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NEW TYPE OF HOMEOMORPHISM IN TOPOLOGICAL SPACE

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ABSTRACT

In this paper we introduce and study new class of homeomorphisms called $\overline{g}$-homeomorphisms and $\overline{g}$c-homeomorphisms. Further we show that the set of all $\overline{g}$c-homeomorphisms form a group under the operation composition of mappings

INTRODUCTION

The notion homeomorphism plays an important role in topology. A homeomorphism is a bijective map $f : X \to Y$ when both f and f^{-1} are continuous. Maki et al[5] introduced and investigated g-homeomorphisms and gc-homeomorphisms. Devi et al[6] introduced and studied sg-homeomorphisms and gs-homeomorphisms. Veera kumar[8] introduced and studied *g-homeomorphisms and *gc-homeomorphisms. Recently the authors [9] introduced and studied $\hat{\hat{g}}$ -homeomorphisms and $\hat{\hat{g}}$ c-homeomorphisms.

In this paper we introduce and study new class of homeomorphisms called $\overline{g}$-homeomorphisms and $\overline{g}$c-homeomorphisms. Further we show that the set of all $\overline{g}$c-homeomorphisms form a group under the operation composition of maps.

PRELIMINARIES

We recall the following definitions:

Definition 2.1

A map $f : (X, \tau) \to (Y, \sigma)$ is called semi-closed map[1] (resp. g-closed [5], gs-closed[6], sg-closed[6], *g-closed[8], ψ-closed[11], $\hat{\hat{g}}$ -closed[9]) map if the image of each closed set in (X, τ) is semi closed set (resp. g-closed set, gs-closed set, sg-closed set, *g-closed set, ψ-closed set, $\hat{\hat{g}}$ -closed set) in (Y, σ).

Definition 2.2

A function $f : (X, \tau) \to (Y, \sigma)$ is called g-continuous[12] (resp. gs-continuous[7], sg-continuous[13], *g-continuous[8], ψ-continuous[15], $\hat{\hat{g}}$ -continuous[14]) if the inverse image of every σ-closed set in Y is g-closed (resp. gs-closed, sg-closed, *g-closed, ψ-closed, $\hat{\hat{g}}$ -closed) in X.

Definition 2.3

A bijection $f: (X, \tau) \to (Y, \sigma)$ is called :

(i) g-homeomorphism[9] if f is both g-continuous and g-open (ii) gc-homeomorphism[9] if f and f^{-1} are g-irresolute (iii) gs-homeomorphism[12] if f is both gs-continuous and gs-open (iv) sg-homeomorphism[12] if f is both sg-continuous and sg-open map (v) *g-homeomorphism[8] if f is both *g-continuous and *g-open (vi) *gc-homeomorphism[8] if f and f^{-1} are *g-irresolute (vii) ψ-homeomorphism[11] if f is both ψ-continuous and ψ-open (viii) $\hat{\hat{g}}$ -homeomorphism[9] if f is both $\hat{\hat{g}}$ -continuous and $\hat{\hat{g}}$ -open (ix) $\hat{\hat{g}}$ c-homeomorphism[9] if f and f^{-1} are $\hat{\hat{g}}$ -irresolute (x) semi-homeomorphism (B)[3] if f is continuous and open map.

3.0 $\overline{g}$ -homeomorphism and $\overline{g}$ c-homeomorphsim

In this section we introduce the following definitions.

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Definition 3.1

A bijection f: *(X, τ) → (Y, σ) is called* $\bar{g}$-homeomorphism if f is both $\bar{g}$-continuous and $\bar{g}$-closed map i.e. both f and f^{-1} are $\bar{g}$-continuous maps.

Theorem 3.2

Every homeomorphism is $\bar{g}$-homeomorphism.

The converse of the above theorem is not necessarily true as it can be seen by the following example.

Example 3.3: Let X = Y = {a, b, c}, τ = {ϕ, {a}, {b, c}, X} and σ = {ϕ, {a}, Y}. Define f: (X, τ) → (Y, σ) by f(a) = a, f(b) = b and f(c) = c then f is $\bar{g}$-homeomorphism but not homeomorphism.

