INTRODUCTION

crossley and Hildebrand[3] introduced and investigated irresolute functions which are stronger than semi-continuous maps but are independent of continuous maps. Since then several researchers have introduced several strong and weak forms of irresolute functions. Di Maio[4], Faro[5], Cammaro[2], Maheshwari[7] and Sundaram[12] have introduced and studied quasi irresolute and strongly irresolute maps, strongly $\alpha$ - irresolute maps, almost irresolute maps, $\alpha$- irresolute maps and gc-irresolute maps respectively.

The notion of homeomorphism has been introduced and generalized by several topologist. Biswas[1], Crossley and Hildebrand[3], Gentry[6], Tadros[13], Um-ehara and Maki et al[14] have introduced and investigated semi homeomorphism, some what homeo-morphism $\alpha$ - homeomorphism,$g^{-}$ - homeomorphism, g-homeomorphism and gc-homeomorphism. Neubrunn[9] and Piotrowski[11] have proved that semi homeomorphism and semi-homeomorphism are independent concepts. Parimelazhagan[10] has introduced sg* - closed set In this paper we introduce new stronger form of irresolute maps and homeomorphism of sg* - closed sets. Also we studied some of its basic properties.

Preliminaries

Before entering into our work, we recall the following definitions which are due to Levine.

**Definition 2.1[1]**: A function $f: X \rightarrow Y$ from a topological space $X$ into a topological space $Y$ is said to be irresolute if the $f^{-1}$ of every semi-open set in $Y$ is semi-open in $X$.

**Definition 2.2[2]**: A function $f: X \rightarrow Y$ from a topological space $X$ into a topological space $Y$ is said to be gc- irresolute if the $f^{-1}$ of every g-open set in $Y$ is g-open in $X$.

**Definition 2.3[7]**: A function $f: X \rightarrow Y$ from a topological space $X$ into a topological space $Y$ is said to be $\alpha$- irresolute if $f^{-1}$ of every $\alpha$- open set in $Y$ is $\alpha$-open in $X$.

**Definition 2.4[11]**: A function $f: X \rightarrow Y$ is said to be semi-homeomorphism if $f$ is continuous, semi open and bijective.

**Definition 2.5[3]**: A function $f: X \rightarrow Y$ is said to be semi-homeomorphism if $f$ is irresolute, pre-semi open and bijective.

**Definition 2.6[8]**: A function $f: X \rightarrow Y$ is said to be generalized homeomorphism (g-homeomorphism) if $f$ is g-continuous, g-open and bijective.

**Definition 2.7[10]**: A function $f: X \rightarrow Y$ is said to be ge-homeomorphism if $f$ is gc- irresolute and $f^{-1}$ is also gc- irresolute.

**Definition 2.8[10]**: Let $f(X, \tau)$ be a topological space and $A$ be its subsect, then $A$ is strongly $g^{*}$-closed set if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is open.

**Strongly g* -irresolute Map**

In this section we have introduced the concept of strongly $g^{*}$ - irresolute in topological spaces.

**Definition 3.1**: Let $X$ and $Y$ be topological spaces. A map $f: X \rightarrow Y$ is said to be strongly $g^{*}$ - irresolute map (sg* - irresolute map) if the inverse image of every sg* -open set in $Y$ is sg* -open in $X$.

**Theorem 3.2**: Let $X, Y, Z$ be topological spaces and let $f: X \rightarrow Y$, $g: Y \rightarrow Z$ be two maps. Their composition gof: $X \rightarrow Z$ is sg* -continuous if $f$ is sg* - irresolute and $g$ is sg* - continuous.

**Proof**: Let $V$ be an open set in $Z$. Then $(gof)^{-1}(V) = g^{-1}(f^{-1}(V)) = f^{-1}(V)$, where $V = g^{-1}(V)$ is sg* -open in $Y$ as $g$ is sg* - continuous. Since $f$ is sg* - irresolute $f^{-1}(V)$ is sg* -open in $X$. Thus gof is sg* - continuous.

**Theorem 3.3**: Let $X, Y, Z$ be topological spaces. Let $f: X \rightarrow Y$, $g: Y \rightarrow Z$ be two sg* - irresolute maps. Then their composition gof: $X \rightarrow Z$ is a sg* - irresolute map.
Theorem 3.4: Let $X,Y,Z$ be topological spaces. Let $f: X \to Y, g: Y \to Z$ be two maps. Then their composition $gof: X \to Z$ is $g$-open if $g$ is $g$-open. If $g$ is surjective then $f$ is $g$-closed.

Example 3.10: Let $X = Y = \{a,b,c\}$ with $\tau = \{\phi, X, \{a\}, \{a,c\}\}$. Assume $\sigma = \{\phi, Y, \{a\}, \{a,b\}\}$. Let $f: (X, \tau) \to (Y, \sigma)$ be denoted by $f(a) = b, f(b) = c, f(c) = a$. Then $f$ is $g$-homeomorphism but not $g$-open.

Remark: The converse of the above theorem need not be true as seen from the following example.

Example 3.6: Let $X = Y = \{a,b,c\}$ with $\tau = \{\phi, X, \{a\}, \{a,c\}\}$ and $\sigma = \{\phi, Y, \{a\}, \{a,b\}\}$. Then $f: (X, \tau) \to (Y, \sigma)$ is denoted by $f(a) = a, f(b) = b, f(c) = c$. Then $f$ is $g$-homeomorphism but not $g$-open.

