natural vibration frequencies for a uniform tower offshore, partially immersed in a fluid, elastically supported at the bottom and carrying an eccentric tip mass with rotary inertia. Moreover in Auciello [15] the free vibration analyses of variable circular cross section column, carrying a tipmass and partially immersed in fluid, has been studied. The closed form solution has been expressed in terms of Bessel functions. Recently, De Rosa *et al*. [16] have investigated the dynamic behaviour of an offshore structure, having the form a column partially immersed in liquid, and have obtained, by means of the improved conventional analytical solution, the roots of the transcendental frequency equations; in the analysis, the influence of the various parameters, as rotary inertia of the concentrated mass and its eccentricity, has been examined.

Many approximated solutions have been developed by numerous authors for Euler- Bernoulli and Timoshenko uniform and tapered beams. For example, Nagaya and Hay [17] propose a method for solving seismic response problems of a pile of variable cross section with a tip inertia subjected to a sea bottom seismic displacement. The method includes use of Fourier series expansion, the Laplace transform, the transfer matrix method and the residue theorem in order to deal with the complex seismic displacement and arbitrarily shaped piles. Chang and Liu studied the natural frequencies of immersed restrained column subjected to an axial force using transfer matrix method and compared the results with some analytical solutions. Recently, the same problem proposed by Uscilowska*et al*. [13] has been studied by Oz [18], who have used the numerical results of the conventional finite element method (FEM) to check the analytical (exact) solutions.

In this paper, assuming the Rayleigh hypotheses, the dynamic behaviour of an offshore tower partially immersed in liquid (water) has been studied. The tower, under consideration, consists of two span beams: for convenience, the immersed beam (in contact with water) is called the "wet" beam, and the other part (a beam without contact with water) is called the "dry" beam, which represents the special case of the "wet" beam.

The frequency equations and mode shapes are obtained by formulating equations of motion for each of two span beams of the column. The roots of the transcendental frequency equations have been obtained by means of the approximate procedure and the solutions have been obtained by employing Rayleigh-Ritz (R-R) method; in the analysis the influence of the various kinematic and inertial parameters is examined. Some of the results are presented in tabular and graphical form and compared with the solutions available for the case under consideration and presented by other authors and reported in bibliography.

Formulation of the problem

Let us consider the variable cross section tower in Fig. 1, whose total span L can be divided into a partial span $L_1 = aL$,totally immersed, and a partial span $L_2 = (1-a)\, L$, which is considered to be dry. The mass density of the immersed part (in contact with water) is equal to $(\rho_w+\rho)$, where ρ_w represents the added mass density of fluid so as defined by Uscilowska and Kolodziej [13], while ρ is the mass density of dry part of the beam. The formulation of the structural system under consideration has been firstly studied, by Chang and Liu, in [19], and developed later by the other authors.

The material is supposed to obey to the Hooke law, with Young modulus E, at the top the tower has an eccentric mass M and rotary inertia J_M, whereas at the bottom the tower is supported by elastic constraints, with rotational stiffness k_R and translational stiffness k_T. For the column under consideration, the cross sectional area A(x) and moment of inertia I(x) are given by the following relations:

$$A(x)=A_1 H(x), \quad I(x)=I_1 G(x), \qquad 0 \le x \le L, \tag{1}$$

where A_1 e I_1 are the cross sectional area and moment of inertia of the beam, for $x=0$, respectively.
Applying the Hamilton Principle, the dynamic behaviour of the column is described by the following relation

$$\delta \int_{t_1}^{t_2} (T-U)\; dt = 0 \tag{2}$$

where T and U are the kinetic and potential energy, respectively.
Assuming the Rayleigh beam model and taking into account the rotary inertia, the kinetic energy is given by:

$$T=\frac{1}{2}\int_0^L m(x)\,\dot{w}^2\, dx+\frac{1}{2}\int_0^L \rho\, I_x \dot{w}_{,x}^2\, dx+\frac{1}{2}M\left[\dot{w}^2\right]_{x=L}+\frac{1}{2}(M\,e^2+J_M)\left[\dot{w}_{,x}^2\right]_{x=L}+$$

$$-M\,e^2\left[\dot{w}^2\right]_{x=L}\left[\dot{w}_{,x}^2\right]_{x=L}=T_1+T_2+T_3 \tag{3}$$

where $\dot{w}=\dfrac{d\,w}{dt}$.

Similarly, the total potential energy of the system $U=U_B+U_F$ is given by sum of two terms: the first is due to bending deformation of the beam and the second is due to constraint, stiffnessesk_R and k_T. In particular

$$U_B=\frac{1}{2}E\int_0^L I(x)\ w_{,xx}^2\ dx+\frac{1}{2}k_T\left[w_x^2\right]_{x=0}+\frac{1}{2}k_R\left[w_x^2\right]_{x=0}, \tag{4}$$

is the elastic energy of the beam and

$$U_F=\frac{F}{2}\int_0^L w_{,x}^2\quad dx_1. \tag{5}$$

is the potential energy due to the axial force F, which can be applied to the free end.