Theorem 3.4

Every $\hat{\hat{g}}$-homeomorphism and so $\hat{\hat{g}}$c-homeomorphism is $\bar{g}$-homeomorphism.

The converse of the above theorem is not necessarily true as it can be seen by the following example.

Example 3.5 : X = Y = {a, b, c}, τ = {ϕ, {b}, {a, c}, X} and σ = {ϕ, {a}, Y}. Define f : (X, τ) → (Y, σ) by identity mapping then f is $\bar{g}$-homeomorphism but not $\hat{\hat{g}}$-homeomorphism and $\hat{\hat{g}}$c-homeomorphism .

Theorem 3.6

Every $\bar{g}$-homeomorphism is g-homeomorphism.

Theorem 3.7

Every *g-homeomorphism is $\bar{g}$-homeomorphism.

The converse of the above theorem is not necessarily true as it can be seen by the following example.

Example 3.8: In example (3.5), Define f : (X, τ) → (Y, σ) by f(a) = b, f(b) = c and f(c) = a then f is $\bar{g}$-homeomorphism but not *g-homeomorphism.

Theorem 3.9

Every $\bar{g}$-homeomorphism is gs-homeomorphism.

The converse of the above theorem is not necessarily true as it can be seen by the following example.

Example 3.10: X = Y = {a, b, c}, τ = {ϕ, {a}, {a, b}, {a, c}, X} and σ = {ϕ, {b}, {a, b}, Y}. Define f: (X, τ) → (Y, σ) by f(a) = b, f(b) = a and f(c) = c then f is gs-homeomorphism but not $\bar{g}$-homeomorphism.

Remark 3.11

$\bar{g}$-homeomorphism and s-homeomorphism (B) (or sg-homeomorphism or ψ-homeomorphism) are independent.

Definition 3.12

A bijection f: (X, τ) → (Y, σ) is called a $\bar{g}$c-homeomorphism if f and f^{-1} are $\bar{g}$-irresolute.

We denote the family of all $\bar{g}$c-homeomorphism of a topological space (X, τ) onto itself by $\bar{g}$c-h(X, τ).

Theorem 3.13

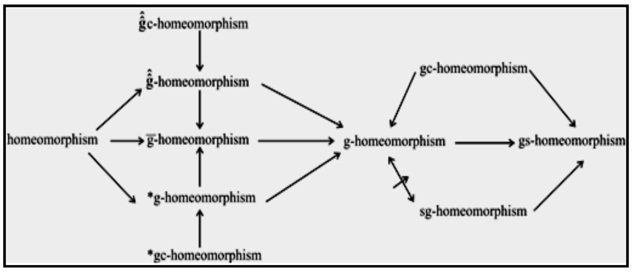
Every $\bar{g}$c-homeomorphism is $\bar{g}$-homeomorphism.

The following example supports that the converse of the above theorem is not true.

Example 3.14: In example (3.3), f is $\bar{g}$-homeomorphism but not $\bar{g}$c-homeomorphism since f^{-1} is not $\bar{g}$-irresolute for {a} is closed set in X but $(f^{-1})^{-1}(\{a\})$ = {a} is not a $\bar{g}$-closed set in Y.

Therefore the class of $\bar{g}$-homeomorphisms properly contains the class of homeomorphisms, the class of $\hat{\hat{g}}$-homeomorphisms, the class of $\hat{\hat{g}}$c-homeomorphisms, the class of *g-homeomorphisms. Also this new class is properly contained in the class of g-homeomorphisms and the class of gs-homeomorphisms.

All the above discussions can be represented by the following diagram.



Theorem 3.15

If f: (X, τ) → (Y, σ) be a bijective and $\bar{g}$-continuous maps then following are equivalent :

(i)f is $\bar{g}$-open map.

(ii)f is $\bar{g}$-homeomorphism.

(iii)f is $\bar{g}$-closed map.

Theorem 3.16

If f: (X, τ) → (Y, σ) and g : (Y, σ) → (Z, η) are $\bar{g}$c-homeomorphism then their composition gof : (X, τ) → (Z, η) is also $\bar{g}$c-homeomorphism.

Theorem 3.17

The set $\bar{g}$c-h(X, τ) is a group under the composition of maps.