Theorem 3.3: If a bijection $f: X \to Y$ is $g$--homeomorphism then $f$ is $g$-continuous.

Proof: Let $Y$ be a $g$-open in $Z$. Consider $(gof)^{-1}(Y) = f^{-1}(Y)$, where $V = g^{-1}(V)$ is $g$-open in $Y$, as $g$ is $g$-irresolute. Hence $f$ is $g$-continuous.

Theorem 4.4: A bijection $f: X \to Y$ is $g$-homeomorphism if and only if $f^{-1}$ is $g$-continuous.

Proof: Let $f$ be a bijection $f: X \to Y$. Then $f^{-1}$ is $g$-continuous if and only if $f$ is $g$-homeomorphism.

Definition 4.1: Let $X$ and $Y$ be two topological spaces. A bijection map $f: X \to Y$ from a topological space $X$ into a topological space $Y$ is called strongly $g$-homeomorphism (strongly $g$-homeomorphism) if $f$ and $f^{-1}$ are $g$-continuous.

Theorem 4.2: Every homeomorphism is $g$-homeomorphism.

Proof: Let $f: X \to Y$ be homeomorphisms from the topological space $X$ to $Y$ then $f$ and $f^{-1}$ are continuous. As every continuous function is $g$-continuous. We have $f$ and $f^{-1}$ are $g$-continuous. Thus $f$ is $g$-homeomorphism.

Remark: The converse of the above theorem need not be true as seen from the following example.

Example 3.3: Let $X = Y = \{a,b,c\}$ with $\tau = \{\phi, X, \{a\}, \{a,c\}\}$ and $\sigma = \{\phi, Y, \{a\}, \{a,b\}\}$. Then $f: (X, \tau) \to (Y, \sigma)$ is defined as $f(a) = \{a\}, f(b) = \{a,c\}$ then $f$ is $g$-homeomorphism but not $g$-closed as the inverse image of the open set $\{a\}$ in $X$ is $\{a\}$ is not open in $Y$.

Theorem 4.3: A bijection $f: X \to Y$ from a topological space $X$ into a topological space $Y$ is $g$-homeomorphism then it is $g$-homeomorphism.

Proof: Since $f$ is $g$-homeomorphism both $f$ and $f^{-1}$ are $g$-continuous. As every continuous functions are $g$-continuous $f$ and $f^{-1}$ are $g$-continuous. Thus $f$ is $g$-homeomorphism.

Remark: The converse of the above theorem need not be true as seen from the following example.

Example 4.3: Let $X = Y = \{a,b,c\}$ with $\tau = \{\phi, X, \{a\}, \{a,c\}\}$ and $\sigma = \{\phi, Y, \{a\}, \{a,b\}\}$. Then $f: (X, \tau) \to (Y, \sigma)$ is defined as $f(a) = \{a\}, f(b) = \{a,c\}$ then $f$ is $g$-homeomorphism but not $g$-closed as the inverse image of the open set $\{a\}$ in $X$ is $\{a\}$ is not open in $Y$.

Theorem 4.6: Let $X$ and $Y$ be topological spaces and let $f$ be a bijection mapping from $X$ onto $Y$. Then the following conditions are equivalent.

(i) $f$ is $g$-open and $g$-continuous.
(ii) $f$ is $g$-closed and $g$-continuous.
(iii) $f$ is $g$-closed.

Proof:
(i) To prove (a) $\Rightarrow$ (b)

(ii) To prove (b) $\Rightarrow$ (a)

(iii) To prove (b) $\Rightarrow$ (c)

Assume that $f$ is $g$-homeomorphism. Let $F$ be a closed set in $X$. Then $(X-F)$ is open and $f^{-1}(g)$ is $g$-continuous. Since $g$ is $g$-continuous, $g^{-1}(X-F)$ is $g$-open and $f^{-1}(X-F) = g^{-1}(F)$ is $g$-open. Thus $g^{-1}(F)$ is $g$-closed. Hence $f$ is $g$-closed.

Strongly $g$-homeomorphisms
In this section we have introduce the concept of strongly $g$-homeomorphisms in topological space.
(iv) To prove (c) \(\Rightarrow\) (b)

If \(f\) is sg*-closed and sg*-continuous then we have to prove \(f^{-1}\) is also sg*-continuous. Let \(G\) be an open set. Then \(X-G\) is closed. Since \(f\) is sg*-closed, \(f(X-G)\) is sg*-closed. i.e. \(g^{-1}(X-G)=Y-g^{-1}(G)\) is sg*-closed, implies \(g^{-1}(G)\) is sg*-open. Thus inverse image under \(g\) of every open set is sg*-open. i.e. \(g = f^{-1}\) is sg*-continuous.

Thus \(f\) is sg* -homeomorphism. Hence (b) \(\Rightarrow\) (c). (v) (c) \(\iff\) (a).

Here we have proved that a sg*-closed and sg*-continuous mapping is sg*-homeomorphism in (a). We have proved that a sg*-homeomorphism is sg*-open and sg*-continuous. Thus sg*-closed and sg*-continuous mapping is also sg*-open and conversely.

References


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