Separating the variables, the displacement function can be written as:

$$w(x,t)=w(x)\,e^{i\mu t}, \tag{6}$$

In order to reduce the continuous problem to a classical holonomic system with a discrete number of Lagrangian coordinates (MDOF), one assumes that the transversal displacements are a linear combination of N independent functions which satisfy the boundary equations. Using a classical Rayleigh-Ritz method (R-R), the displacement functions are chosen in form of orthogonal polynomials $\varphi_i$obtained by the Gram-Schmidt method, as shown in Rektorys[20].

In the approximate formulation, the transversal displacements assume the following form:

$$w(x)=\left[\ddot{}_1(x)\quad \ddot{}_2(x)\qquad \ddot{}_n(x)\right]\begin{bmatrix} q_1\\ q_2\\ \vdots\\ q_n\end{bmatrix}=\boldsymbol{\varphi}^T\mathbf{q}\ , \tag{7}$$

where q_i are generalized coordinates.
The substitution of Eq. (7) into Eq. (3) leads to the following form for the elastic energy:

$$\begin{aligned}U_B=&\frac{1}{2}E\,I_1\int_0^{L_1}G(x)\left[\left(\boldsymbol{\varphi}^T\mathbf{q}\right)_{,xx}^T\left(\boldsymbol{\varphi}^T\mathbf{q}\right)_{,xx}\right]dx+\frac{1}{2}E\,I_1\int_{L_1}^{L_2}G(x)\left[\left(\boldsymbol{\varphi}^T\mathbf{q}\right)_{,xx}^T\left(\boldsymbol{\varphi}^T\mathbf{q}\right)_{,xx}\right]dx\\ &+\frac{1}{2}k_T\left[\left(\boldsymbol{\varphi}^T\mathbf{q}\right)^T\left(\boldsymbol{\varphi}^T\mathbf{q}\right)\right]_0+\frac{1}{2}k_R\left[\left(\boldsymbol{\varphi}^T\mathbf{q}\right)_{,x}^T\left(\boldsymbol{\varphi}^T\mathbf{q}\right)_{,x}\right]_0,\end{aligned} \tag{8}$$

or in synthetic form:

$$U_B=\frac{1}{2}\,\mathbf{q}^T\left\{E\,I_1\left(\mathbf{B}_1+\mathbf{B}_2\right)+k_T\mathbf{B}_T+k_R\mathbf{B}_R\right\}\mathbf{q}=\frac{1}{2}\,\mathbf{q}^T\mathbf{K}_B\,\mathbf{q}, \tag{10}$$

where

$$\begin{aligned}&\mathbf{B}_1=\int_0^{L_1}G(x)\boldsymbol{\varphi}_{,xx}\,\boldsymbol{\varphi}_{,xx}^T\,dx,\quad \mathbf{B}_2=\int_{L_1}^{L_2}G(x)\boldsymbol{\varphi}_{,xx}\,\boldsymbol{\varphi}_{,xx}^T dx,\\ &\mathbf{B}_T=\left[\boldsymbol{\varphi}\boldsymbol{\varphi}^T\right]_{x=0},\quad \mathbf{B}_R=\left[\boldsymbol{\varphi}_{,x}\,\boldsymbol{\varphi}_{,x}^T\right]_{x=0},\end{aligned} \tag{11}$$

while the potential energy, due to the axial force F, assumes the form:

$$U_F=\frac{1}{2}\,F\,\mathbf{q}^T\int_0^L\boldsymbol{\varphi}_{,x}\,\boldsymbol{\varphi}_{,xx}^T\,dx=\frac{1}{2}\,F\,\mathbf{q}^T\,K_F\,\mathbf{q} \tag{12}$$

The total potential energy can be written as:

$$U = \frac{1}{2}\mathbf{q}^T\left(\mathbf{K}_B - F\,\mathbf{K}_F\right)\mathbf{q} = \frac{1}{2}\mathbf{q}^T\mathbf{K}_U\,\mathbf{q}, \tag{13}$$

where the stiffness matrix K_U contains the contribution of the bending strain energy, of the constraints stiffness and the flexural deformation energy, stiffness of constraints and axial force F.