Proof: Define a binary operation $*$: $\bar{g}$c-h(X, τ) × $\bar{g}$c-h(X, τ) → $\bar{g}$c-h(X, τ) by $f*g$ = gof for all f and g ∈ $\bar{g}$c-h(X, τ), then by theorem (3.17) gof ∈ $\bar{g}$c-h(X, τ). Again composition of maps is associated and the identity map I: (X, τ) → (X, τ) belonging to $\bar{g}$c-h(X, τ) is identity element of $\bar{g}$c-h(X, τ). If f ∈ $\bar{g}$c-h(X, τ) then f^{-1} ∈ $\bar{g}$c-h(X, τ) s.t. $fof^{-1} = f^{-1}of = I$ so inverse exist for all element of $\bar{g}$c-h(X, τ). Thus ($\bar{g}$c-h(X, τ), o) is a group under the operation of composition of maps.

Theorem 3.18

Let $f : (X, \tau) \to (Y, \sigma)$ be a $\bar{g}$c-homeomorphism then f induces an isomorphism from the group $\bar{g}$c-h(X, τ) onto the group $\bar{g}$c-h(Y, σ).

Proof : Define $\theta_f : \bar{g}c\text{-}h(X, \tau) \to \bar{g}c\text{-}h(Y, \sigma)$ by $\theta_f (h) = fohof^{-1}$ for every $h \in \bar{g}c\text{-}h(X, \tau)$. Then θ_f is a bijection. Again for all $h_1, h_2 \in \bar{g}c\text{-}h(X, \tau)$, $\theta_f (h_1oh_2) = fo (h_1oh_2)of^{-1} = (foh_1of^{-1}) o (foh_2f^{-1}) = \theta_f (h_1)o\theta_f (h_2)$ so θ_f is a homeomorphism and so it is an isomorphism induced by f.

Theorem 3.19

$\bar{g}$C-homeomorphism is an equivalence relation in the collection of all topological spaces.

Theorem 3.20

If $f : (X, \tau) \to (Y, \sigma)$ is a $\bar{g}$c-homeomorphism then $\bar{g}\text{-cl}(f^{-1}(A)) = f^{-1}(\bar{g}\text{-cl}(A))$ for all $A \subseteq Y$.

Corollary 3.21

If $f : (X, \tau) \to (Y, \sigma)$ is $\bar{g}$c-homemorphism then $\bar{g}\text{-cl}(f(A)) = f(\bar{g}\text{-cl}(A))$ for all $A \subseteq X$.

Proof: Follows from theorem (3.20).

Definition 3.22

Let (X, τ) be a topological space and $A \subseteq X$. We define the $\bar{g}$-interior of A ($\bar{g}$-int(A)) to be the union of all $\bar{g}$-open sets contained in A.

Lemma 3.23

For any $A \subseteq X$, $\text{int}(A) \subseteq \bar{g}\text{-int}(A) \subseteq A$.

Proof: Since every open set is $\bar{g}$-open so proof follows immediately.

Theorem 3.24

If $f: (X, \tau) \to (Y, \sigma)$ is a $\bar{g}$-open mapping then for a subset A of (X, τ), $f (\text{int}(A)) \subseteq \bar{g}\text{-int}(f(A))$.

Theorem 3.25

If $f: (X, \tau) \to (Y, \sigma)$ is a $\bar{g}$c-homeomorphism then $f (\bar{g}\text{-int}(A)) = \bar{g}\text{-int}(f(A))$ for all $A \subseteq X$.

Proof: For any set A of X, $\bar{g}\text{-int}(A) = (\bar{g}\text{-cl}(A^C))^C$.

Thus $f(\bar{g}\text{-int}(A)) = f((\bar{g}\text{-cl}(A^C))^C) = (f(\bar{g}\text{-cl}(A^C)))^C$

$= (\bar{g}\text{-cl}(f(A^C)))^C$ by corollary (3.21)

$= (\bar{g}\text{-cl}((f(A))^C))^C = \bar{g}\text{-int}(f(A))$.

Corollary 3.26

If $f: (X, \tau) \to (Y, \sigma)$ is $\bar{g}$c-homeomorphism then $f^{-1}(\bar{g}\text{-int}(A)) = \bar{g}\text{-int}(f^{-1}(A))$ for all $A \subseteq Y$.

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