Substituting Eq. (7) into Eq. (3), the kinetic energy T becomes:

$$T_1 = \frac{1}{2}\int_0^L m(x)\,\dot{w}^2\,dx = \frac{\mu^2}{2}(\circ + \circ_w)A_1\int_0^{L_1} H(x)\left[\left(\boldsymbol{\varphi}^T\mathbf{q}\right)^T\left(\boldsymbol{\varphi}^T\mathbf{q}\right)\right]dx + $$
$$+\frac{\mu^2}{2}\circ A_1\int_{L_1}^{L_2} H(x)\left[\left(\boldsymbol{\varphi}^T\mathbf{q}\right)^T\left(\boldsymbol{\varphi}^T\mathbf{q}\right)\right]dx \tag{14}$$

$$T_2 = \frac{1}{2}\int_0^L \circ I_x\dot{w}_{,x}^2\,dx = \frac{\mu^2}{2}(\circ + \circ_w)I_1\int_{L_1}^{L_1} G(x)\left[\left(\boldsymbol{\varphi}^T\mathbf{q}\right)_{,x}^T\left(\boldsymbol{\varphi}^T\mathbf{q}\right)_{,x}\right]dx$$
$$+\frac{\mu^2}{2}\circ I_1\int_{L_1}^{L_2} G(x)\left[\left(\boldsymbol{\varphi}^T\mathbf{q}\right)_{,x}^T\left(\boldsymbol{\varphi}^T\mathbf{q}\right)_{,x}\right]dx \tag{15}$$

$$T_3 = \frac{\mu^2}{2}M\left[\left(\boldsymbol{\varphi}^T\mathbf{q}\right)^T\left(\boldsymbol{\varphi}^T\mathbf{q}\right)\right]_{x=L} + \frac{\mu^2}{2}(M\,e^2 + J_M)\left[\left(\boldsymbol{\varphi}^T\mathbf{q}\right)_{,x}^T\left(\boldsymbol{\varphi}^T\mathbf{q}\right)_{,x}\right]_{x=L} +$$
$$-\mu^2 M\,e^2\left[\left(\boldsymbol{\varphi}^T\mathbf{q}\right)^T\left(\boldsymbol{\varphi}^T\mathbf{q}\right)\right]_{x=L}\left[\left(\boldsymbol{\varphi}^T\mathbf{q}\right)_{,x}^T\left(\boldsymbol{\varphi}^T\mathbf{q}\right)_{,x}\right]_{x=L}. \tag{16}$$

and in matrix form, it can be written:

$$T_1 = \frac{\mu^2}{2}\mathbf{q}^T\left(\mathbf{M}_1 + \mathbf{M}_2\right)\mathbf{q}, \qquad T_2 = \frac{\mu^2}{2}\mathbf{q}^T\left(\tilde{\mathbf{M}}_1 + \tilde{\mathbf{M}}_2\right)\mathbf{q}, \tag{17-18}$$

$$T_3 = \frac{\mu^2}{2}\mathbf{q}^T(\mathbf{M}_3 + \mathbf{M}_4 + 2\mathbf{M}_5)\mathbf{q}, \tag{19}$$

where

$$\mathbf{M}_1 = (\circ + \circ_w)A_1\int_0^{L_1} H(x)\boldsymbol{\varphi}\boldsymbol{\varphi}^T dx, \quad \mathbf{M}_2 = \circ A_1\int_{L_1}^{L_2} H(x)\boldsymbol{\varphi}\boldsymbol{\varphi}^T dx, \tag{20-21}$$

$$\tilde{\mathbf{M}}_1 = (\circ + \circ_w)\Gamma_1\int_0^{L_1}\boldsymbol{\varphi}_{,x}\boldsymbol{\varphi}_{,x}^T dx, \qquad \tilde{\mathbf{M}}_2 = \circ\int_{L_1}^{L_2} I_2(x)\boldsymbol{\varphi}_{,x}\boldsymbol{\varphi}_{,x}^T dx, \tag{22-23}$$

and

$$\mathbf{M}_3 = M\left[\boldsymbol{\varphi}\;\boldsymbol{\varphi}^T\right]_{x=L}, \quad \mathbf{M}_3 = (M\,e^2 + J_M)\left[\boldsymbol{\varphi}_{,x}\;\boldsymbol{\varphi}_{,x}^T\right]_{x=L}, \tag{24-25}$$

$$\mathbf{M}_5 = M\,e^2\left[\boldsymbol{\varphi}\;\boldsymbol{\varphi}^T\right]_{x=L}\left[\boldsymbol{\varphi}_{,x}\;\boldsymbol{\varphi}_{,x}^T\right]_{x=L}. \tag{26}$$

Finally, the kinetic energy can be written as:

$$T = \frac{\mu^2}{2}\mathbf{q}^T(\mathbf{M}_1 + \mathbf{M}_2 + \tilde{\mathbf{M}}_1 + \tilde{\mathbf{M}}_2 + \mathbf{M}_3 + \mathbf{M}_4 + 2\mathbf{M}_5)\mathbf{q} = \frac{\mu^2}{2}\mathbf{q}^T\mathbf{M}\,\mathbf{q}, \tag{27}$$

Finally, the functional Φ is written as:

$$\Phi = \frac{1}{2}\mathbf{q}^T\left(\mathbf{K}_U - \mu^2\mathbf{M}\right)\mathbf{q}. \tag{28}$$

The stationary condition of the functional of Eq. (2) leads to the following eigenvalues problem:

$$\left(\mathbf{K}_U - \mu^2\mathbf{M}\right)\mathbf{q} = 0. \tag{29}$$

The free vibration frequencies are given by calculating the roots of the characteristic polynomial as follows:

$$\det\left(\mathbf{K}_U - \mu^2\mathbf{M}\right) = 0. \tag{30}$$

NUMERICAL RESULTS AND DISCUSSION

From a numerical point of view, it is convenient to introduce the following non-dimensional parameters:

$$¶ = \frac{x}{L}, \quad d = \frac{e}{L}, \quad r_H = \left(\frac{A_1 L^2}{I_1}\right)^{1/2}, « = \frac{M}{M_t}, \quad k^2 = \frac{J_M}{M L^2},$$

$$K_R = \frac{k_R L}{E I_1}, \quad K_T = \frac{k_T L^3}{E I_1}, \quad a = \mu^2 \frac{(°_w + °)A_1 L^4}{E I_1}, \quad P = \frac{F L^2}{E I_1}, \tag{31}$$

where M is the eccentric mass, with rotary inertia J_M , applied to free end. The whole mass of the beam is written as:

$$M_t = ° A_1 L. \tag{32}$$

The polynomial functions φ_i of the Eq. (7), obtained by means of the fundamental and normality conditions, are given by the following relations:

$$k_T w - w = 0, \quad k_R w_{,¶} - \frac{\partial w}{\partial ¶} = 0, \quad \text{at} \quad ¶ = 0, \tag{33}$$

From Eq. (28), the first polynomial φ_1 is obtained, and by employing the Gram-Schmidt procedure, all the other polynomial functions and free frequencies values can be obtained by a general code developed in *Mathematica* [21].

Table 1 Comparison of first three natural frequencies of uniform Euler-Bernoulli (E-B) beam; ε=0, ν=0.887, d=0, r_H=0 with K_T=K_R=0.

				Uściłowska	***et al.* [13]**			**Present**	
ε	k	μ	a	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3
0	0	1	0	1,28589	4,15381	7,35122	1,28589	4,15381	7,35115
	0	2	0,5	1,10878	4,05978	7,20006	1,10878	4,10377	7,31880
	$\sqrt{1/2}$	2	0,5	0,91261	1,74004	4,89985	0,91261	1,74004	4,89985
	$\sqrt{1/2}$	2	1	0,91147	1,73393	4,84047	0,91148	1,73393	4,84047

Table 2 Non-dimensional frequencies for uniform Rayleigh as a function of the slenderness ratio r_H; ε=0, a=0; ν=0.887, d=0, K_T=K_R=0 and μ=1.

k=0	**(E-B)**	**r_H=1/30**	**r_H=1/10**	**r_H=1/5**	**r_H=1/2**
λ_1	1,28589	1,2856	1,2828	1,2738	1,2182
λ_2	4,15381	4,1383	4,0239	3,7270	2,8756
λ_3	7,35124	7,2624	6,7013	5,6915	3,9559
λ_4	10,5723	10,3077	8,9365	7,1382	4,7527
λ_5	13,8295	13,2549	10,8318	8,3395	5,4535
k=1	(E-B)	r_H=1/30	r_H=1/10	r_H=1/5	r_H=1/2
λ_1	0,9599	0,9598	0,9592	0,9571	0,9429
λ_2	1,8973	1,8970	1,8938	1,8833	1,8187
λ_3	5,0495	5,0302	4,8844	4,5091	3,4490
λ_4	8,2096	8,1044	7,4414	6,2717	4,3204
λ_5	11,4266	11,1286	9,6018	7,6493	5,1015

In the present paper, the dynamic behaviour of the column, partially immersed in liquid (water), has been studied by means of the Rayleigh model. The numerical results are validated by making comparisons with the results in literature and reported in bibliography.

For numerical computations, one considers a variable cross-section beam (tapered beam) whose cross sectional area and moment of inertia are represented by the following expressions:

$$A(\P) = A_1 H(\P), \quad I(\P) = I_1 G(\P), \quad 0 \le \P \le 1. \tag{34}$$

where

$$H(\P) = [1 - \pounds\,\P]^2, \quad G(\P) = [1 - \pounds\,\P]^4. \tag{35}$$

The coefficient ε can assume the following values:

- ε=0 the uniform cross section beam for total length;
- ε> 0 the cross section beam assumes a decreasing function;
- ε< 0 the cross section beam assumes an increasing function.

Uniform beam for ε=0

Assuming the Euler-Bernoulli hypothesis, the first numerical example deals with the dynamic analysis of an uniform beam. Neglecting the axial force effect, ($P=0$), and setting $r_H \rightarrow 0$ (slenderness ratio) and $\varepsilon=0$, the free frequencies values are calculated. The same structure has been already solved in Uściłowska and Kołodziej, [13], using an exact approach and the first three natural frequencies values are reported in Table 1 and they are compared with the results obtained by using the present approximate procedure. As shown, the approximate procedure gives free frequencies values which numerically coincide with the results obtained in [13]. Moreover, for the higher vibration modes, the discrepancies between the exact and approximate procedure do not appear to be relevant. In the Table 2, for μ=1 and a=0, the non-dimensional free frequencies values for an uniform Rayleigh beam model, as a function of the slenderness ratio r_H, are reported. One can see that the natural frequencies decrease for increasing values of the slenderness ratio parameter r_H; in particular, the Rayleigh beam model gives the non dimensional natural frequencies values lower bounds to the corresponding values obtained by using the Euler-Bernoulli (E-B) model. In Table 3, the first five free frequencies values are listed for tapered ratio a equal to $1/2$. In the Euler-Bernoulli beam model (E-B) and for r_H>0, the free frequencies values are higher than the values obtained with the Rayleigh beam model and for r_H =1/2. Taking into account the first free frequencies λ_1 the discrepancies between the Euler- Bernoulli theory and the Rayleigh theory are 5,26 %, if $a=0$,and 5,33 %, if $a=1/2$.

Table 3 Non-dimensional frequencies for uniform Rayleigh as a function of the slenderness ratio r_H; ε=0, a=1/2, ν=0.887, d=0, K_T=K_R=0 and μ=1.

k=0	(E-B)	r_H=1/30	r_H=1/10	r_H=1/5	r_H=1/2
λ_1	1,2855	1,2851	1,2822	1,2731	1,2163
λ_2	4,1089	4,0936	3,9807	3,6874	2,8432
λ_3	7,2328	7,1465	6,6005	5,6141	3,9077
λ_4	10,4227	10,1610	8,8044	7,0267	4,6751
λ_5	13,6101	13,0467	10,6661	8,2115	5,3688
k=1	(E-B)	r_H=1/30	r_H=1/10	r_H=1/5	r_H=1/2
λ_1	0,9599	0,9598	0,9592	0,9572	0,9435
λ_2	1,8960	1,8956	1,8923	1,8815	1,8153
λ_3	4,9722	4,9530	4,8110	4,4448	3,4069
λ_4	8,0911	7,9862	7,3300	6,1749	4,2562
λ_5	11,2535	10,594	9,4582	7,5373	5,0329

Table 4 Non-dimensional frequencies for non-uniform Rayleigh beam; k=0, ν=0.887, d=0, K_T=K_R=0 and μ=1.

kZM=		**Chang *et al*. (E-B), [19]**			**Present**	**(E-B)**		**Present**	**r_H=1/30**		**Present**	**r_H=1/10**	
ε	a=	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3
0	0	1,2858	4,1537	7,3508	1,2858	4,1538	7,3512	1,2855	4,1383	7,2624	1,2828	4,0239	6,7013
	0,5	1,2855	4,1077	7,2322	1,2855	4,1089	7,2381	1,2852	4,0936	7,1465	1,2824	3,9807	6,6005
	1	1,2781	4,0424	7,1414	1,2781	4,0425	7,1414	1,2778	4,0272	7,0550	1,2748	3,9140	6,5077
0,2	0	1,2369	3,9940	6,9910	1,2369	3,9941	6,9914	1,2368	3,9831	6,9250	1,2351	3,9001	6,4855
	0,5	1,2367	3,9522	6,8798	1,2367	3,9533	6,8804	1,2365	3,9423	6,8160	1,2348	3,8591	6,3885
	1	1,2317	3,8843	6,7896	1,2317	3,8845	6,7899	1,2315	3,8736	6,7253	1,2297	3,7920	6,2974
0,4	0	1,1678	3,8205	6,5986	1,1678	3,8206	6,5989	1,1677	3,8128	6,5511	1,1669	3,7531	6,2211
	0,5	1,1677	3,7832	6,4959	1,1677	3,7843	6,4966	1,1676	3,7763	6,4502	1,1667	3,7153	6,1291
	1	1,1647	3,7130	6,4067	1,1647	3,7131	6,4069	1,1646	3,7054	6,3605	1,1637	3,6471	6,0396
0,8	0	0,9042	3,3922	5,6143	0,9042	3,3924	5,6147	0,9042	3,3884	5,5948	0,9041	3,3558	5,4450
	0,5	0,9041	3,3669	5,5364	0,9042	3,3678	5,5375	0,9042	3,3635	5,5179	0,9041	3,3290	5,3704
	1	0,9039	3,2928	5,4488	0,9038	3,2930	5,4493	0,9038	3,2898	5,4324	0,9038	3,2574	5,2849
1	0				0,4655	3,1238	4,9315	0,4655	3,1205	4,9207	0,4655	3,0031	4,6949
	0,5				0,4655	3,1069	4,8754	0,4655	3,1034	4,8644	0,4655	3,0759	4,7808
	1				0,4655	3,0316	4,7849	0,4655	3,0283	4,7754	0,4655	3,0031	4,6949

In Figure 2, the first three free frequencies are plotted, for different values of the r_H parameter. If r_H->0, the discrepancies between the Euler- Bernoulli theory and the Rayleigh theory, if $r_H \neq 0$, appear to be relevant. When the slenderness ratio increases, the higher free frequencies decrease; if μ, *k* parameters, relative to tip mass, increase, the free frequencies decrease.

Table 5 Non-dimensional frequencies for non-uniform Rayleigh beam; *k*=1, ν=0.887, d=0, K_T=K_R=0 and μ=1.

kZN=		**Chang *et al.*[(E-B), [19]**			**Present**	**(E-B)**		**Present**	**r_H=1/30**		**Present**	**r_H=1/10**	
ε	*a*=	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3
0	0	0,9599	1,8974	5,0500	0,9599	1,8974	5,0500	0,9599	1,8970	5,0303	0,9593	1,8940	4,8849
	0,5	0,9599	1,8960	4,9706	0,9599	1,8960	4,9722	0,9598	1,8956	4,9530	0,9592	1,8923	4,8110
	1	0,9586	1,8835	4,9177	0,9586	1,8835	4,9178	0,9585	1,8831	4,8983	0,9579	1,8798	4,7548
0,2	0	0,8730	1,7123	4,7412	0,8730	1,7124	4,7413	0,8729	1,7121	4,7270	0,8727	1,7100	4,6194
	0,5	0,8729	1,7115	4,6716	0,8729	1,7115	4,6731	0,8729	1,7112	4,6589	0,8727	1,7090	4,5521
	1	0,8723	1,7027	4,6140	0,8723	1,7027	4,6141	0,8723	1,7024	4,5999	0,8720	1,7001	4,4942
0,4	0	0,7579	1,5124	4,4118	0,7578	1,5123	4,4118	0,7578	1,5122	4,4016	0,7578	1,5110	4,3238
	0,5	0,7578	1,5119	4,3528	0,7578	1,5119	4,3541	0,7578	1,5117	4,3438	0,7578	1,5104	4,2649
	1	0,7577	1,5066	4,2899	0,7576	1,5066	4,2899	0,7576	1,5065	4,2799	0,7576	1,5051	4,2038
0,8	0	0,3832	1,0270	3,6364	0,3831	1,0269	3,6366	0,3831	1,0269	3,6317	0,3831	1,0268	3,5930
	0,5	0,3832	1,0270	3,6030	0,3831	1,0269	3,6041	0,3831	1,0269	3,5989	0,3831	1,0268	3,5580
	1	0,3832	1,0264	3,5303	0,3831	1,0264	3,5305	0,3831	1,0263	3,5257	0,3831	1,0262	3,4882
1	0				0,1055	0,6794	3,2544	0,1055	0,6795	3,3504	0,1055	0,6795	3,3187
	0,5				0,1055	0,6794	3,3300	0,1055	0,6795	3,3257	0,1055	0,6795	3,2920
	1				0,1055	0,6794	3,2555	0,1055	0,6794	3,2515	0,1055	0,6794	3,2207

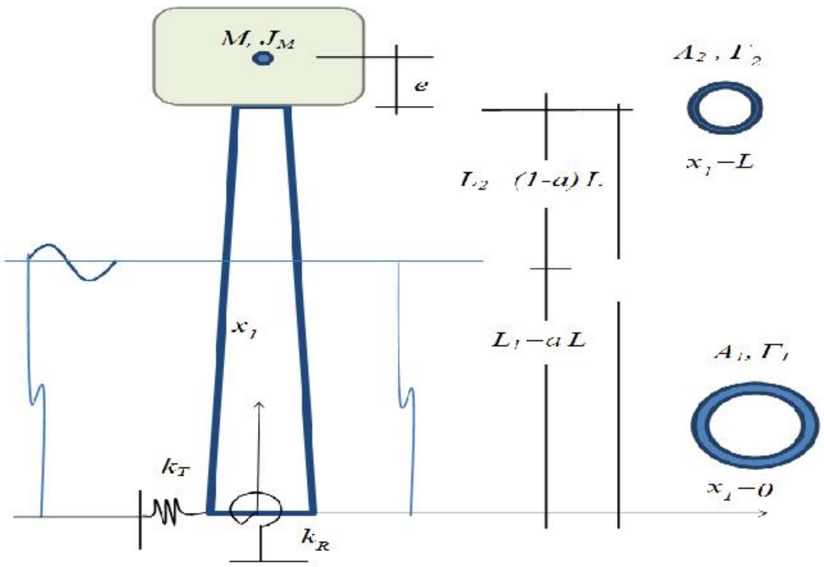

Fig. 1 Offshore structure under consideration

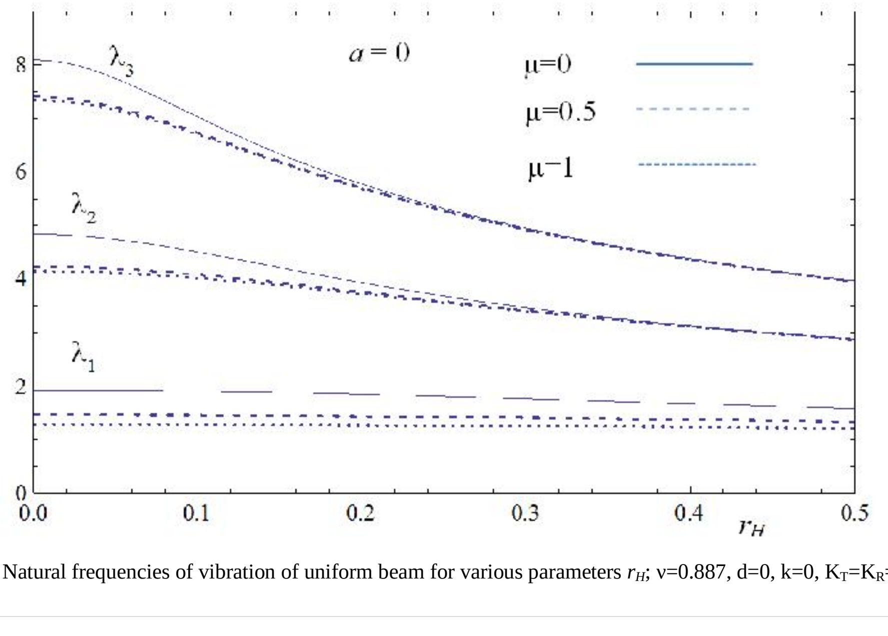

Fig. 2 Natural frequencies of vibration of uniform beam for various parameters r_H; ν=0.887, d=0, k=0, K_T=K_R=0.

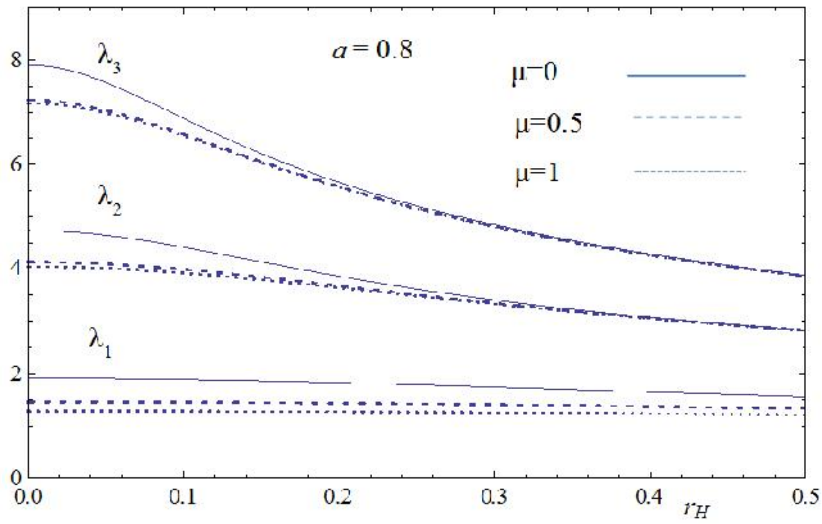

Fig. 3 Natural frequencies of vibration of uniform beam for various parameters r_H; ν=0.887, d=0, k=0, K_T=K_R=0.

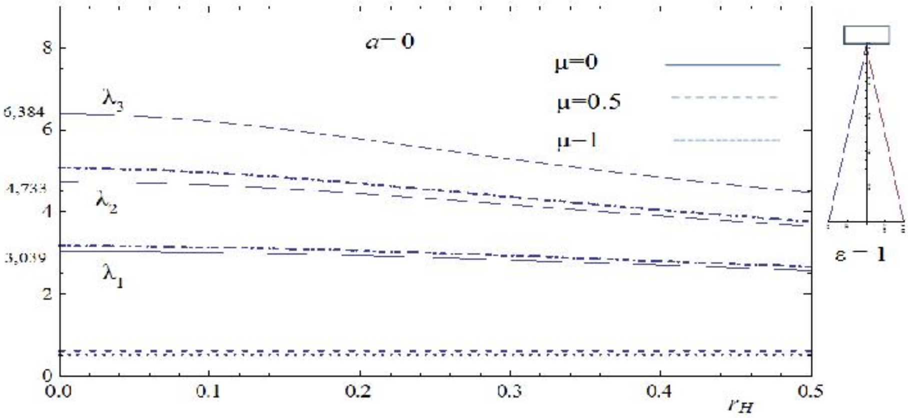

Fig. 4 Natural frequencies of vibration of tapered beam for various parameters r_H; ν=0.887, d=0, k=0, K_T=K_R=0.

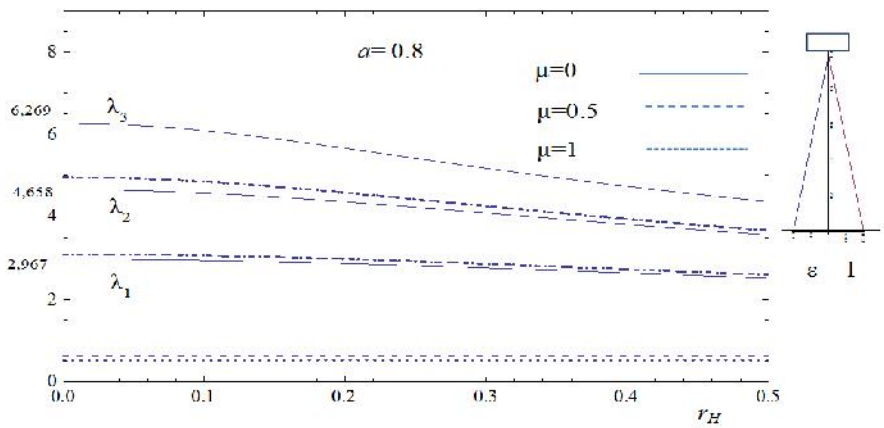

Fig. 5 Natural frequencies of vibration of tapered beam for various parameters r_H; ν=0.887, d=0, k=0, K_T=K_R=0.

Tapered beam for ε>0

Assuming Euler-Bernoulli hypothesis, let us consider a variable cross section beam with positive taper parameter. The problem of vibration frequencies has been solved by Chang *et al*. [19] employing transfer matrix method. The obtained results are validated by making comparisons with the results calculated using the presented method and by assuming r_H=0, μ=1 and for different values of *k*. The numerical comparison is illustrated in Table 4 and the obtained results show an excellent agreement, if r_H=0. For the Rayleigh beam theory, with r_H≠0, the natural frequencies are always lower bounds and the discrepancies increase if r_H parameter increases. This behaviour occurs also for different values of μ and *a* parameters, so as shown in Figures 4 and 5. Considering the tapered beam, with ε≠0, the first natural freee frequencies, λ_1, depend on the proposed beam model. Comparing Euler- Bernoulli (E-B) model, with

r_H->0, and Rayleigh beam model, with r_H=1/10, the frequencies decrease up to about 1,62 %. As shown and taking into account the rotary of inertia effect, the Rayleigh beam theory gives a model closer to the real structure.

Finally, in Table 5 the dynamic behaviour of Rayleigh beam has been analyzed varying the rotary inertia of tip mass. In particular, for μ =1 and for k > 0, the first three frequencies decrease and the phenomeno not does depend on the taper law of the cross section.

CONCLUSION

In this paper the dynamic analysis of tapered column partially immersed in water has been deduced. Based on the Rayleigh beam theory, the free vibration problem is solved by employing the approximated procedure which is based upon the Hamilton Principle and assuming as tentative functions the set of the orthogonal polynomials satisfying the fundamental conditions. The equations of motion are written by numerical code developed in *Mathematica*, andthe free frequencies are calculated by means of the Newton bisection method and so that the obtained results are validated by making comparisons with the results in literature and reported in bibliography.

Also it is demonstrated in the numerical routine that, the present technique is quite simple and converges quickly to the exact solution with very small computational effort and resources. Furthermore, the Rayleigh theory (R-B) is proved to give very accurate results in comparison with the Euler-Bernoulli (E-B) theory. Thus, this study demonstrates the reliability and convenience of the application of the Rayleigh theory (R-B). The natural frequencies are in excellent agreement with published results. Though for comparison purposes, the natural frequencies are kept accurate to the fourth decimal places, the precision of the natural frequencies can be increased and made as high as desired.

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